

THEME A · SPACE, TIME AND MOTION

HL ONLY

≈ 35 min read

IB Physics A.5 Galilean and Special Relativity — Notes (HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). **All of A.5 is Additional Higher Level content:** there is no SL content in this sub-topic, so it appears only on HL papers.



HL A.5 questions



Flashcards



Special relativity simulation

What this page covers

- ✓ Reference frames, inertial frames and events
- ✓ Galilean relativity: Newton's laws hold in every inertial frame
- ✓ The Galilean transformations for position, time and velocity
- ✓ The two postulates of special relativity
- ✓ The Lorentz transformations and the Lorentz factor γ
- ✓ Relativistic addition of velocities
- ✓ The space–time interval as a quantity all inertial observers agree on
- ✓ Proper time interval and proper length
- ✓ Time dilation and length contraction
- ✓ Why simultaneity depends on the reference frame
- ✓ Space–time diagrams: world lines, the angle θ with $\tan \theta = v/c$, and the axes of a moving frame
- ✓ Muon decay experiments as evidence for time dilation and length contraction

In one minute

- In Galilean relativity time is the same for everyone: $x' = x - vt$, $t' = t$, $u' = u - v$. It works well when all speeds are much less than c .

- Special relativity rests on two postulates: the laws of physics are the same in all inertial frames, and every inertial observer measures the same speed of light c .
- The Lorentz transformations replace the Galilean ones. Time and space mix, and the Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$ is always at least 1.
- Different observers disagree about Δt and Δx , but they all agree on the space–time interval $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$.
- Moving clocks run slow ($\Delta t = \gamma\Delta t_0$) and moving objects are shorter along their motion ($L = L_0/\gamma$). Events that are simultaneous in one frame need not be simultaneous in another.
- On a space–time diagram, a moving frame's ct' and x' axes both tilt towards the 45° light line by the angle θ , where $\tan \theta = v/c$.
- Muons made high in the atmosphere reach the ground in large numbers. Earth explains this with time dilation and the muon with length contraction, and both give the same count.

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What is a reference frame?

A reference frame is a set of coordinate axes and synchronised clocks that an observer uses to give each event a position and a time. An inertial

reference frame is one that is not accelerating: it is at rest or moves with constant velocity.

- An **event** happens at one point in space at one instant, for example a lamp flashing or a particle decaying. In one dimension its coordinates are (x, t) .
- An **observer** is anyone at rest in a particular frame. "Observer S measures ..." means "the clocks and rulers at rest in frame S measure ...". It is *not* what one person sees with their eyes. Delays caused by light travelling to the observer have already been allowed for.
- In an inertial frame, Newton's first law holds: a body with no resultant force on it stays at rest or keeps moving at constant velocity. In an accelerating frame (a braking bus, say) loose objects seem to accelerate with no force acting, so that frame is not inertial.
- The standard set-up used throughout this page: frame S' moves at constant velocity v along the positive x -direction of frame S. The axes are parallel, and the origins coincide at $t = t' = 0$.

What is Galilean relativity?

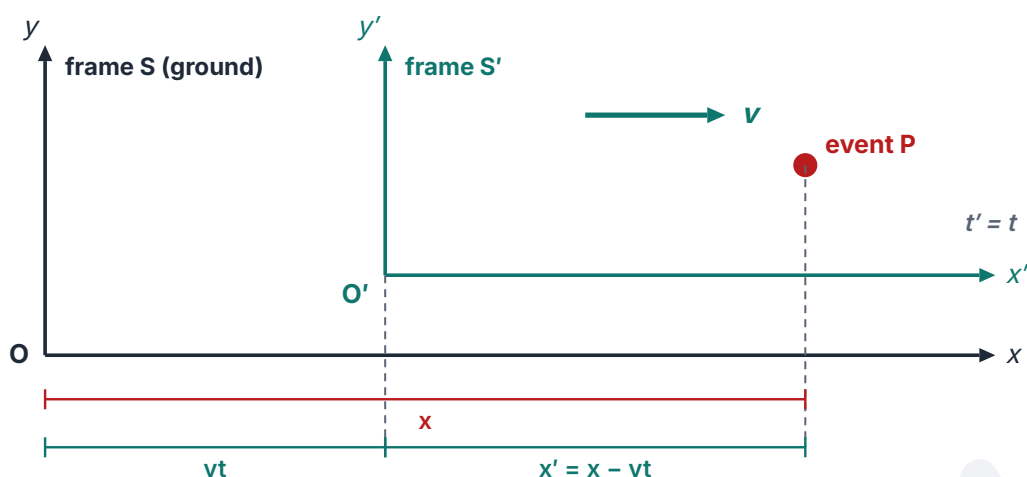
Galilean relativity: Newton's laws of motion are the same in all inertial reference frames. No mechanics experiment carried out inside a frame can show whether that frame is at rest or moving with constant velocity.

In Galilean relativity, time is **absolute**: every observer's clocks agree. The coordinates of an event in S and in S' are therefore linked by the Galilean transformation:

$$x' = x - vt \quad t' = t \quad u' = u - v$$

- x, t = position (m) and time (s) of the event measured in S
- x', t' = position (m) and time (s) of the same event measured in S'
- v = velocity of S' relative to S (m s^{-1})
- u, u' = velocity of a moving object measured in S and in S' (m s^{-1})

All three equations are in the data booklet.



At time t the origin O' of frame S' has moved a distance vt from O , so the event P is a distance $x - vt$ from O' . In Galilean relativity both frames give the event the same time.

- The velocity equation follows by dividing the position equation by time:

$$\frac{\Delta x'}{\Delta t} = \frac{\Delta x}{\Delta t} - v.$$
- Velocities differ between frames, but a **change** in velocity is the same in both (v is constant). So both frames measure the same acceleration and, with the same mass, the same resultant force. That is why Newton's laws hold in every inertial frame.
- Lengths and time intervals are also the same for all observers in Galilean relativity. Special relativity will show that both statements fail at high speed.

WORKED EXAMPLE 1 Easy · Calculate

A car moves along a straight road at 25.0 m s^{-1} relative to the ground. Frame S is fixed to the ground and frame S' to the car, with the positive direction the direction of the car's motion. The origins coincide at $t = 0$. A roadside lamp at $x = 400 \text{ m}$ flashes at $t = 6.0 \text{ s}$. A cyclist rides in the same direction at 8.0 m s^{-1} relative to the ground.

Calculate, in the car's frame, (a) the position and time of the flash, (b) the velocity of the cyclist and (c) the velocity of the lamp.

(a) $x' = x - vt = 400 - 25.0 \times 6.0 = \mathbf{250 \text{ m}}; t' = t = \mathbf{6.0 \text{ s}}$

(b) $u' = u - v = 8.0 - 25.0 = -17.0 \text{ m s}^{-1}$. In the car's frame the cyclist moves backwards at 17.0 m s^{-1} .

(c) The lamp is at rest in S, so $u = 0$ and $u' = 0 - 25.0 = -25.0 \text{ m s}^{-1}$

What are the two postulates of special relativity?

Maxwell's theory of electromagnetism predicts that light travels through a vacuum at one fixed speed, c . It does not say this speed is measured relative to anything. If $u' = u - v$ were correct, observers moving at different speeds would measure different speeds of light, and Maxwell's equations would hold in only one special frame. Experiments such as Michelson and Morley's found no such frame. Einstein (1905) resolved the conflict by keeping the principle of relativity and giving up absolute time.

Postulate 1: the laws of physics are the same in all inertial reference frames.

Postulate 2: the speed of light in a vacuum, c , is the same for all inertial observers, whatever the motion of the source or of the observer.

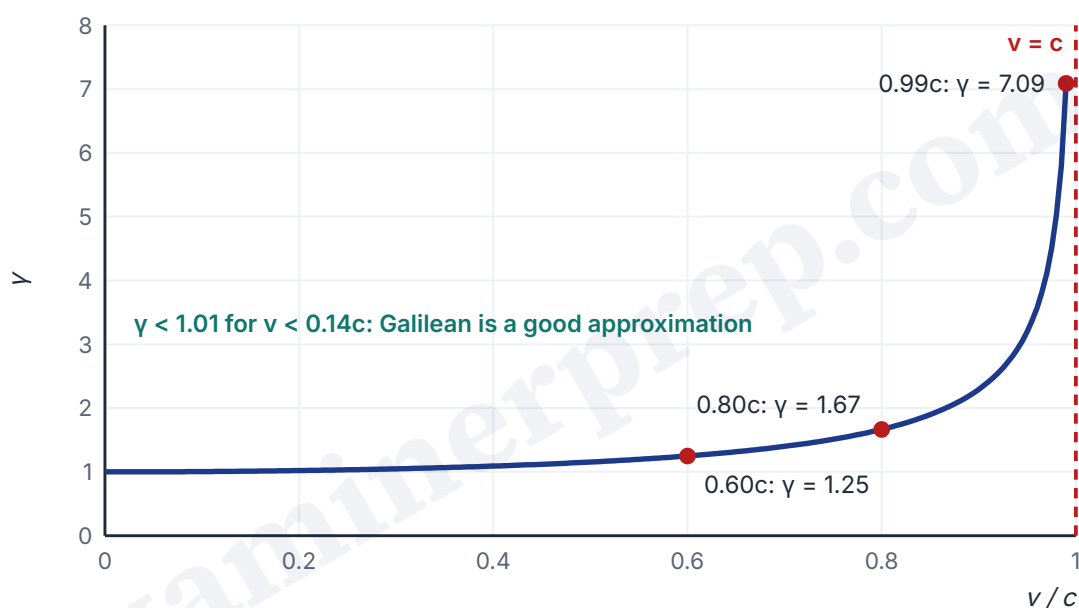
- Postulate 1 extends Galilean relativity from mechanics to *all* of physics, including electromagnetism. There is no experiment that can detect absolute rest.
- Postulate 2 is the surprising one. A light pulse sent forward from a fast spacecraft travels at c relative to the spacecraft *and* at c relative to the ground.
- Together they force time to be relative. Different observers measure different time intervals and lengths, and may disagree about whether two events are simultaneous.

What are the Lorentz transformations?

The postulates lead to a new link between the coordinates of an event in S and in S'. You do not need to derive these equations. They are in the data booklet:

$$x' = \gamma (x - vt) \quad t' = \gamma \left(t - \frac{vx}{c^2} \right) \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- x , t and x' , t' = coordinates of the same event in S and S' (m, s)
- v = velocity of S' relative to S (m s⁻¹)
- c = speed of light in a vacuum = 3.00×10^8 m s⁻¹
- γ = Lorentz factor (no unit), always ≥ 1



The Lorentz factor stays very close to 1 until v is a sizeable fraction of c , then rises steeply and tends to infinity as v approaches c .

- **Low speeds:** when $v \ll c$, $\gamma \approx 1$ and $vx/c^2 \approx 0$, so the equations become $x' = x - vt$ and $t' = t$. Galilean relativity is the low-speed limit of special relativity.
- **Time and space mix:** t' depends on where the event happens (x), not only on when. This term causes the relativity of simultaneity.
- **Going back from S' to S:** swap primed and unprimed symbols and change the sign of v : $x = \gamma(x' + vt')$, $t = \gamma\left(t' + \frac{vx'}{c^2}\right)$.
- **Pairs of events:** the same form applies to differences, $\Delta x' = \gamma(\Delta x - v\Delta t)$ and $\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right)$. These are the most useful forms in exam questions.

- As $v \rightarrow c$, $\gamma \rightarrow \infty$. The relative speed of two inertial frames must therefore be less than c .

Relativistic velocity addition

Applying the Lorentz transformations to a moving object gives the equation that replaces $u' = u - v$. It is in the data booklet, and you do not need to derive it:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

- u = velocity of the object measured in S; u' = its velocity measured in S'; v = velocity of S' relative to S. All carry signs.
- **Check with light:** put $u = c$. Then $u' = \frac{c - v}{1 - v/c} = c$. Every observer measures c , as postulate 2 requires.
- Combining two speeds below c always gives a speed below c .

WORKED EXAMPLE 2 Medium · Calculate / Explain

Two spacecraft, A and B, approach each other along the same straight line. Relative to a space station, A moves at $0.70c$ to the right and B moves at $0.70c$ to the left.

(a) Calculate the velocity of A relative to B. (b) A sends a light pulse towards B. State the speed of the pulse measured by B. (c) Explain why the Galilean answer to (a) cannot be correct.

(a) Let S be the station frame (right positive) and S' the frame of B, so $v = -0.70c$. For A, $u = +0.70c$.

$$u' = \frac{0.70c - (-0.70c)}{1 - \frac{(0.70c)(-0.70c)}{c^2}} = \frac{1.40c}{1.49} = \mathbf{0.94c \text{ towards B}} \quad (0.940c)$$

(b) c ($3.00 \times 10^8 \text{ m s}^{-1}$). The speed of light is the same for all inertial observers.

(c) Galilean addition gives $u' = 0.70c + 0.70c = 1.40c$. That is faster than light, which is impossible: B could then measure light from A moving at a

speed other than c .

Note: in the station frame the gap between A and B does shrink at $1.40c$. That is a rate of change of separation measured by a third observer, not the speed of any object, so it is allowed.

TRY IT YOURSELF

A.5 Special Relativity simulation: change the relative speed of two frames and watch γ , the transformed coordinates and the space–time diagram respond.

What is the space–time interval, and what are proper time and proper length?

Observers in different frames disagree about the time Δt and the distance Δx between two events. One combination of the two, however, comes out the same in every inertial frame.

The space–time interval Δs between two events is given by $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$. It is invariant: every inertial observer calculates the same value.

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 = (c\Delta t')^2 - (\Delta x')^2$$

- Δt , Δx = time (s) and distance (m) between the two events in one frame; Δs is in metres. This equation is in the data booklet.
- Compare Pythagoras in ordinary space: rotating your axes changes the x - and y -components of a length, but not the length itself. Here the minus sign is the important difference.
- For two events on the world line of a light pulse, $\Delta x = c\Delta t$, so $\Delta s = 0$ in every frame. This is another way of saying that everyone measures c .

The proper time interval Δt_0 between two events is the time measured in the frame in which both events happen at the same place. The proper length L_0 of an object is its length measured in the frame in which the object is at rest.

- A proper time interval is measured by **one clock** that is present at both events, such as a clock carried by a particle from its creation to its decay.
- In that frame $\Delta x = 0$, so $(\Delta s)^2 = (c\Delta t_0)^2$. The proper time is therefore $\Delta t_0 = \Delta s/c$, and you can calculate it from measurements made in *any* frame.
- Of all inertial frames, the proper time interval is the **shortest** time between the two events, and the proper length is the **longest** length of the object.

What are time dilation and length contraction?

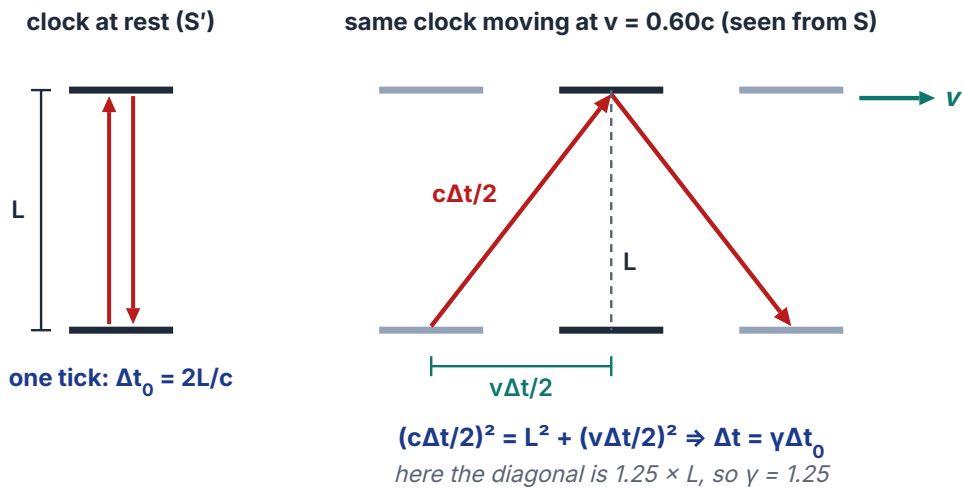
Time dilation

Time dilation: a clock moving relative to an observer is measured to run slow. The time interval Δt measured by that observer is longer than the proper time interval Δt_0 : $\Delta t = \gamma \Delta t_0$.

$$\Delta t = \gamma \Delta t_0$$

- Δt_0 = proper time interval (s), measured in the frame where the events are at the same place
- Δt = time interval between the same events measured in a frame moving at speed v relative to that frame (s)

The derivation is not required, but a **light clock** shows where the equation comes from. A light pulse bounces between two mirrors a distance L apart, and each round trip is one tick.



Seen from S , the moving clock's light pulse travels along the longer diagonal path, still at speed c (postulate 2). Each tick therefore takes longer. The triangle is drawn to scale for $v = 0.60c$.

- Time dilation affects time itself, not just light clocks: atomic clocks, heartbeats and particle decays in the moving frame are all slowed by the same factor.
- The effect is **symmetric**. Each observer measures the *other's* clock running slow. Nobody notices anything odd about their own clock.

Length contraction

Length contraction: an object moving relative to an observer is measured to be shorter, along its direction of motion, than its proper length: $L = L_0/\gamma$.

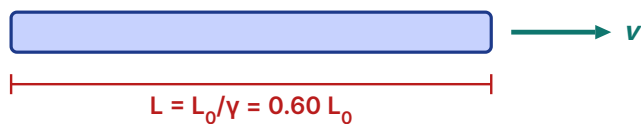
$$L = \frac{L_0}{\gamma}$$

- L_0 = proper length (m), measured in the object's rest frame
- L = length measured by an observer for whom the object moves at speed v (m)
- To measure a moving object's length, you must record the positions of both ends **at the same time in your own frame**. Because the two frames disagree about simultaneity, they disagree about the length.

In the rod's rest frame S' : the rod is at rest



In frame S : the rod moves at $v = 0.80c$



only the length along the motion contracts; the thickness is unchanged

Drawn to scale for $v = 0.80c$ ($\gamma = 1.67$). Only the dimension along the motion is contracted.

	Proper value	Measured in a frame where it moves	Comparison
Time between two events	Δt_0 : the events are at the same place	$\Delta t = \gamma \Delta t_0$	longer
Length of an object	L_0 : the object is at rest	$L = L_0/\gamma$	shorter

WORKED EXAMPLE 3 Medium · Show that / Determine

A spacecraft travels at $0.80c$ from Earth to a star that is 6.0 ly away in the Earth frame. Earth and the star are at rest relative to each other. (1 ly is the distance light travels in 1 year, so $c = 1 \text{ ly yr}^{-1}$.)

(a) Show that $\gamma \approx 1.7$. (b) Calculate the time taken for the journey in the Earth frame. (c) Determine the time taken for the journey according to the crew. (d) Calculate the Earth–star distance measured by the crew, and use it to confirm your answer to (c).

$$\text{(a)} \quad \gamma = \frac{1}{\sqrt{1 - 0.80^2}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.60} = \mathbf{1.67} \approx 1.7$$

$$\text{(b)} \quad \Delta t = \frac{6.0 \text{ ly}}{0.80 \text{ ly yr}^{-1}} = \mathbf{7.5 \text{ yr}}$$

(c) Departure and arrival both happen at the spacecraft, so the crew's clock measures the proper time: $\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{7.5}{1.667} = \mathbf{4.5 \text{ yr}}$

(d) The Earth–star distance is a proper length in the Earth frame, so the crew measure $L = \frac{6.0}{1.667} = 3.6 \text{ ly}$.

For the crew, the star approaches at $0.80c$ and covers 3.6 ly : $\Delta t_0 = 3.6/0.80 = 4.5 \text{ yr}$. ✓

Interval check: $\sqrt{(c\Delta t)^2 - (\Delta x)^2} = \sqrt{7.5^2 - 6.0^2} = 4.5 \text{ ly}$, so $\Delta t_0 = 4.5 \text{ yr}$. All three methods agree.

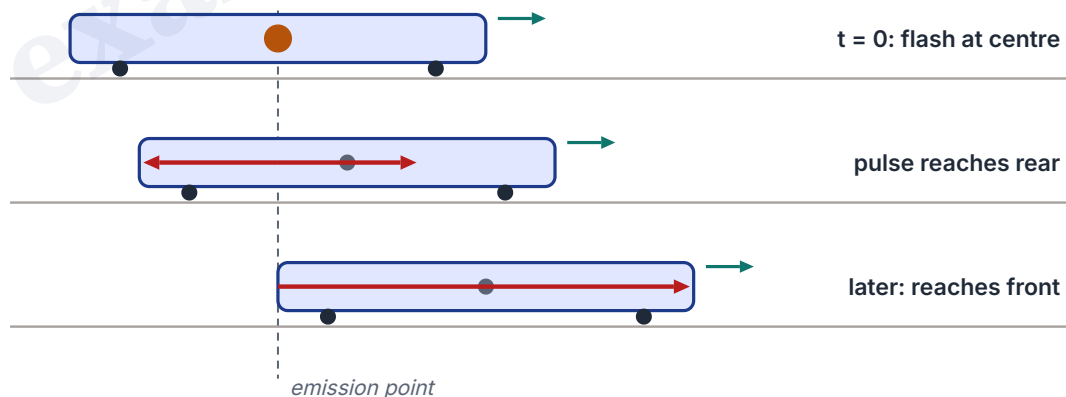
Why is simultaneity relative?

Relativity of simultaneity: two events at different places that are simultaneous in one inertial frame are not simultaneous in a frame moving relative to it along the line joining them.

In the train's frame S' (train at rest)



In the platform's frame S (train moves right at $v = 0.50c$)



A lamp at the centre of a carriage flashes. In the train frame the light reaches both ends together. In the platform frame (drawn to scale for $0.50c$, with the carriage length-contracted) the light still travels at c each way, but the rear moves towards it and the front moves away, so the rear is reached first.

How to explain it in an exam, step by step:

1. In the train frame, the lamp is equidistant from both ends and light travels at c both ways, so the arrivals are simultaneous.

2. By postulate 2, light also travels at c both ways in the platform frame.
3. In the platform frame the rear moves towards the emission point and the front moves away from it, so the light travels a shorter distance to the rear.
4. So, in the platform frame, the light reaches the rear **before** the front. The two arrival events are not simultaneous.

The Lorentz transformation gives the same result. If two events are simultaneous in S ($\Delta t = 0$) and a distance Δx apart, then in S'

$$\Delta t' = \gamma \left(0 - \frac{v\Delta x}{c^2} \right) = -\frac{\gamma v\Delta x}{c^2}$$

- $\Delta t'$ is zero only if $\Delta x = 0$. Events at the *same place* and same time are simultaneous for everyone.
- In S' (moving in the $+x$ -direction), the event further along $+x$ happens **first**.
- Neither frame is "right". Simultaneity is not absolute; it depends on the observer's frame.

How do you draw and read space–time diagrams?

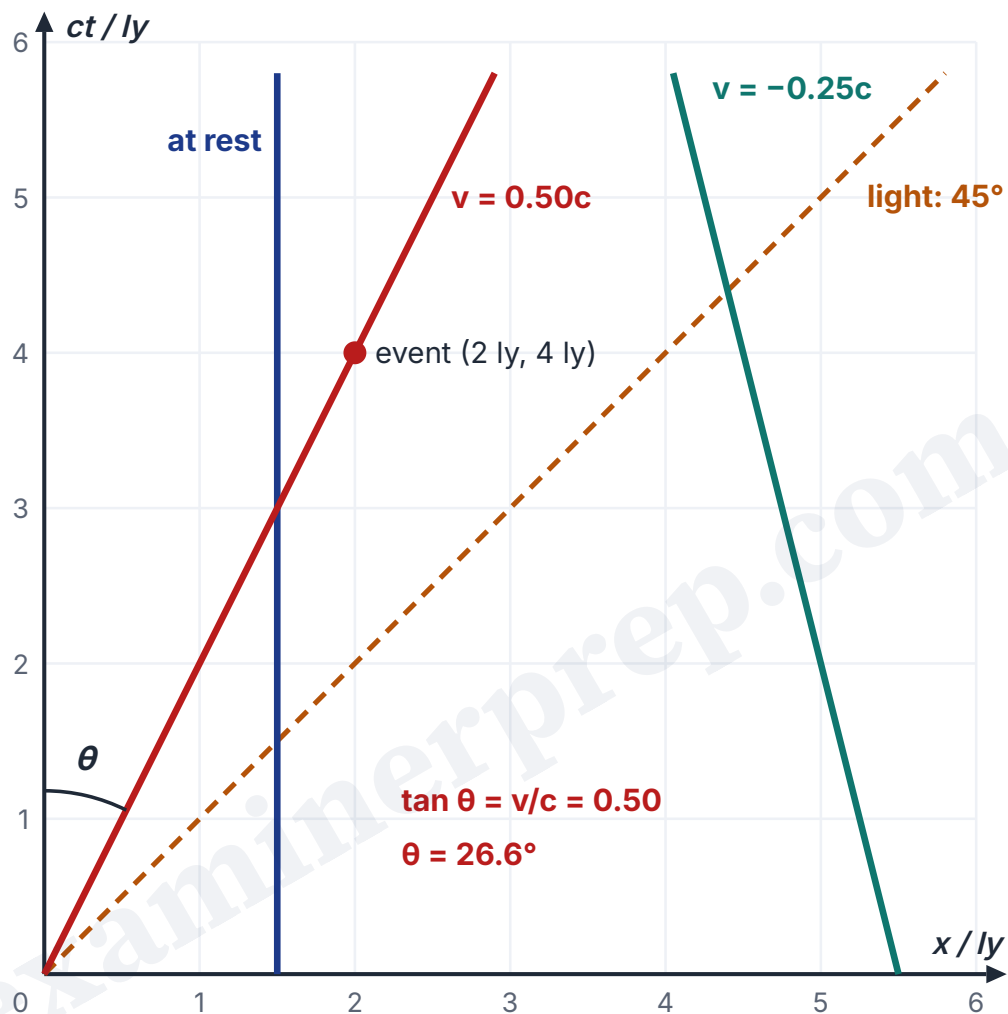
A space–time (Minkowski) diagram plots events in one frame. Position x is on the horizontal axis, and the time axis is labelled ct so that both axes are in metres (or both in light-years). Each event is a point.

A world line is the line on a space–time diagram that shows the history of a body. For a body moving with constant velocity it is a straight line at angle θ to the ct axis, where $\tan \theta = v/c$.

$$\tan \theta = \frac{v}{c}$$

- θ = angle between the world line and the ct axis; v = speed of the body in that frame. This equation is in the data booklet.
- A body at rest has a vertical world line ($\theta = 0$). Light has $\theta = 45^\circ$. Every material body has $\theta < 45^\circ$.

- Only world lines with constant velocity (straight lines) are needed in IB questions.

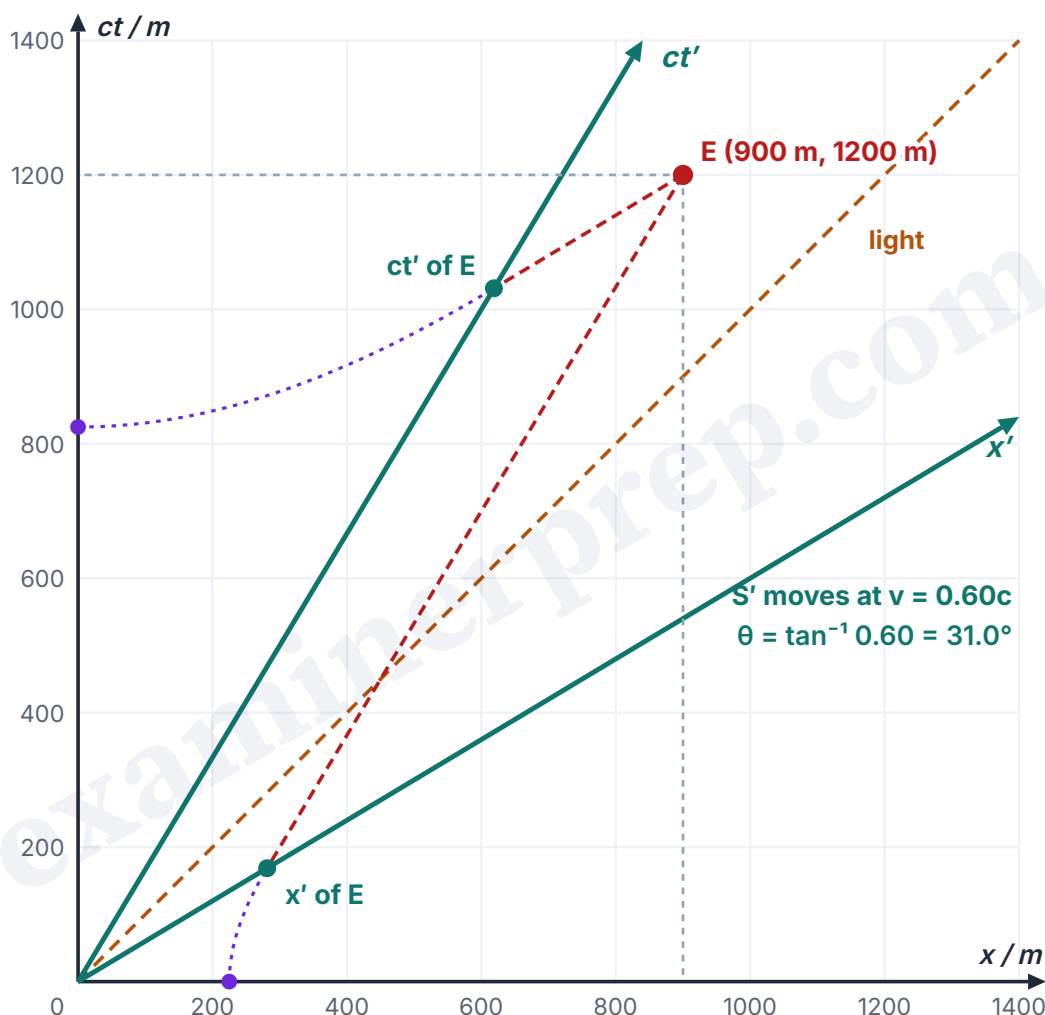


World lines in frame S : a body at rest, a body moving at $0.50c$ ($\theta = 26.6^\circ$), a body moving in the $-x$ direction at $0.25c$ and a light ray at 45° .

Adding the axes of a moving frame S'

- The ct' **axis** is the world line of the origin of S' ($x' = 0$). It is tilted from the ct axis by θ , where $\tan \theta = v/c$.
- The x' **axis** is the set of events with $t' = 0$. It is tilted up from the x axis by the *same* angle θ . Both primed axes close in symmetrically on the 45° light line.
- Lines parallel to the x' axis are lines of simultaneity in S'** (constant t'). Lines parallel to the ct' axis join events at constant x' .
- To read the S' coordinates of an event, draw a line through it *parallel to the x' axis* to meet the ct' axis, and a line *parallel to the ct' axis* to meet the x' axis.

- **The scales are not the same.** One metre on the ct' or x' axis is drawn longer than one metre on the ct or x axis. The scales are matched using curves of constant space–time interval: $(ct)^2 - x^2 = \text{constant}$ crosses the ct and ct' axes at the same time reading, and $x^2 - (ct)^2 = \text{constant}$ crosses the x and x' axes at the same distance.



Frame S' moves at $0.60c$. The red dashed lines through E are parallel to the primed axes and give its S' coordinates. The purple dotted hyperbolae have constant space–time interval: each crosses an unprimed axis and the matching primed axis at the same reading, so one metre on a primed axis is drawn longer than one metre on an unprimed axis.

WORKED EXAMPLE 4 Medium · Determine / Show that

Frame S' moves at $0.60c$ relative to frame S , and the origins coincide at $t = t' = 0$. In S , an event E occurs at $x = 900$ m and $t = 4.00$ μs . The space–time diagram above shows E .

(a) Determine the coordinates x' and t' of E. (b) Show that the space-time interval between the origin event O and E is the same in both frames. (c) Outline how the diagram gives the S' coordinates of E.

(a) $\gamma = 1/\sqrt{1 - 0.60^2} = 1.25$. In S, $ct = 3.00 \times 10^8 \times 4.00 \times 10^{-6} = 1200$ m and $vt = 0.60 \times 1200 = 720$ m.

$$x' = 1.25 (900 - 720) = \mathbf{225 \text{ m}}$$

$$t' = 1.25 \left(4.00 \times 10^{-6} - \frac{0.60 \times 900}{3.00 \times 10^8} \right) = 1.25 (4.00 - 1.80) \times 10^{-6} = \mathbf{2.75 \mu s} \quad (ct' = 825 \text{ m})$$

(b) S: $(\Delta s)^2 = 1200^2 - 900^2 = 6.30 \times 10^5 \text{ m}^2$

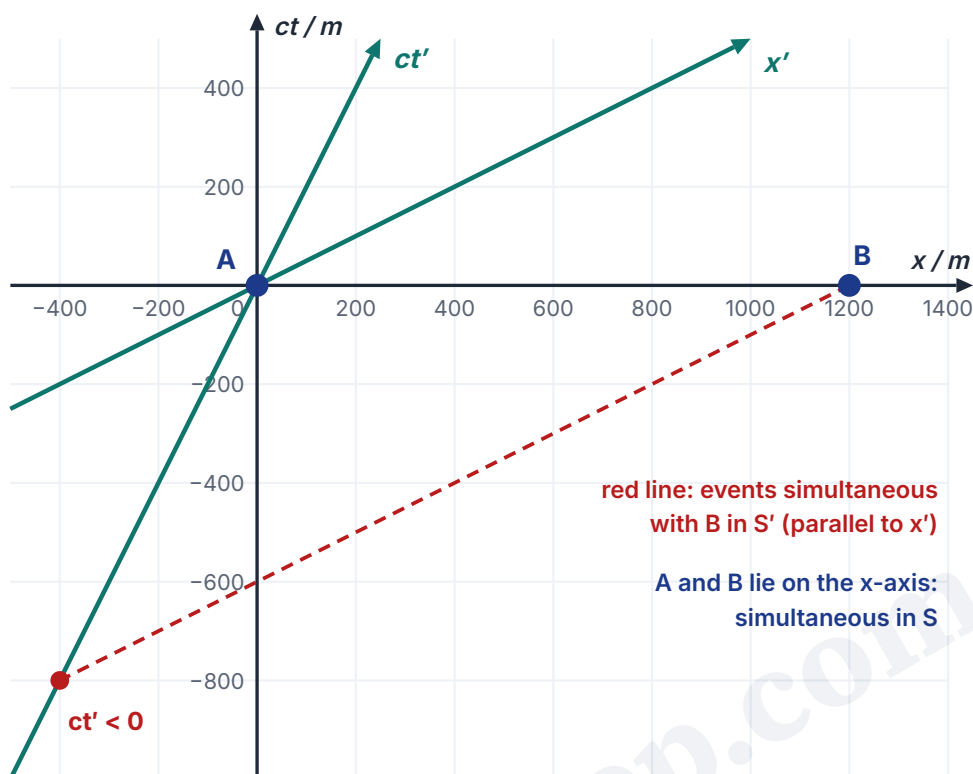
S': $(\Delta s)^2 = 825^2 - 225^2 = 6.30 \times 10^5 \text{ m}^2$. **The values are equal, so $\Delta s = 794$ m in both frames.**

(c) A line through E parallel to the x' axis meets the ct' axis at $ct' = 825$ m. A line through E parallel to the ct' axis meets the x' axis at $x' = 225$ m. The distances along the primed axes must be read using the primed scale, which the hyperbolae calibrate.

Galilean comparison: $x' = 900 - 720 = 180$ m and $t' = 4.00 \mu s$. At $0.60c$ the Galilean values are badly wrong.

Simultaneity on a space-time diagram

Events on a horizontal line are simultaneous in S. Events on a line parallel to the x' axis are simultaneous in S' . Because these two families of lines are not parallel, the two frames disagree about which events happen at the same time.



A and B lie on the x axis, so they are simultaneous in S . Frame S' moves at $0.50c$. The line through B parallel to the x' axis cuts the ct' axis below the origin, so in S' event B happens before event A.

WORKED EXAMPLE 5 Hard · Determine / Explain

Two beacons, A and B, are fixed 1200 m apart on a long space station (frame S). They flash simultaneously in S . A probe flies past the station at $0.50c$ in the direction from A towards B.

(a) Determine the time between the two flashes in the probe's frame. (b) State which flash happens first in the probe's frame, and explain your answer using the space–time diagram above.

(a) Take A at $x = 0$ and B at $x = +1200$ m, with the probe frame S' moving at $v = +0.50c$; $\gamma = 1/\sqrt{1 - 0.50^2} = 1.155$.

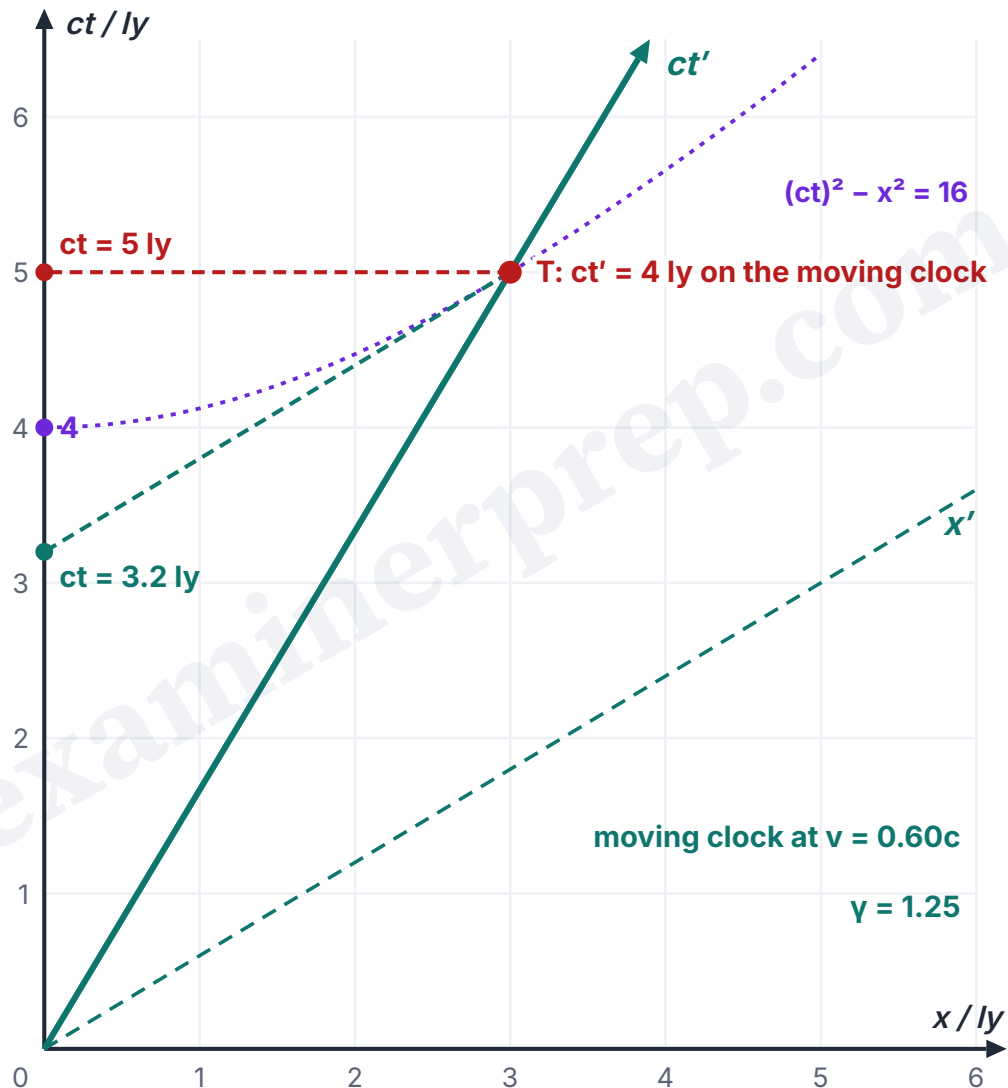
$$\Delta t' = t'_B - t'_A = \gamma \left(0 - \frac{v\Delta x}{c^2} \right) = -\frac{1.155 \times 0.50 \times 1200}{3.00 \times 10^8} = -2.31 \times 10^{-6} \text{ s}$$

The flashes are **2.3 μs apart** in the probe's frame.

(b) $t'_B - t'_A$ is negative, so **B flashes first**. On the diagram, A is at the origin, so $t'_A = 0$. The line of simultaneity in S' through B is parallel to the x' axis

and meets the ct' axis at $ct' = c\Delta t' = -693$ m, below A. In S' , B is earlier. This agrees with the rule that the event further along the direction of the probe's motion happens first.

Time dilation and length contraction on a space–time diagram



A clock at the origin of S' moves at $0.60c$, so its world line is the ct' axis. The hyperbola shows that event T , where the clock reads $t' = 4$ yr ($ct' = 4$ ly), is at the same interval from O as $ct = 4$ ly on the ct axis.

- **As judged in S :** the horizontal red line through T is S 's line of simultaneity. S 's clocks read $t = 5$ yr ($ct = 5$ ly) at T , while the moving clock reads 4 yr. So $\Delta t = 5$ yr $= \gamma \times 4$ yr: the moving clock runs slow.
- **As judged in S' :** S' 's line of simultaneity through T (teal, parallel to x') meets the ct axis at $ct = 3.2$ ly $= 4/1.25$. So S' finds that S 's clock runs slow by the

same factor. The effect is symmetric, and there is no contradiction because the two frames disagree about simultaneity.

- **Length contraction** works the same way. The two ends of a moving rod have parallel world lines. S measures the length along a horizontal line (both ends at the same t), while the rod's own frame measures along a line parallel to its x' axis. After calibrating with $x^2 - (ct)^2 = \text{constant}$, the S measurement is shorter by the factor γ .

How do muon decay experiments support special relativity?

Muons are unstable particles created when cosmic rays collide with nuclei high in the atmosphere, typically 10–15 km up. They travel down at speeds very close to c . Their proper half-life is about $1.5 \mu\text{s}$, so even at c a muon travels only about 460 m in one half-life. With no relativity, almost none would survive a journey of several kilometres. In fact, a large fraction reaches the ground.

Muon decay experiments are evidence for special relativity: far more muons reach the ground than non-relativistic physics predicts. In the Earth frame this is explained by time dilation, and in the muon frame by length contraction.

- **Earth frame:** the atmosphere has its proper thickness. The muon's internal "clock" is moving, so its half-life is dilated to $\gamma \times$ the proper half-life.
- **Muon frame:** the muon is at rest and decays with its proper half-life. The atmosphere and the ground rush towards it, so the thickness of the atmosphere is contracted to $1/\gamma$ of its proper value.
- Both frames must agree on how many muons reach the ground, because "muon reaches the detector" is a single event that every frame records.

WORKED EXAMPLE 6 Hard · Determine / Show that

A batch of 1.0×10^6 muons is created 12.0 km above the ground. The muons move vertically downwards at $0.992c$ relative to the Earth. The proper half-

life of a muon is $1.52 \mu\text{s}$.

(a) Calculate γ . (b) Determine, in the Earth frame, the time taken to reach the ground and the number of muons expected to arrive. (c) Determine the number expected if relativistic effects are ignored. (d) Show that a calculation in the muon frame gives the same result as (b).

$$\text{(a)} \quad \gamma = \frac{1}{\sqrt{1 - 0.992^2}} = \mathbf{7.92}$$

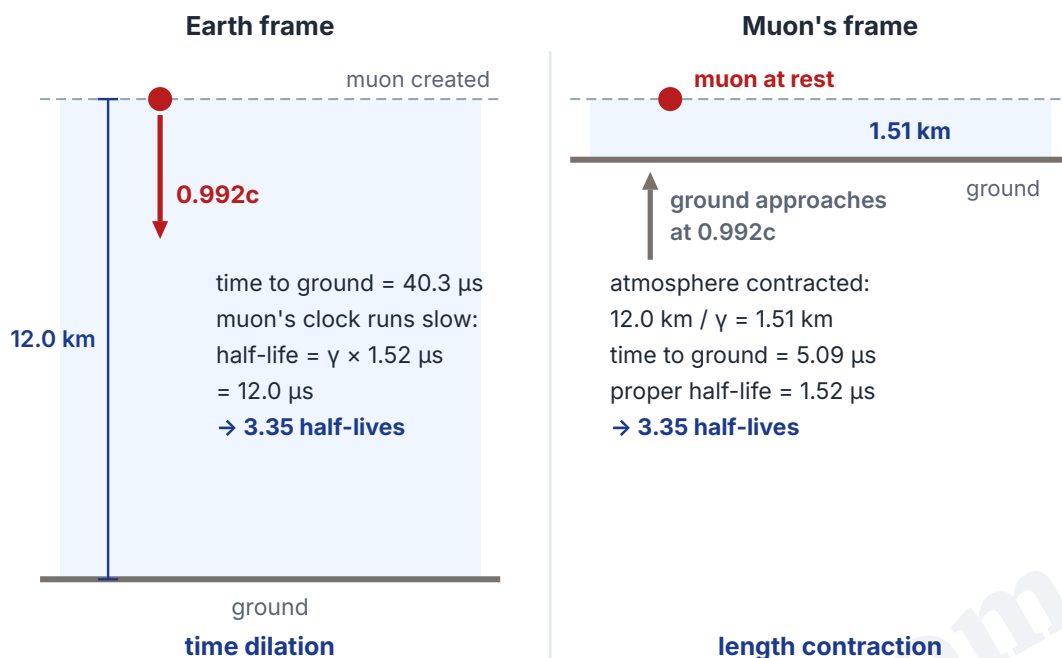
$$\text{(b)} \quad t = \frac{12.0 \times 10^3}{0.992 \times 3.00 \times 10^8} = 4.03 \times 10^{-5} \text{ s} = 40.3 \mu\text{s}.$$

Dilated half-life = $7.92 \times 1.52 = 12.04 \mu\text{s}$ ($\approx 12.0 \mu\text{s}$), so the number of half-lives is $40.3/12.04 = 3.35$.

$$N = 1.0 \times 10^6 \times \left(\frac{1}{2}\right)^{3.35} = \mathbf{9.8 \times 10^4 \text{ muons}}$$
 (about 10 %)

(c) Without time dilation the number of half-lives is $40.3/1.52 = 26.5$, so $N = 1.0 \times 10^6 \times \left(\frac{1}{2}\right)^{26.5} \approx 0.01$: **essentially none**.

(d) In the muon frame the atmosphere is contracted: $L = \frac{12.0 \text{ km}}{7.92} = 1.51 \text{ km}$. Time for the ground to arrive = $\frac{12.0 \times 10^3}{7.92 \times 0.992 \times 3.00 \times 10^8} = 5.09 \mu\text{s}$. Number of proper half-lives = $5.09/1.52 = 3.35$. This is the same as in (b), so **N = 9.8×10^4** again.



Same prediction in both frames: about 9.8 % of the muons reach the ground.
 $\gamma = 7.92$; heights drawn to scale.

The same muon journey described in two frames (numbers from worked example 6). Each frame uses one relativistic effect, and both predict the same survival fraction.

Exam tips for A.5

EXAM TIP 1 — FIND THE PROPER QUANTITY FIRST

Before using γ , write down which frame measures the proper time: the frame in which the two events happen **at the same place**, timed by one clock present at both. "The observer at rest" is not an answer, because every observer is at rest in their own frame. For lengths, the proper length belongs to the frame in which the **object** is at rest.

EXAM TIP 2 — SANITY-CHECK THE DIRECTION

γ is always ≥ 1 . A dilated time must come out **longer** than the proper time, and a contracted length **shorter** than the proper length. If yours does not, you have multiplied where you should have divided. When the data allow, check your answer with the space–time interval.

EXAM TIP 3 — “EXPLAIN” ANSWERS ON SIMULTANEITY

Credit goes to a chain of physics, not to “because the train is moving”. State that light travels at c in **both** frames (postulate 2). Then say which way each receiver moves relative to the emission point in the second frame, and conclude which event happens first *in which frame*.

EXAM TIP 4 — READING PRIMED COORDINATES

On a space–time diagram, read ct' with a line **parallel to the x' axis** and x' with a line **parallel to the ct' axis**, never horizontally or vertically. The primed scale differs from the unprimed one, so when a numerical answer is needed, use the Lorentz transformation and the diagram only to check it.

EXAM TIP 5 — ONE FRAME PER MUON ARGUMENT

Work in one frame at a time. In the Earth frame, use the full atmosphere thickness with the *dilated* half-life. In the muon frame, use the *contracted* thickness with the proper half-life. Using a contracted distance together with a dilated half-life applies the effect twice and gives the wrong survival fraction.

Common misconceptions

Misconception	Why it is wrong	Correct idea
Moving clocks run slow because the motion damages them.	Every process in the moving frame is slowed by the same factor, including the body processes of anyone travelling with the clock.	Time itself differs between frames. Nobody notices their own clock running slow.
Relativistic effects are optical illusions caused by light-travel delays.	The effects remain after delays have been corrected for.	They are measured results in that frame, not what the eye sees.
Two objects approaching at $0.70c$ each have a relative speed of $1.40c$.	The speed of one in the other's frame must use relativistic velocity addition.	Relative speed = $0.94c$. The separation can shrink at $1.40c$ in a third frame.
Length contraction squashes a moving object in every direction.	Only distances parallel to the relative velocity are affected.	Perpendicular dimensions are unchanged.
Relativity only applies to things moving close to c .	The Lorentz transformations apply at all speeds.	At low speeds $\gamma \approx 1$, so the effects are too small to notice and the Galilean equations are an excellent approximation.
Simultaneity is relative, so cause and effect can be reversed.	The order of two events can differ between frames only if light could not travel from one to the other in the time between them.	Events that could be connected by a signal keep the same order in every frame.

How this topic connects

Kinematics within a frame, transformations between frames

Within any **single** inertial frame, the A.1 kinematics still holds: a body moving at constant velocity covers distance = speed $\times$ time, measured with that frame's own rulers and clocks (worked examples 3 and 6). What changes is how quantities are carried *between* frames. Positions and times transform by the Lorentz equations rather than $x' = x - vt$, $t' = t$. Velocities combine by $u' = \frac{u - v}{1 - uv/c^2}$ instead of $u' = u - v$. Time intervals and lengths are no longer the same for every observer.

Doppler shift for light versus sound: the role of a medium

Sound needs a medium. Its speed is fixed relative to the air, so a moving source and a moving observer produce different frequency shifts, and the equations treat them separately. Light needs no medium, and by postulate 2 its speed is c for every observer. Only the **relative** velocity of source and observer can matter, so for light (C.5) there is a single equation: $\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{v}{c}$ for $v \ll c$.

Natural limits in physics, from the speed of light to absolute zero

Other natural limits include **absolute zero**: a temperature of 0 K can be approached but never reached (B.1). The **efficiency of a heat engine** is always less than 1, and is capped by the Carnot efficiency for given reservoir temperatures (B.4). There are also **limits on measurement**: every measured value carries an uncertainty, which better technique can reduce but never remove. Like c , absolute zero and the Carnot limit are not failures of technology. They are built into the theories, and finding them has repeatedly pushed physics forward.

Tools & nature of science — evidence for relativity

- **Falsification**: the Michelson–Morley experiment looked for a change in the speed of light due to Earth's motion and found none. The Galilean picture of light failed, and special relativity accounts for the null result.
- **Muon counts**: in 1963 Frisch and Smith counted muons of known speed near the top of Mount Washington (about 1.9 km high) and at sea level.

Many more survived the descent than non-relativistic decay predicts, in agreement with time dilation.

- **Accelerators:** fast unstable particles in particle accelerators live longer, by the factor γ , than identical particles at rest. This is the same test made under controlled conditions.
- **Units tool:** use light-years and years together ($c = 1 \text{ ly yr}^{-1}$), or metres for both x and ct . This keeps numbers small and avoids power-of-ten slips.
- **Theories as limits:** Galilean relativity is not thrown away. It is the $v \ll c$ limit of special relativity, which is why A.1–A.4 still work for everyday motion.

Key equations

Equation	Use	In data booklet?
$x' = x - vt, \quad t' = t$	Galilean transformation of an event	yes
$u' = u - v$	Galilean velocity addition ($v \ll c$)	yes
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$	Lorentz factor	yes
$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$	Lorentz transformation of an event (same form for $\Delta x, \Delta t$)	yes
$u' = \frac{u - v}{1 - uv/c^2}$	relativistic velocity addition	yes
$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$	invariant space-time interval	yes
$\Delta t = \gamma\Delta t_0$	time dilation (Δt_0 = proper time)	yes
$L = L_0/\gamma$	length contraction (L_0 = proper length)	yes
$\tan \theta = v/c$	angle between a world line (or the ct' axis) and the ct axis	yes
$x = \gamma(x' + vt'), \quad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$	inverse Lorentz transformation	no — swap primes and the sign of v

Check your understanding

1. Calculate the Lorentz factor for a relative speed of $0.40c$.

$$\gamma = 1/\sqrt{1 - 0.40^2} = 1/\sqrt{0.84} = 1.09$$

2. A spacecraft moving at $0.60c$ relative to Earth launches a probe forwards at $0.50c$ relative to the spacecraft. Calculate the speed of the probe relative to Earth.

Take S' as Earth's frame, moving at $v = -0.60c$ relative to the spacecraft frame S , in which the probe has $u = +0.50c$. Then $u' = \frac{0.50c + 0.60c}{1 + 0.30} = \frac{1.10c}{1.30} = 0.85c$ ($0.846c$), not $1.10c$.

3. A rod has a proper length of 2.0 m. Calculate its length measured by an observer for whom it moves along its length at $0.80c$.

$\gamma = 1.67$, so $L = 2.0/1.67 = 1.2$ m.

4. On a space–time diagram, a world line makes an angle of 20° with the ct axis. Calculate the speed of the body.

$v = c \tan 20^\circ = 0.36c = 1.1 \times 10^8 \text{ m s}^{-1}$

5. Two events occur at the same place and at the same time in frame S . Could they be non-simultaneous in another inertial frame?

No. With $\Delta x = 0$ and $\Delta t = 0$, the transformation gives $\Delta t' = \gamma(0 - 0) = 0$ in every frame. Only events at *different* places can lose their simultaneity.

Practise A.5 with exam-style questions

Original HL Paper 1A, 1B and 2 questions on Galilean and special relativity, with full mark schemes. Tick the questions you want and print them as your own test.

HL A.5 questions

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