

THEME A · SPACE, TIME AND MOTION

HL ONLY

≈ 30 min read

IB Physics A.4 Rigid Body Mechanics — Notes (HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). **A.4 is entirely Additional Higher Level content** — there is no SL material in this sub-topic, and it is examined only on HL papers. It takes the ideas of A.1–A.3 (kinematics, forces, momentum and energy) and applies them to bodies that rotate.

[HL A.4 questions](#)[Flashcards](#)[Rotational dynamics simulation](#)[Angular momentum simulation](#)

What this page covers

- ✓ Torque as the turning effect of a force about an axis, $\tau = Fr \sin \theta$, including its sense (clockwise or anticlockwise)
- ✓ Rotational equilibrium: zero resultant torque
- ✓ Why an unbalanced torque gives a rigid body an angular acceleration
- ✓ Angular displacement, angular velocity and angular acceleration
- ✓ The four equations of motion for uniform angular acceleration
- ✓ Moment of inertia: how it depends on the way mass is spread about the axis, and $I = \sum mr^2$ for point masses
- ✓ Newton's second law for rotation, $\tau = I\alpha$
- ✓ Angular momentum $L = I\omega$, its conservation, and angular impulse $\Delta L = \tau \Delta t$
- ✓ Rotational kinetic energy $E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$, and rolling without slipping

In one minute

- **Torque** is force $\times$ perpendicular distance from the axis to the line of action: $\tau = Fr \sin \theta$, unit N m.
- Zero resultant torque $\rightarrow$ rotational equilibrium (no angular acceleration). A resultant torque $\rightarrow$ angular acceleration, $\tau = I\alpha$.
- Every linear quantity has a rotational partner: $s \rightarrow \theta$, $v \rightarrow \omega$, $a \rightarrow \alpha$, $m \rightarrow I$, $F \rightarrow \tau$, $p \rightarrow L$. The equations have the same form.
- **Moment of inertia** I is rotational inertia. It depends on the mass *and* on how far that mass is from the axis. For point masses $I = \sum mr^2$; for other shapes you are given the formula.
- With no external torque, **angular momentum** $L = I\omega$ is conserved — but rotational kinetic energy usually is not.
- A body **rolling without slipping** has $v = \omega r$ and shares its kinetic energy between translation and rotation, so it reaches the bottom of a slope more slowly than a sliding body with no friction.

Contents

- | | |
|-------------------------------------|--|
| 1. Torque | 7. Angular momentum and angular impulse |
| 2. Rotational equilibrium | 8. Conservation of angular momentum |
| 3. Angular quantities | 9. Rotational kinetic energy and rolling |
| 4. Equations of rotational motion | 10. Exam tips |
| 5. Moment of inertia | 11. Common misconceptions |
| 6. Newton's second law for rotation | 12. How this topic connects |
| | 13. Check your understanding |

What is torque and how do you calculate it?

A **rigid body** is an extended body whose shape does not change: the distance between any two of its points stays fixed. Unlike a point particle, it can rotate, and whether a force makes it rotate depends on *where* and in *which direction* the force is applied — not only on its size.

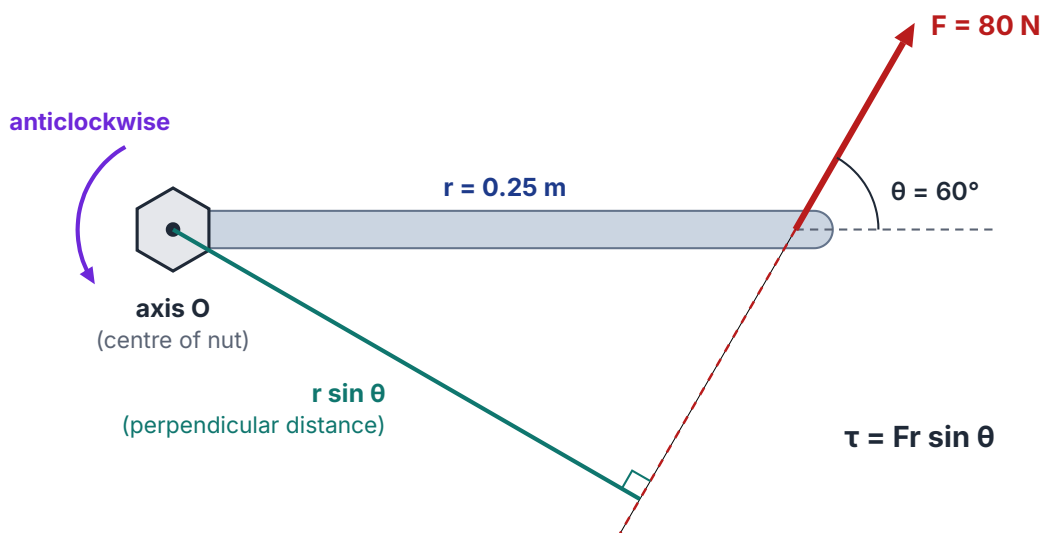
The torque of a force about an axis is the product of the force and the perpendicular distance from the axis to the line of action of the force: $\tau = Fr \sin \theta$.

$$\tau = Fr \sin \theta$$

- τ = torque (N m)
- F = magnitude of the force (N)
- r = distance from the axis to the point where the force is applied (m)
- θ = angle between the force and the line joining the axis to the point of application

This equation is in the data booklet. There are two equivalent ways to read it:

- $F \times (r \sin \theta)$: the force multiplied by the **perpendicular distance** $r \sin \theta$ from the axis to its line of action.
- $(F \sin \theta) \times r$: the **component of the force perpendicular to** r , multiplied by r . The component along r ($F \cos \theta$) points straight at the axis and has no turning effect.
- Torque is greatest when $\theta = 90^\circ$ and is zero when the line of action passes through the axis ($\theta = 0^\circ$ or 180°).
- **Sense:** you must state whether a torque is **clockwise** or **anticlockwise**. Choose one sense as positive and keep it, exactly as you choose a positive direction in A.1. (A full vector treatment of torque is not required.)
- The unit is the **newton metre, N m**. Do not write joules: torque is not energy, even though the base units are the same.



A force applied at 60° to a spanner. Only the perpendicular distance from the axis to the line of action (teal) matters, so the torque is less than it would be if the force were at 90° .

WORKED EXAMPLE 1 Easy · Calculate

A mechanic tightens a nut by pulling on a spanner with a force of 80 N. The force is applied 0.25 m from the centre of the nut, at an angle of 60° to the spanner, as shown in the diagram above.

(a) Calculate the torque exerted about the centre of the nut. (b) Calculate the smallest force that would produce the same torque when applied at the same point.

(a) $\tau = Fr \sin \theta = 80 \times 0.25 \times \sin 60^\circ = \mathbf{17 \text{ N m}}$ (17.3 N m), anticlockwise.

(b) The force is smallest when it is perpendicular to the spanner ($\sin \theta = 1$):

$$F = \frac{\tau}{r} = \frac{17.3}{0.25} = \mathbf{69 \text{ N}}$$

Check: the perpendicular distance in (a) is $0.25 \sin 60^\circ = 0.2165 \text{ m}$, and $80 \times 0.2165 = 17.3 \text{ N m}$.

What does an unbalanced torque do?

An unbalanced (resultant) torque acting on a rigid body causes an angular acceleration: the body's rate of rotation changes.

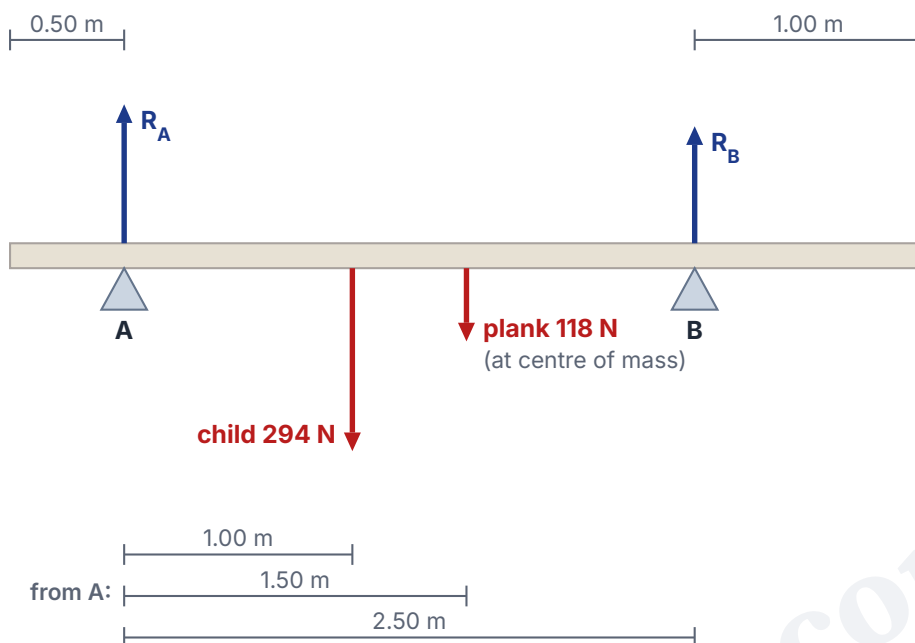
- This is the rotational version of Newton's first and second laws: no resultant torque, no change in rotation; a resultant torque, a change in rotation.
- A resultant **force** changes the motion of the centre of mass; a resultant **torque** changes the rotation. They are separate conditions. Two equal and opposite forces whose lines of action are separated (for example, two hands turning a steering wheel) give zero resultant force but a resultant torque — the wheel gains angular acceleration while its centre stays put.
- For linear motion of an extended body you may treat its whole mass as concentrated at the **centre of mass**, and its weight acts there. You will not be asked to calculate the position of a centre of mass; for a uniform body it is at the geometric centre.

What is rotational equilibrium?

A body is in rotational equilibrium when the resultant torque on it is zero. Its angular velocity is then constant — it is either not rotating or rotating at a constant rate.

- For rotational equilibrium: **sum of clockwise torques = sum of anticlockwise torques** about the axis. If the resultant force is also zero, this holds about any point.
- A body in complete **static** equilibrium (for example a beam at rest) also has zero resultant force (A.2). Use both conditions: torques give one equation, forces give another.
- You may take torques about *any* point. Choose the point where an unknown force acts — that force then has zero torque and drops out of the equation.
- The weight of a uniform beam acts at its centre (its centre of mass).

uniform plank: length 4.00 m, mass 12 kg



Free-body diagram for a uniform plank on two supports with a child standing on it. Arrow lengths are to scale. Distances below the plank are measured from support A.

WORKED EXAMPLE 2 Medium · Determine

A uniform plank of length 4.00 m and mass 12 kg rests horizontally on two supports. Support A is 0.50 m from the left-hand end and support B is 1.00 m from the right-hand end. A child of mass 30 kg stands 1.00 m to the right of A (see the diagram above).

(a) Determine the forces exerted on the plank by each support. (b) The child walks slowly to the right, beyond B. Determine how far beyond B the child can stand before the plank starts to tip.

Weights: child $30 \times 9.8 = 294 \text{ N}$; plank $12 \times 9.8 = 117.6 \text{ N}$, acting at the centre, 1.50 m from A.

(a) Take torques about A, so R_A drops out. Clockwise torques = anticlockwise torques:

$$294 \times 1.00 + 117.6 \times 1.50 = R_B \times 2.50 \Rightarrow R_B = \frac{470.4}{2.50} = \mathbf{188 \text{ N}}$$

Resultant force is zero: $R_A = 294 + 117.6 - 188.2 = \mathbf{223 \text{ N}}$

(b) When the plank is about to tip about B, it lifts off A, so $R_A = 0$. Take torques about B. The plank's weight acts 1.00 m to the left of B

(anticlockwise); the child is a distance d to the right (clockwise):

$$294d = 117.6 \times 1.00 \Rightarrow d = \mathbf{0.40 \text{ m beyond B}}$$

That is 0.60 m short of the right-hand end, so the child cannot reach the end of the plank.

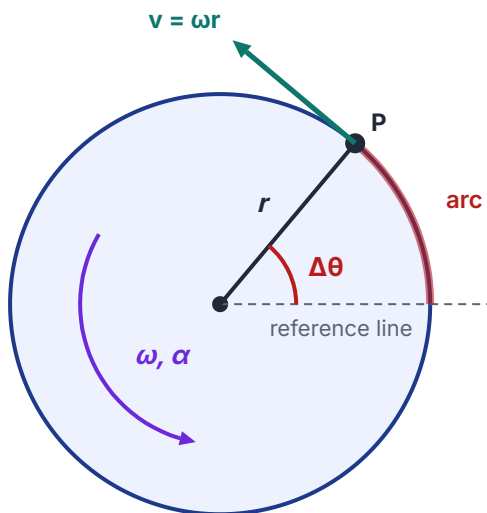
Notice that g cancels: $30d = 12 \times 1.00$.

How do you describe rotation?

Every point of a rotating rigid body turns through the **same angle** in the same time, even though points further from the axis travel further. So rotation is described with angles, not distances.

Quantity	Symbol	Definition	SI unit	Linear partner
Angular position	θ	angle of a reference radius from a fixed reference line	rad	position x
Angular displacement	$\Delta\theta$	change in angular position	rad	displacement s
Angular velocity	ω	rate of change of angular displacement, $\omega = \frac{\Delta\theta}{\Delta t}$	rad s ⁻¹	velocity v
Angular acceleration	α	rate of change of angular velocity, $\alpha = \frac{\Delta\omega}{\Delta t}$	rad s ⁻²	acceleration a

- Angles are measured in **radians**. One revolution = 2π rad, so ω in rad s⁻¹ = $2\pi \times$ (revolutions per second).
- The term angular velocity is used, but you only need its size and sense (clockwise or anticlockwise), not a formal vector treatment. Its magnitude is the angular speed.
- A point at distance r from the axis moves along an arc of length $s = r\Delta\theta$ and has linear speed $v = \omega r$ (data booklet, A.2). Its tangential acceleration is $a = \alpha r$ (not in the data booklet; it follows from $v = \omega r$).



angular displacement $\Delta\theta$ / rad
angle turned through by any radius

angular velocity $\omega = \Delta\theta/\Delta t$
 rad s^{-1} — same for every point

angular acceleration $\alpha = \Delta\omega/\Delta t$
 rad s^{-2}

linear speed of a point $v = \omega r$
grows with distance r from the axis

Every radius of a rigid disc turns through the same angle $\Delta\theta$. A point P on the rim sweeps an arc $r\Delta\theta$ and moves at $v = \omega r$, tangent to the circle.

Rotational graphs

The graph rules of A.1 carry straight across:

Graph	Gradient gives	Area under the graph gives
angular displacement–time (θ – t)	angular velocity ω	no physical meaning
angular velocity–time (ω – t)	angular acceleration α	angular displacement $\Delta\theta$
torque–time (τ – t)	not required	angular impulse ΔL (see below)

What are the equations of motion for rotation?

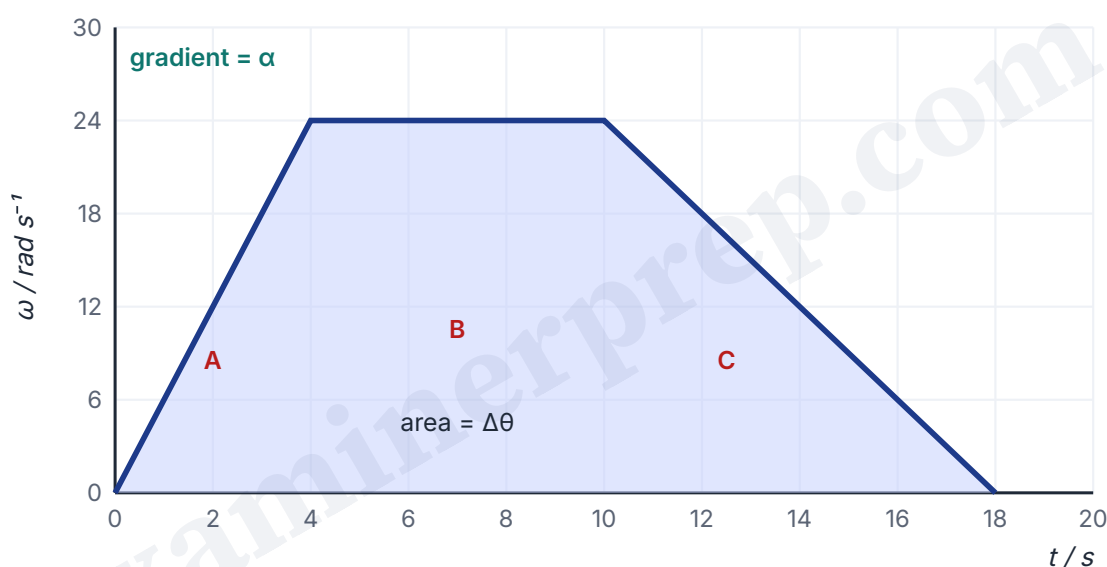
When the angular acceleration is **uniform** (constant), the four equations of motion of A.1 apply with every linear quantity replaced by its rotational partner. All four are in the data booklet.

$$\Delta\theta = \frac{\omega_f + \omega_i}{2} t \quad \omega_f = \omega_i + \alpha t \quad \Delta\theta = \omega_i t + \frac{1}{2} \alpha t^2 \quad \omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

- $\Delta\theta$ = angular displacement (rad)

- ω_i = initial angular velocity (rad s^{-1})
- ω_f = final angular velocity (rad s^{-1})
- α = angular acceleration (rad s^{-2}), constant
- t = time taken (s)

Use the same method as for “suvat”: list the five quantities, identify the one you neither know nor need, and choose the equation without it. Always convert revolutions to radians first, and convert back at the end if the question asks for revolutions ($\text{revolutions} = \Delta\theta/2\pi$).



Angular velocity–time graph for a turntable: uniform angular acceleration (A), constant angular velocity (B) and uniform angular deceleration (C). The shaded area is the total angular displacement.

WORKED EXAMPLE 3 Medium · Determine

The graph above shows how the angular velocity ω of a turntable varies with time t after it is switched on at $t = 0$ and switched off at $t = 10$ s.

(a) Determine the angular acceleration during stages A and C. (b) Determine the total number of revolutions made by the turntable. (c) Use an equation of motion to confirm the angular displacement during stage C.

(a) Gradient of stage A: $\alpha = \frac{24 - 0}{4.0} = \mathbf{6.0 \text{ rad s}^{-2}}$. Stage C: $\alpha = \frac{0 - 24}{18 - 10} = \mathbf{-3.0 \text{ rad s}^{-2}}$ (slowing down).

(b) Angular displacement = area under the graph:

$$\Delta\theta = \frac{1}{2}(4.0)(24) + (6.0)(24) + \frac{1}{2}(8.0)(24) = 48 + 144 + 96 = 288 \text{ rad}$$

$$\text{Revolutions} = \frac{288}{2\pi} = \mathbf{46 \text{ revolutions}} \text{ (45.8)}$$

$$\mathbf{(c)} \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta: 0 = 24^2 + 2(-3.0)\Delta\theta \Rightarrow \Delta\theta = \frac{576}{6.0} = \mathbf{96 \text{ rad}},$$

matching the area of triangle C.

What is moment of inertia?

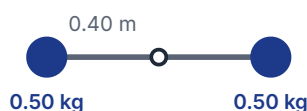
In linear motion, mass measures how hard it is to change a body's velocity. In rotation the matching quantity is the **moment of inertia**, and it depends not only on how much mass there is but on **where** that mass is.

The moment of inertia of a body about an axis measures its resistance to angular acceleration. It depends on the mass of the body and on how that mass is distributed about the axis of rotation.

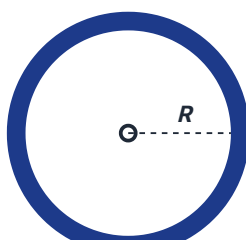
$$I = \Sigma mr^2$$

- I = moment of inertia (kg m^2)
- m = mass of each point mass (kg)
- r = perpendicular distance of that mass from the axis of rotation (m)
- This equation (in the data booklet) applies to a system of **point masses**. Add the mr^2 contributions of every mass.
- Because r is squared, mass far from the axis contributes much more. Doubling the distance of a mass from the axis quadruples its contribution.
- The same body has **different** moments of inertia about different axes. Always state the axis.
- For extended bodies such as discs, rings, rods and spheres, the formula **will be given** in the question when it is needed — you do not have to remember or derive it. Typical given results: ring MR^2 , uniform disc or solid cylinder $\frac{1}{2}MR^2$, solid sphere $\frac{2}{5}MR^2$ (each about an axis through the centre).

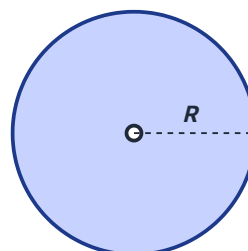
ring and disc: $M = 2.0 \text{ kg}$, $R = 0.20 \text{ m}$
all axes through the centre, perpendicular to the page



two point masses
 $I = \sum mr^2$
see worked example 4



ring (hoop)
all mass at distance R
 $I = MR^2 = 0.080 \text{ kg m}^2$



uniform disc
mass spread from 0 to R
 $I = \frac{1}{2}MR^2 = 0.040 \text{ kg m}^2$

A ring and a disc of the same mass and radius: in the ring all the mass is at distance R , so its moment of inertia is twice that of the disc. The point-mass system on the left is the one used in worked example 4.

What is Newton's second law for rotation?

The resultant torque on a rigid body equals the product of its moment of inertia and its angular acceleration: $\tau = I\alpha$.

$$\tau = I\alpha$$

- τ = resultant (average) torque about the axis (N m)
- I = moment of inertia about the same axis (kg m^2)
- α = angular acceleration (rad s^{-2})
- This is in the data booklet. It is the rotational form of $F = ma$: a given torque produces a smaller angular acceleration in a body with a larger moment of inertia.
- Use the **resultant** torque — subtract any opposing (for example frictional) torque from the driving torque.
- If the torque varies, $\tau = I\alpha$ gives the average angular acceleration when τ is the average torque.
- Many questions couple a rotating body to a body moving in a line (a pulley and a hanging mass, a winch and a load). Write **one equation for each body** — $F = ma$ for the linear one, $\tau = I\alpha$ for the rotating one — and link them with $a = \alpha r$ if the string does not slip.

WORKED EXAMPLE 4 Medium · Calculate

Two small masses, each of 0.50 kg, are fixed to the ends of a light rod of length 0.80 m. The masses may be treated as point masses.

(a) Calculate the moment of inertia of the system about an axis through the centre of the rod, perpendicular to it. (b) Calculate the moment of inertia about a parallel axis through one of the masses. (c) The system is free to rotate about the central axis and is initially at rest. A constant resultant torque of 0.40 N m acts on it for 3.0 s. Calculate the angular acceleration and the final angular velocity.

(a) Each mass is 0.40 m from the axis: $I = \Sigma mr^2 = 2 \times 0.50 \times 0.40^2 = \mathbf{0.16 \text{ kg m}^2}$

(b) One mass is on the axis ($r = 0$) and the other is 0.80 m away: $I = 0.50 \times 0^2 + 0.50 \times 0.80^2 = \mathbf{0.32 \text{ kg m}^2}$

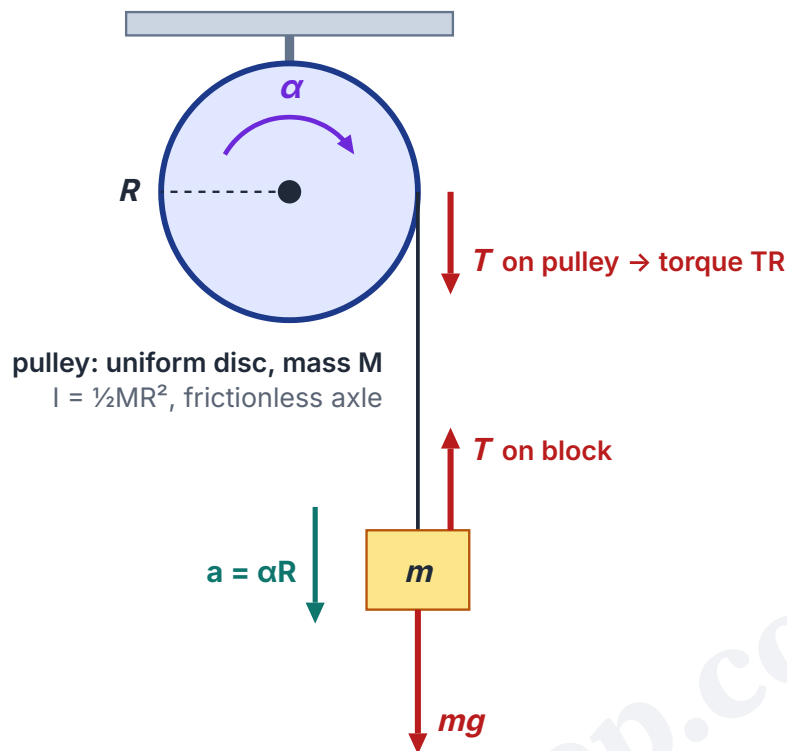
(c) $\alpha = \frac{\tau}{I} = \frac{0.40}{0.16} = \mathbf{2.5 \text{ rad s}^{-2}}$

$\omega_f = \omega_i + \alpha t = 0 + 2.5 \times 3.0 = \mathbf{7.5 \text{ rad s}^{-1}}$

Same masses, same rod — but moving the axis doubles I , so the same torque would give only half the angular acceleration (1.25 rad s^{-2}).

TRY IT YOURSELF

A.4 Rotational Dynamics simulation: apply a torque to rings, discs and rods, change where the mass sits, and watch how the angular acceleration responds.



A block hangs from a light string wrapped round a pulley. The same tension T pulls up on the block and down on the rim of the pulley, where it provides the torque TR .

WORKED EXAMPLE 5 Hard · Show that / Determine

A pulley is a uniform disc of mass 2.4 kg and radius 0.15 m, free to rotate about a frictionless horizontal axle. Its moment of inertia is $I = \frac{1}{2}MR^2$. A light string is wrapped round the pulley and a block of mass 0.50 kg hangs from its free end (see the diagram above). The block is released from rest and the string does not slip.

(a) Show that the acceleration of the block is about 2.9 m s^{-2} . (b) Determine the tension in the string. (c) Determine the speed of the block after it has fallen 1.20 m. (d) Show that energy is conserved during this fall.

(a) $I = \frac{1}{2}(2.4)(0.15)^2 = 0.027 \text{ kg m}^2$.

Block (down positive): $mg - T = ma$.

Pulley: $TR = I\alpha = I\frac{a}{R} \Rightarrow T = \frac{I}{R^2}a = \frac{1}{2}Ma = 1.2a$.

Add the two equations: $mg = (m + \frac{1}{2}M)a \Rightarrow a = \frac{0.50 \times 9.8}{0.50 + 1.2} = 2.88 \text{ m s}^{-2} \approx 2.9 \text{ m s}^{-2}$

(b) $T = 1.2 \times 2.88 = 3.5 \text{ N}$ (3.46 N). Check: $T = m(g - a) = 0.50(9.8 - 2.88) = 3.46 \text{ N}$ — less than the block's weight of 4.9 N, as it must be for the block to accelerate downwards.

(c) $v^2 = u^2 + 2as = 0 + 2(2.88)(1.20) \Rightarrow v = 2.6 \text{ m s}^{-1}$ (2.63 m s⁻¹)

(d) Loss of gravitational potential energy: $mgh = 0.50 \times 9.8 \times 1.20 = 5.88 \text{ J}$.

Translational kinetic energy of the block: $\frac{1}{2}mv^2 = \frac{1}{2}(0.50)(2.63)^2 = 1.73 \text{ J}$

Rotational kinetic energy of the pulley: $\omega = \frac{v}{R} = \frac{2.63}{0.15} = 17.5 \text{ rad s}^{-1}$,
 $\frac{1}{2}I\omega^2 = \frac{1}{2}(0.027)(17.53)^2 = 4.15 \text{ J}$.

Total kinetic energy = $1.73 + 4.15 = 5.88 \text{ J} = \text{loss of } E_p \checkmark$

Without the pulley's inertia the block would fall freely and reach $\sqrt{2gh} = 4.8 \text{ m s}^{-1}$. Most of the energy here goes into turning the heavy pulley.

What are angular momentum and angular impulse?

The angular momentum of a rigid body rotating about an axis is the product of its moment of inertia and its angular velocity: $L = I\omega$.

$$L = I\omega$$

- L = angular momentum (kg m² s⁻¹)
- I = moment of inertia (kg m²); ω = angular velocity (rad s⁻¹)

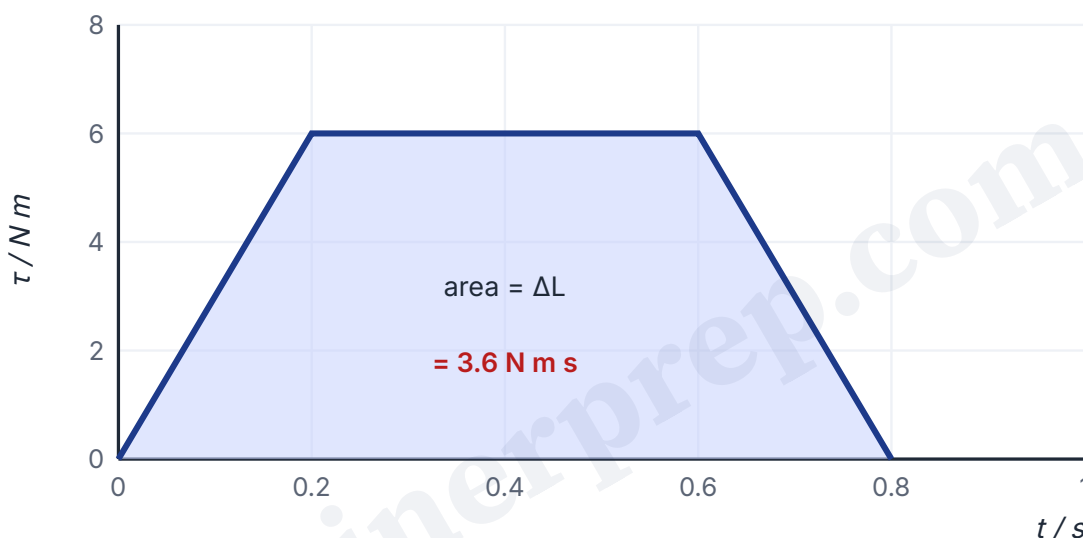
A resultant torque acting for a time changes the angular momentum. This is the rotational version of impulse ($J = F\Delta t = \Delta p$).

Angular impulse is the product of the resultant torque and the time for which it acts. It equals the change in angular momentum: $\Delta L = \tau\Delta t = \Delta(I\omega)$.

$$\Delta L = \tau\Delta t \quad \Delta L = \Delta(I\omega)$$

- Both forms are in the data booklet. The unit N m s is the same as kg m² s⁻¹.

- $\Delta(I\omega)$ reminds you that angular momentum can change because ω changes, because I changes, or both.
- If the torque varies, the angular impulse is the **area under the torque–time graph**.
- In worked example 4, the angular impulse is $0.40 \times 3.0 = 1.2 \text{ N m s}$, and the final angular momentum is $I\omega = 0.16 \times 7.5 = 1.2 \text{ kg m}^2 \text{ s}^{-1}$ — the same, as it must be.



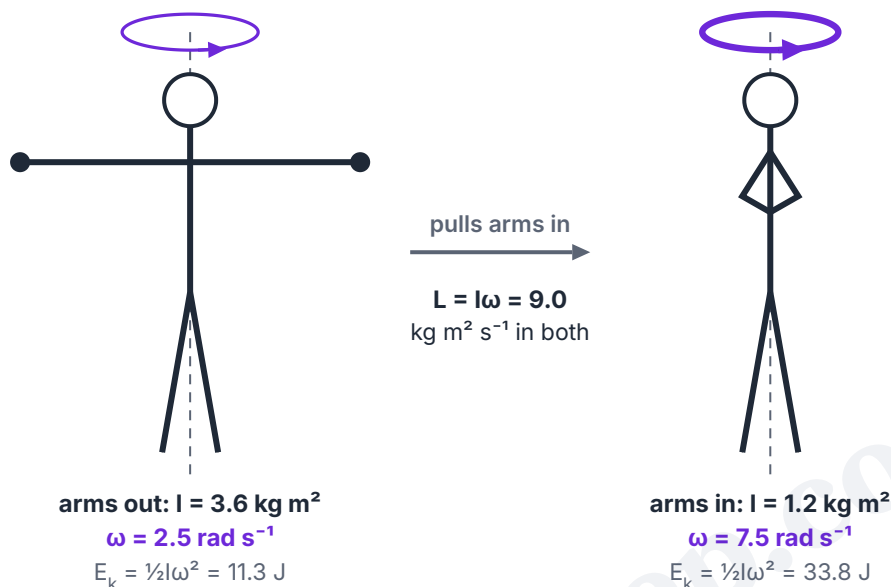
A varying torque acting on a wheel. The area under the torque–time graph, 3.6 N m s , is the angular impulse — the change in the wheel's angular momentum.

When is angular momentum conserved?

The angular momentum of a body or system remains constant unless a resultant external torque acts on it.

- If $\tau = 0$ then $\Delta L = 0$, so $I_i\omega_i = I_f\omega_f$.
- **Change of moment of inertia in one body:** if a spinning body changes shape and moves mass towards the axis, I decreases and ω increases. Examples: a skater pulling in her arms, a diver tucking, a collapsing star.
- **Coupled pairs of bodies:** when two rotating bodies interact (one disc dropped onto another, a person stepping onto a roundabout), the torques they exert on each other are equal and opposite. The total angular momentum of the pair is conserved, though angular momentum is transferred between them.

- Angular momentum is conserved in these events, but **rotational kinetic energy usually is not** (see below).



A skater spinning on ice (negligible friction torque). Pulling her arms in reduces I to one third, so ω trebles; L stays $9.0 \text{ kg m}^2 \text{ s}^{-1}$. The rotational kinetic energy trebles — the extra 22.5 J comes from work done by her muscles.

TRY IT YOURSELF

A.4 Angular Momentum simulation: change a spinning body's moment of inertia and see how its angular velocity and rotational kinetic energy respond while L stays fixed.

WORKED EXAMPLE 6 Hard · Calculate / Show that / Explain

A turntable of moment of inertia 0.060 kg m^2 rotates freely at 3.5 rad s^{-1} about a vertical axis. A uniform disc of moment of inertia 0.025 kg m^2 , not rotating, is dropped gently onto the turntable so that their centres coincide. After a short time of slipping, the two rotate together.

(a) Calculate the final angular velocity. (b) Show that about 30 % of the initial kinetic energy is transferred to other forms. (c) Explain why angular momentum is conserved but kinetic energy is not.

(a) No external torque about the axis, so L is conserved:

$$0.060 \times 3.5 = (0.060 + 0.025) \omega_f \Rightarrow \omega_f = \frac{0.21}{0.085} = \mathbf{2.5 \text{ rad s}^{-1}} \text{ (2.47 rad s}^{-1}\text{)}$$

(b) Initial $E_k = \frac{1}{2}(0.060)(3.5)^2 = 0.368 \text{ J}$. Final $E_k = \frac{1}{2}(0.085)(2.47)^2 = 0.259 \text{ J}$.

$$\text{Fraction transferred} = \frac{0.368 - 0.259}{0.368} = \mathbf{0.294 \approx 30 \%}$$

Quicker: L is constant, so $E_k = \frac{L^2}{2I} \propto \frac{1}{I}$. The fraction remaining is $\frac{0.060}{0.085} = 0.706$, so 0.294 is transferred.

(c) The friction torques that the turntable and the disc exert on each other are equal and opposite, so they are internal to the system and the total angular momentum does not change. However, while the surfaces slip, friction does work that is dissipated as internal (thermal) energy, so kinetic energy decreases.

How do you calculate rotational kinetic energy?

A rotating body has rotational kinetic energy $E_k = \frac{1}{2}I\omega^2$, which can also be written $E_k = \frac{L^2}{2I}$.

$$E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$

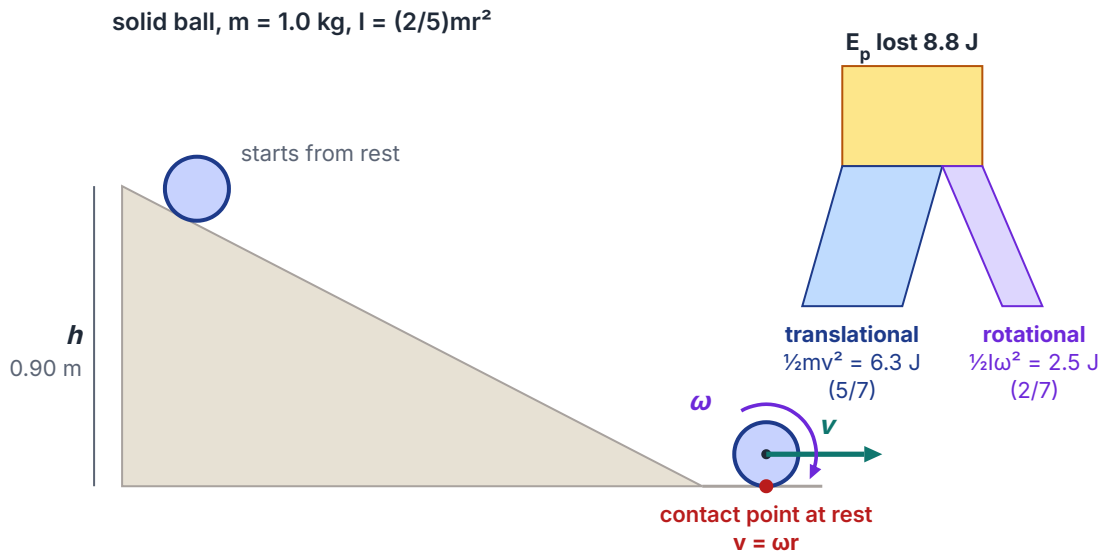
- E_k = rotational kinetic energy (J)
- I = moment of inertia (kg m^2); ω = angular velocity (rad s^{-1}); L = angular momentum ($\text{kg m}^2 \text{ s}^{-1}$)
- Both forms are in the data booklet, and they mirror $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ from A.3.

- Use $\frac{L^2}{2I}$ whenever L is conserved: kinetic energy is then inversely proportional to I .
- Work done by a constant torque turning through an angle is $\tau\Delta\theta$; the work done by the resultant torque equals the change in rotational kinetic energy. (This follows from $W = Fs$ and is not in the data booklet.)
- A flywheel stores energy as rotational kinetic energy. For a given mass and angular velocity, it stores more if its mass is concentrated at the rim, because I is larger.

Rolling without slipping

The only combined rotation-and-translation you need is **rolling without slipping** — a ball, cylinder or wheel rolling along a surface so that the point in contact with the surface is momentarily at rest.

- Condition for rolling without slipping: $v = \omega r$, where v is the speed of the centre of mass and r is the radius of the rolling body. Also $a = \alpha r$.
- Total kinetic energy = translational + rotational: $E_k = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.
- Friction at the contact point provides the torque that makes the body spin up. Because the contact point does not slide, this friction does **no work**, so mechanical energy is conserved.
- Rolling down a slope from rest: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. Substitute $\omega = v/r$ and the given expression for I , and the mass and radius cancel.



A solid ball rolling without slipping. Of the 8.8 J of gravitational potential energy lost, five-sevenths becomes translational kinetic energy and two-sevenths rotational kinetic energy, so it arrives at 3.55 m s^{-1} .

For the solid ball in the diagram ($I = \frac{2}{5}mr^2$, $h = 0.90 \text{ m}$):

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2 \Rightarrow v = \sqrt{\frac{10}{7}gh} = \sqrt{\frac{10}{7} \times 9.8 \times 0.90}$$

A block sliding down a frictionless slope from the same height reaches $\sqrt{2gh} = 4.20 \text{ m s}^{-1}$. The rolling ball is slower because part of its energy is rotational. The larger I is compared with mr^2 , the larger the rotational share and the slower the body.

	■ translational E_k	■ rotational E_k	
block, sliding (no friction)	100%		$v = 4.20 \text{ m s}^{-1}$
solid sphere $I = \frac{2}{5}mr^2$	71%	29%	$v = 3.55 \text{ m s}^{-1}$
uniform disc / cylinder $I = \frac{1}{2}mr^2$	67%	33%	$v = 3.43 \text{ m s}^{-1}$
ring / hoop $I = mr^2$	50%	50%	$v = 2.97 \text{ m s}^{-1}$

Bodies released from rest at a height of 0.90 m. A ring loses the race because half of its kinetic energy is rotational. The results do not depend on the mass or radius — only on the shape.

Exam tips for A.4

EXAM TIP 1 — SAY “ROTATIONAL” KINETIC ENERGY

When you describe energy changes in a spinning body (a collapsing star, a flywheel, a rolling ball), write **rotational kinetic energy**. Writing just “kinetic energy” is ambiguous and it may not be credited when the rotational form was the point of the question.

EXAM TIP 2 — FIND THE RIGHT ANGLE

In $\tau = Fr \sin \theta$, θ is the angle between the force and the line from the axis to the point of application. If the question gives the angle between the force and the *perpendicular* to that line, you need $\cos$. When in doubt, sketch the perpendicular distance from the axis to the line of action and work it out geometrically.

EXAM TIP 3 — RADIANS, ALWAYS

Convert revolutions per minute or per second to rad s^{-1} before using any A.4 equation ($\times 2\pi$, and $\div 60$ for per minute). A number of revolutions is $\Delta\theta/2\pi$. Leaving ω in rev s^{-1} in $E_k = \frac{1}{2}I\omega^2$ loses a factor of $4\pi^2$.

EXAM TIP 4 — CHOOSE YOUR PIVOT, STATE THE SENSE

In equilibrium problems, take torques about the point where an unknown force acts, and label each torque clockwise or anticlockwise. Then use the force condition for the second unknown. “Tipping” means the reaction at the other support has just become zero.

EXAM TIP 5 — CONSERVATION OF ANGULAR MOMENTUM FIRST

Begin with “no external torque acts, so angular momentum is conserved” — the condition earns credit. Then find ω from $I_i\omega_i = I_f\omega_f$, and only

afterwards find the energy, using $E_k = L^2/2I$. Never assume kinetic energy is conserved when a body changes shape or two bodies couple together.

Common misconceptions

Misconception	Why it is wrong	Correct idea
Torque = force $\times$ distance from the axis to where the force acts.	Only the perpendicular distance to the line of action counts.	$\tau = Fr \sin \theta$; a force through the axis has zero torque.
Moment of inertia depends only on mass.	The same mass placed further from the axis is harder to spin up.	I depends on the mass distribution and on the chosen axis.
Zero resultant torque means the body is not rotating.	Zero torque means zero angular <i>acceleration</i> .	The body may rotate at constant ω .
A skater who pulls in her arms keeps the same kinetic energy.	L is constant, so $E_k = L^2/2I$ rises as I falls.	Her muscles do work on her arms; that work becomes rotational kinetic energy.
A rolling ball reaches the bottom of a slope as fast as a sliding block.	Some of the ball's energy is rotational.	A rolling body is slower; the bigger I/mr^2 , the slower.
Friction makes a rolling ball lose energy.	With no slipping, the contact point is at rest, so friction does no work.	Friction supplies the torque for rotation without dissipating energy.

How this topic connects

Rotational twins: how torque, angular momentum and rotational energy mirror linear motion

Every linear equation has a rotational twin with the same form: $F = ma \leftrightarrow \tau = I\alpha$, $p = mv \leftrightarrow L = I\omega$, $F\Delta t = \Delta p \leftrightarrow \tau\Delta t = \Delta L$, $\frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}I\omega^2$, and the four suvat equations become the four equations in θ, ω, α . Conservation of linear momentum (no resultant external force) is matched by conservation of angular momentum (no resultant external torque). The one real difference is that I , unlike m , can change — a body can alter its own moment of inertia and hence its angular velocity without any external torque.

Linear	Rotational
displacement s / m	angular displacement $\Delta\theta$ / rad
velocity v / m s ⁻¹	angular velocity ω / rad s ⁻¹
acceleration a / m s ⁻²	angular acceleration α / rad s ⁻²
mass m / kg	moment of inertia I / kg m ²
force F / N	torque τ / N m
momentum $p = mv$	angular momentum $L = I\omega$
impulse $F\Delta t = \Delta p$	angular impulse $\tau\Delta t = \Delta L$
$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	$E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

Orbits as rotation: angular momentum of satellites and circling charges

A satellite or a charged particle moving in a circle can be treated as a point mass with $I = mr^2$ and angular velocity $\omega = v/r$, so its angular momentum is $L = I\omega = mvr$. The gravitational force on a satellite (D.1) and the magnetic force on a charged particle (D.3) both act along the radius, towards the centre. A force through the axis exerts no torque, so the angular momentum stays constant: the satellite or particle keeps the same angular velocity around a circular path.

Turning coils into current: the torque behind a generator

In a generator (D.4), a turbine applies a torque to a coil, making it rotate in a magnetic field. The changing flux linkage induces an emf. When current flows,

the magnetic force on the coil produces a torque that opposes the rotation (Lenz's law), so a driving torque must keep acting to maintain constant ω . The work done by that torque, $\tau\Delta\theta$, is the energy transferred to the circuit.

Tools & inquiry — measuring the moment of inertia of a flywheel

- Wrap a string round the axle (radius r) and hang a mass m from it. Time the fall through a height h from rest: $a = 2h/t^2$, $\alpha = a/r$, tension $T = m(g - a)$ and torque $\tau = Tr$.
- Repeat for several masses and plot τ against α . For $\tau - \tau_f = I\alpha$, the **gradient is I** and the intercept on the τ axis is the frictional torque τ_f — a systematic error that the graph separates out instead of ignoring.
- Light gates or video analysis reduce the reaction-time uncertainty of hand timing. Measure the axle radius with vernier callipers: r appears twice, so its percentage uncertainty matters.
- **NOS — analogy and models:** the rigid body is an idealisation (real bodies flex), and the rotational equations were built by analogy with linear ones. Analogy is a powerful tool for constructing models, but each model must still be tested against experiment.

Key equations

Equation	Use	In data booklet?
$\tau = Fr \sin \theta$	torque of a force about an axis	yes
$\omega = \frac{\Delta\theta}{\Delta t}, \alpha = \frac{\Delta\omega}{\Delta t}$	definitions; average values	no — learn them
$\Delta\theta = \frac{\omega_f + \omega_i}{2}t, \omega_f = \omega_i + \alpha t,$ $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2, \omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	uniform angular acceleration	yes
$v = \omega r$	linear speed of a point; rolling without slipping	yes (A.2)
$a = \alpha r$	string on a pulley; rolling without slipping	no — follows from $v = \omega r$
$I = \Sigma mr^2$	moment of inertia of point masses	yes
$\tau = I\alpha$	Newton's second law for rotation	yes
$L = I\omega$	angular momentum	yes
$\Delta L = \tau\Delta t = \Delta(I\omega)$	angular impulse	yes
$E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$	rotational kinetic energy	yes

Check your understanding

1. Calculate the angular velocity of the minute hand of a clock.

It turns through 2π rad in 3600 s: $\omega = \frac{2\pi}{3600} = 1.7 \times 10^{-3} \text{ rad s}^{-1}$ (1.75×10^{-3}).

2. Explain why a door handle is fitted at the edge of the door furthest from the hinges.

Torque = $Fr \sin \theta$. Placing the handle as far from the hinge axis as possible maximises r , so a given force produces the largest torque and the largest angular acceleration. Pushing near the hinges, or along the plane of the door ($\theta = 0$), produces little or no torque.

3. The torque–time graph in the angular impulse section acts on a wheel of moment of inertia 0.30 kg m^2 , initially at rest, on a frictionless axle. Determine the final angular velocity.

Area under the graph = $\frac{1}{2}(0.2)(6) + (0.4)(6) + \frac{1}{2}(0.2)(6) = 3.6 \text{ N m s} = \Delta L$. Then $\omega = \frac{\Delta L}{I} = \frac{3.6}{0.30} = 12 \text{ rad s}^{-1}$.

4. A ring and a uniform disc are released together from rest at the top of the same slope and roll without slipping. Which reaches the bottom first? Explain.

The disc. Both lose the same gravitational potential energy per kilogram, but the ring has a larger moment of inertia relative to mr^2 (mr^2 against $\frac{1}{2}mr^2$), so a larger fraction of its kinetic energy is rotational (one half against one third). The disc has more translational kinetic energy per kilogram, so its centre moves faster at every point. Mass and radius do not affect the result.

5. A student on a freely rotating stool pulls two dumbbells inwards so that the moment of inertia of the system halves. State what happens to the angular velocity and to the rotational kinetic energy.

No external torque, so L is constant: ω **doubles**. Since $E_k = \frac{L^2}{2I}$, the rotational kinetic energy also **doubles**. The extra energy comes from the work done by the student's arms pulling the dumbbells inwards.

Practise A.4 with exam-style questions

Original Paper 1A, 1B and 2 questions on rigid body mechanics, with full mark schemes. A.4 is HL only, so there is no SL bank for this sub-topic. Tick

the questions you want and print them as your own test.

HL A.4 questions

Full question bank

Revise and explore

- **Flashcards** — choose IB Physics and filter to A.4 Rigid body mechanics. Free, no sign-up.
- **A.4 Rotational Dynamics simulation**
- **A.4 Angular Momentum simulation**
- **Full IB Physics question bank (all sub-topics, SL & HL)**

← Previous: A.3 Work, energy and power · Next: A.5 Galilean and special relativity (HL only) →

© ExaminerPrep · examinerprep.com · Original material by Evan Kotronis. ExaminerPrep is an independent tutoring service. It has been developed independently from and is not endorsed by the International Baccalaureate Organization. "International Baccalaureate", "IB" and "IB Diploma Programme" are registered trademarks of the IBO, used here for descriptive purposes only. Pearson Edexcel and Cambridge are trademarks of their owners; ExaminerPrep is not affiliated with any exam board.