

THEME B · THE PARTICULATE NATURE OF MATTER

SL & HL ≈ 35 min read

IB Physics B.3 Gas Laws — Notes (SL & HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). B.3 is identical for SL and HL — there is no additional HL content in this sub-topic. It links the large-scale behaviour of a gas (pressure, volume, temperature) to the motion of its molecules.

 SL B.3 questions HL B.3 questions Flashcards Gas laws simulation

What this page covers

- ✓ Pressure as the perpendicular force per unit area, $P = F/A$
- ✓ Amount of substance in moles, $n = N/N_A$
- ✓ The ideal gas as a model, built on kinetic theory, that approximates real gases
- ✓ The three empirical gas laws (constant temperature, constant pressure, constant volume) and how they combine into $PV/T = \text{constant}$
- ✓ The ideal gas equation in both forms, $PV = Nk_B T$ and $PV = nRT$, and changes of state on a pressure–volume diagram
- ✓ How molecular collisions with a wall produce pressure, leading to $P = \frac{1}{3}\rho\overline{v^2}$
- ✓ Internal energy of an ideal monatomic gas, $U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$
- ✓ The conditions of temperature, pressure and density under which a real gas behaves almost ideally

In one minute

- Pressure is perpendicular force per unit area: $P = F/A$, in pascals (1 Pa = 1 N m⁻²).

- One mole contains $N_A = 6.02 \times 10^{23}$ particles, so $n = N/N_A$.
- For a fixed amount of gas: PV is constant at constant T ; $V \propto T$ at constant P ; $P \propto T$ at constant V . Together: $PV/T = \text{constant}$. **T is always in kelvin.**
- Ideal gas equation: $PV = nRT = Nk_B T$, with $R = N_A k_B$.
- Gas pressure comes from the momentum change of molecules bouncing off the walls: $P = \frac{1}{3}\rho\overline{v^2}$.
- An ideal gas has no intermolecular forces, so no potential energy: $U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$ (monatomic).
- Real gases behave ideally at **low pressure, high temperature and low density**.

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What is pressure?

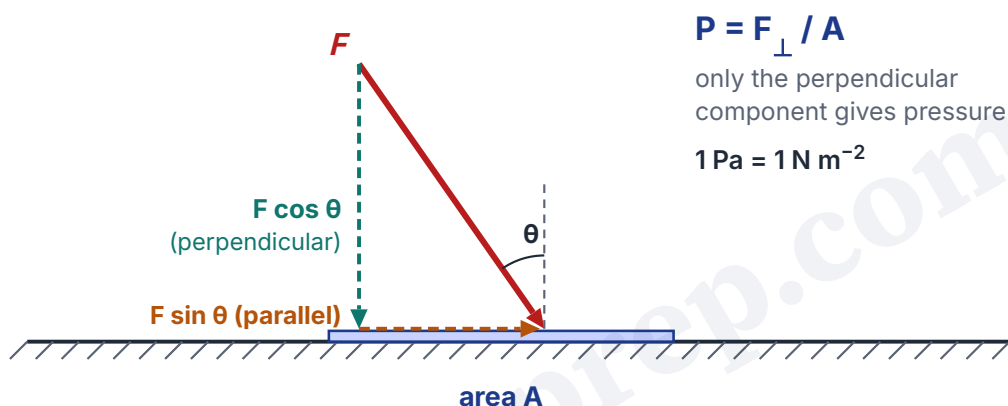
Pressure is the force acting perpendicular to a surface per unit area of that surface.

$$P = \frac{F}{A}$$

- P = pressure (Pa, where $1 \text{ Pa} = 1 \text{ N m}^{-2}$)

- F = force exerted **perpendicular** (normal) to the surface (N)
- A = area of the surface (m^2)

This equation is in the data booklet. Pressure is a **scalar**: a gas at pressure P pushes outwards with a force PA on each area A of its container walls, at right angles to that surface, whichever way the surface faces.



When a force acts at an angle θ to the normal, only the perpendicular component $F \cos \theta$ produces pressure on the surface. The parallel component $F \sin \theta$ tends to slide the surface instead.

- Atmospheric pressure at sea level is about $1.01 \times 10^5 \text{ Pa}$ (101 kPa). The air pushes on every square metre of your body with a force of about 100 kN, balanced by the pressure of the fluids inside you.
- Useful conversions: $1 \text{ kPa} = 10^3 \text{ Pa}$; $1 \text{ cm}^2 = 10^{-4} \text{ m}^2$; $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$; 1 litre = 10^{-3} m^3 .

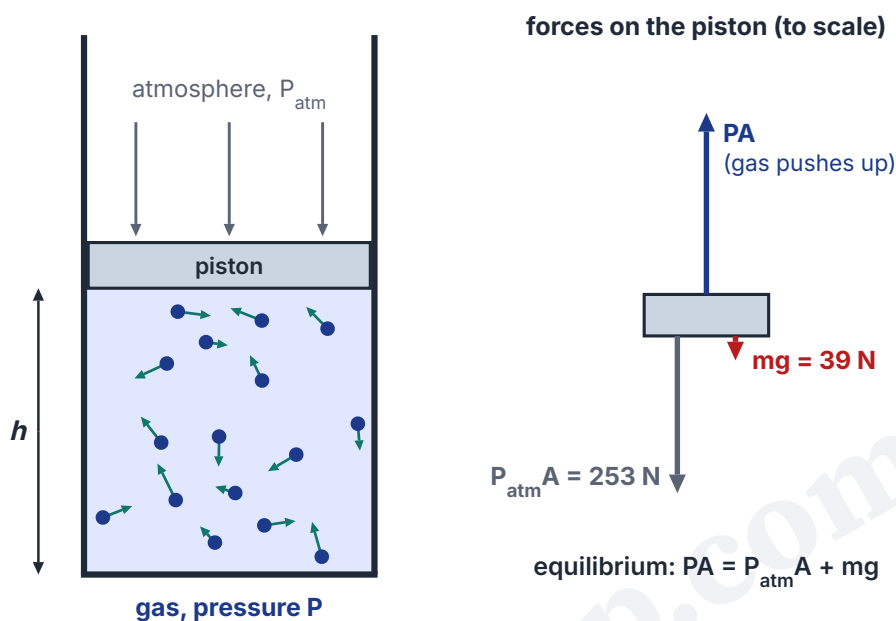
Gas trapped by a piston

Many gas questions use a cylinder closed by a piston that can move without friction. When the piston is at rest, the resultant force on it is zero (A.2). For a vertical cylinder with the piston on top:

$$PA = P_{\text{atm}}A + mg \quad \Rightarrow \quad P = P_{\text{atm}} + \frac{mg}{A}$$

If the gas is then heated slowly, the piston rises but the forces on it are unchanged once it settles, so the gas stays at **constant pressure**. This is the

standard way to set up a constant-pressure change.



Gas in a vertical cylinder under a 4.0 kg piston of area $2.5 \times 10^{-3} \text{ m}^2$ (worked example 2). The upward force from the gas balances the weight of the piston plus the force from the atmosphere above it. Forces drawn to scale.

What is the amount of substance?

The amount of substance n is the number of particles in a sample divided by the Avogadro constant. Its SI unit is the mole (mol).

$$n = \frac{N}{N_{\text{A}}}$$

- n = amount of substance (mol)
- N = number of particles (molecules or atoms) — a pure number, no unit
- N_{A} = Avogadro constant = $6.02 \times 10^{23} \text{ mol}^{-1}$ (data booklet)

The equation is in the data booklet. One mole of any substance contains 6.02×10^{23} particles. The **molar mass** M is the mass of one mole (for example 4.0 g mol^{-1} for helium, 28 g mol^{-1} for nitrogen gas). A question will give it to you; then $\text{mass} = nM$, and the mass of one molecule is M/N_{A} .

Quick example. A balloon holds 0.25 mol of helium. Number of atoms $N = nN_A = 0.25 \times 6.02 \times 10^{23} = 1.5 \times 10^{23}$. Mass = $nM = 0.25 \times 4.0 = 1.0$ g.

EXAM TIP — N OR n?

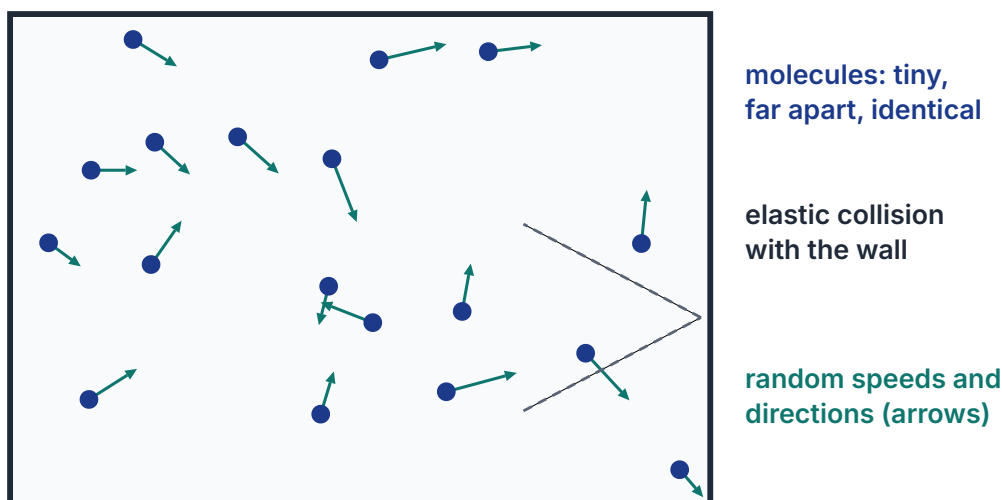
Lower-case n is moles; capital N is the number of molecules. Use R with n and k_B with N . Mixing them gives answers wrong by a factor of 6.02×10^{23} — usually obvious if you stop and ask whether the number is sensible.

What is an ideal gas?

An ideal gas is a modelled system that obeys the ideal gas equation $PV = nRT$ exactly at all pressures, volumes and temperatures. Its behaviour is described by the kinetic theory of gases, and real gases can be treated as approximately ideal.

The kinetic model pictures the gas as a very large number of molecules in constant random motion. The ideal gas is defined by these assumptions:

- The gas is made of a **very large number** of identical molecules in **random motion**, with a range of speeds.
- The molecules are point particles: the total **volume of the molecules is negligible** compared with the volume of the container.
- There are **no intermolecular forces** except during collisions, so molecules travel in straight lines at constant velocity between collisions, and the gas has **no intermolecular potential energy**.
- All collisions, between molecules and with the walls, are **elastic**: kinetic energy is conserved.
- The **duration of a collision is negligible** compared with the time between collisions.
- The molecules obey Newton's laws of motion.



no forces between molecules except during collisions

The kinetic model of an ideal gas. Each molecule moves in a straight line until it hits another molecule or a wall; arrows show velocities, which are random in direction and size.

No real gas is exactly ideal — real molecules do have size and do attract each other. The model is useful because it is simple, it explains the measured gas laws, and real gases follow it closely under the conditions described in the final section.

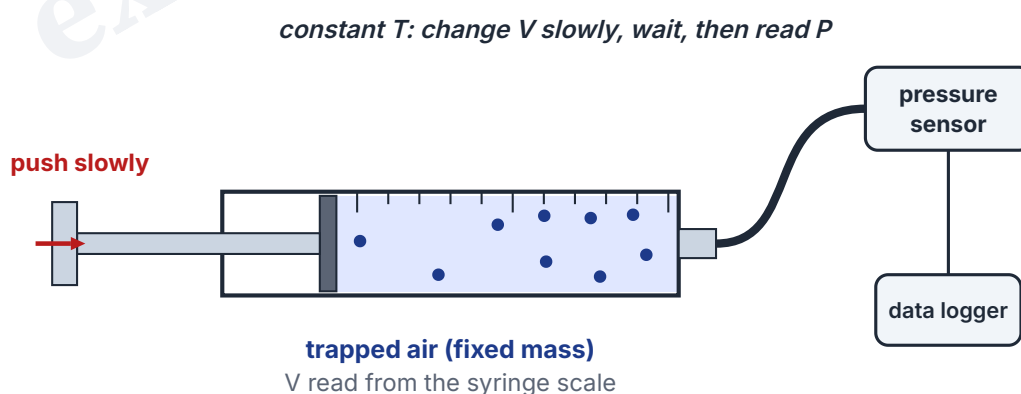
What are the three empirical gas laws?

The gas laws were found by experiment (they are **empirical**) long before kinetic theory explained them. Each holds one variable constant and applies to a **fixed amount** of gas. Temperatures must be in **kelvin**: $T/\text{K} = \theta/^{\circ}\text{C} + 273$ (data booklet).

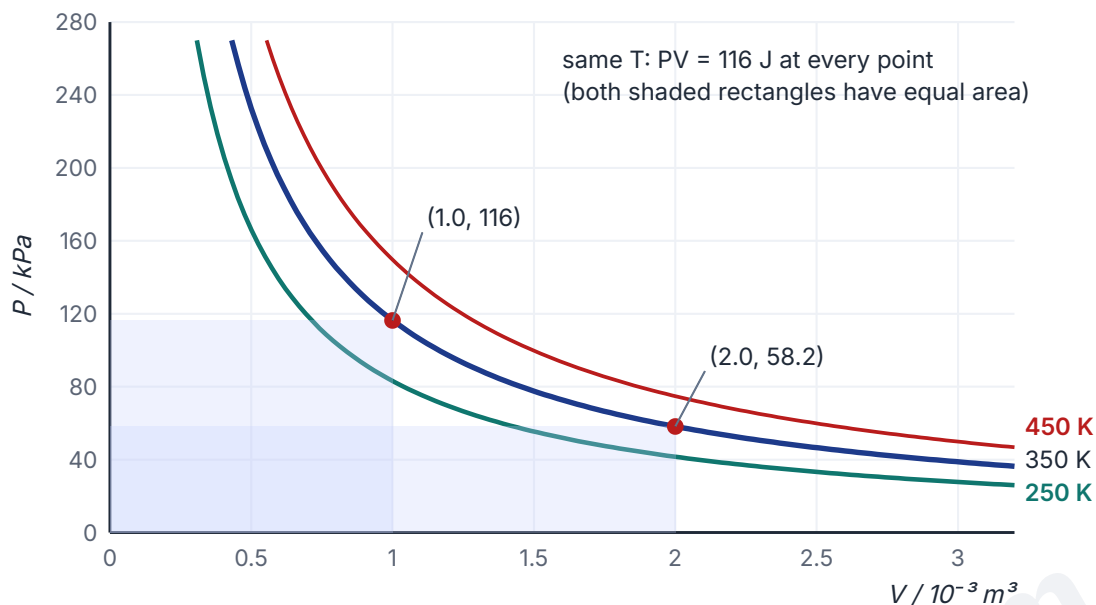
Law	Held constant	Statement	Equation	Straight-line graph
Boyle's law	T and n	pressure is inversely proportional to volume	$PV = \text{constant};$ $P_1 V_1 = P_2 V_2$	P against $1/V$ through the origin
Charles's law	P and n	volume is directly proportional to kelvin temperature	$\frac{V}{T} = \text{constant};$ $\frac{V_1}{T_1} = \frac{V_2}{T_2}$	V against T through the origin
Pressure law (Gay-Lussac's law)	V and n	pressure is directly proportional to kelvin temperature	$\frac{P}{T} = \text{constant};$ $\frac{P_1}{T_1} = \frac{P_2}{T_2}$	P against T through the origin

Constant temperature: Boyle's law

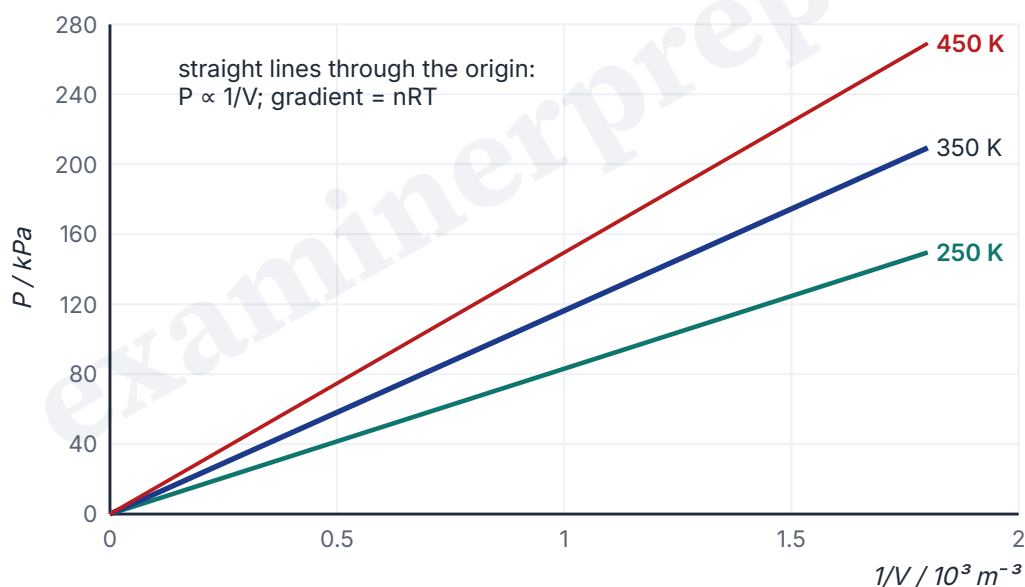
For a fixed amount of gas at constant temperature, the pressure is inversely proportional to the volume.



Boyle's law apparatus. The plunger is moved in steps; after each step the air is left to return to room temperature before the pressure is recorded.



Isotherms for 0.040 mol of an ideal gas at 250 K, 350 K and 450 K. Each curve is a hyperbola, $P = \text{constant}/V$. A hotter gas has a higher isotherm, further from the origin.



The same data plotted as P against $1/V$: straight lines through the origin, which confirm inverse proportionality. The gradient is $PV = nRT$, so a steeper line means a higher temperature.

WORKED EXAMPLE 1 Easy · Calculate

A syringe holds 60 cm^3 of air at a pressure of $1.0 \times 10^5 \text{ Pa}$. The nozzle is sealed and the plunger is pushed in slowly so that the temperature does not change.

- (a) Calculate the pressure when the volume is 24 cm^3 . (b) Calculate the volume at which the pressure would be $3.0 \times 10^5 \text{ Pa}$.

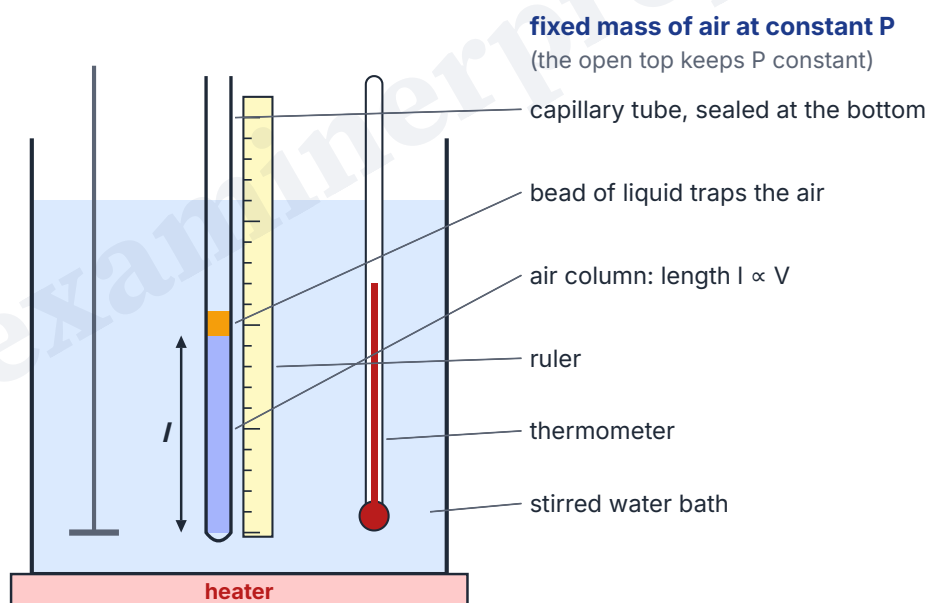
(a) Fixed amount, constant T : $P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{1.0 \times 10^5 \times 60}{24} = 2.5 \times 10^5 \text{ Pa}$

(b) $V_2 = \frac{P_1V_1}{P_2} = \frac{1.0 \times 10^5 \times 60}{3.0 \times 10^5} = 20 \text{ cm}^3$

The volumes appear on both sides of the equation, so they can stay in cm^3 . That is only true for ratios — in $PV = nRT$ you must use m^3 .

Constant pressure: Charles's law

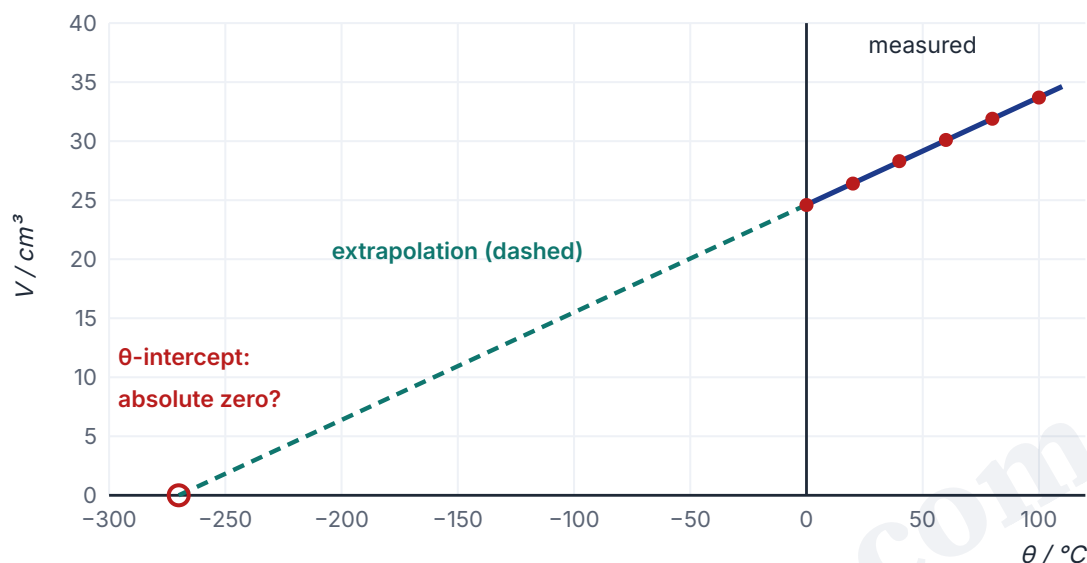
For a fixed amount of gas at constant pressure, the volume is directly proportional to the absolute (kelvin) temperature.



Charles's law apparatus. The tube has a uniform cross-section, so the length l of the air column is proportional to its volume. The top of the tube is open to the atmosphere, so the trapped air stays at constant pressure (atmospheric pressure plus the small, constant pressure due to the bead).

Plotted against Celsius temperature, the volume of any gas gives a straight line that does *not* pass through the origin. Extrapolated backwards, the lines for every gas meet the temperature axis at the same point, close to $-273\text{ }^\circ\text{C}$. This is **absolute zero**, 0 K — the temperature at which an ideal gas would have zero volume at constant pressure (or zero pressure at constant volume), and the

molecules zero kinetic energy. The kelvin scale starts there, which is why the gas laws only take a simple form in kelvin.



Volume against Celsius temperature for the air in worked example 3. The best-fit line, extended (dashed) to zero volume, meets the θ -axis close to absolute zero; worked example 3 finds the value. A real gas would liquefy long before reaching it.

WORKED EXAMPLE 2 Medium · Determine (diagram)

Gas is trapped in the vertical cylinder in the diagram in the pressure section by a piston of mass 4.0 kg and area $2.5 \times 10^{-3} \text{ m}^2$, which moves without friction. Atmospheric pressure is $1.01 \times 10^5 \text{ Pa}$. The gas column has height 0.20 m at 17°C .

(a) Determine the pressure of the gas. (b) The gas is heated slowly to 87°C . Explain why the pressure of the gas does not change, and determine the distance the piston rises.

(a) Piston in equilibrium:
$$P = P_{\text{atm}} + \frac{mg}{A} = 1.01 \times 10^5 + \frac{4.0 \times 9.8}{2.5 \times 10^{-3}} = 1.01 \times 10^5 + 1.57 \times 10^4 = \mathbf{1.17 \times 10^5 \text{ Pa}}$$

(b) The piston settles where the resultant force on it is zero. Its weight, its area and the atmospheric pressure do not change, so the gas pressure must still be $P_{\text{atm}} + mg/A$: it is constant.

The cross-section is constant, so $V \propto h$. Charles's law with $T_1 = 290 \text{ K}$, $T_2 = 360 \text{ K}$:

$$h_2 = h_1 \frac{T_2}{T_1} = 0.20 \times \frac{360}{290} = 0.248 \text{ m, so the piston rises by } \mathbf{0.048 \text{ m (4.8 cm)}}$$

WORKED EXAMPLE 3 Medium · Determine (graph)

A student traps air in a capillary tube (as in the Charles's law apparatus) and measures its volume at six temperatures at constant pressure.

$\theta / ^\circ\text{C}$	0	20	40	60	80	100
V / cm^3	24.6	26.4	28.3	30.1	31.9	33.7

- (a) Use the graph above to determine the gradient of the line, with its unit.
 (b) Determine the temperature at which the volume would become zero. (c) Explain why this gives an estimate of absolute zero, and state why the gas could never actually reach zero volume.

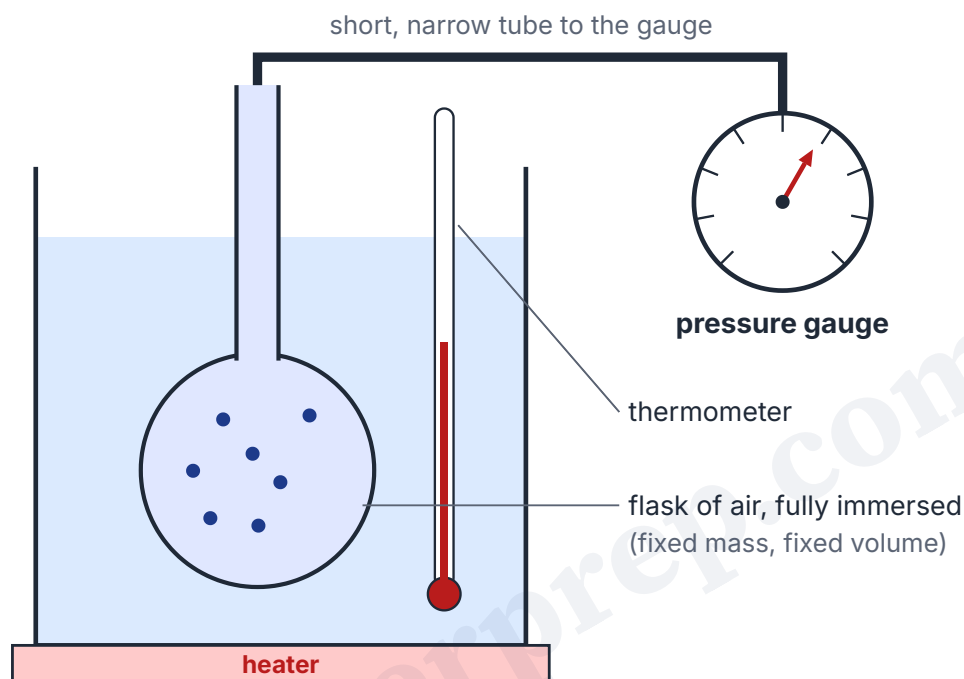
(a) Gradient = $\frac{\Delta V}{\Delta \theta} = \frac{33.7 - 24.6}{100 - 0} = \mathbf{0.091 \text{ cm}^3 \text{ } ^\circ\text{C}^{-1}}$ (the best-fit line through all six points gives $0.0911 \text{ cm}^3 \text{ } ^\circ\text{C}^{-1}$)

(b) The intercept on the volume axis is 24.6 cm^3 . At $V = 0$: $\theta = -\frac{24.6}{0.0911} = \mathbf{-270 \text{ } ^\circ\text{C}}$ (accept $-268 \text{ } ^\circ\text{C}$ to $-272 \text{ } ^\circ\text{C}$)

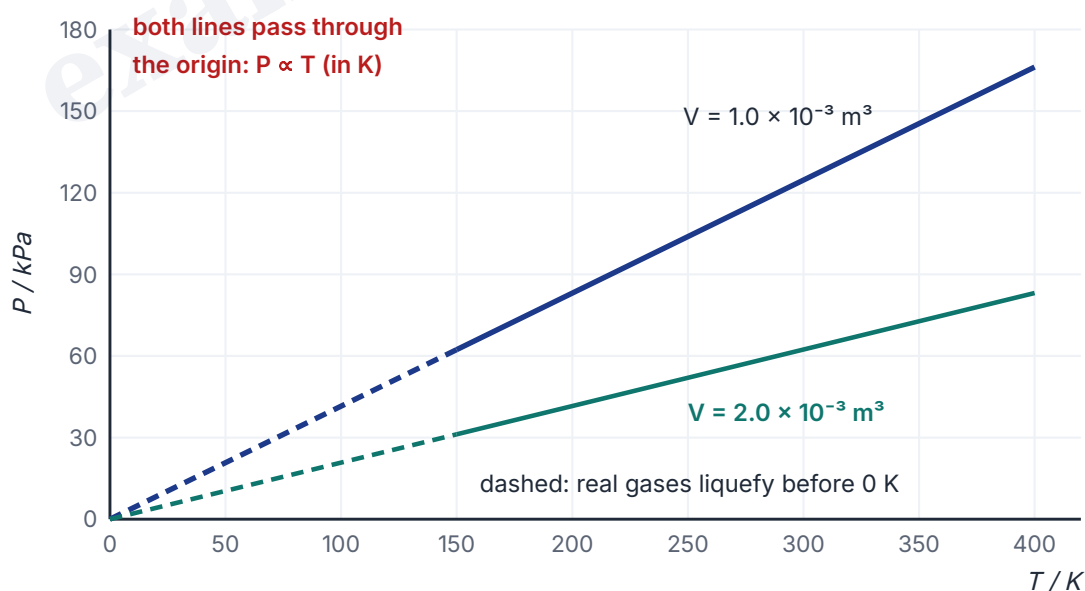
(c) For an ideal gas $V \propto T$ in kelvin, so $V = 0$ at $T = 0 \text{ K}$; the θ -intercept is therefore where absolute zero lies on the Celsius scale. The accepted value is $-273 \text{ } ^\circ\text{C}$, so this estimate is within about 1 %. A real gas liquefies (and the molecules have a volume of their own) long before 0 K , so the line is valid only as an **extrapolation** of the ideal-gas behaviour.

Constant volume: the pressure law

For a fixed amount of gas at constant volume, the pressure is directly proportional to the absolute (kelvin) temperature.



Pressure law apparatus. The flask is fully under water so all the air is at the bath temperature; the tube to the gauge is short so that only a tiny fraction of the air is outside the bath.



Pressure against kelvin temperature for 0.050 mol of gas held at two fixed volumes. Both lines pass through the origin; the gas in the smaller container is at the higher pressure at every temperature.

TRY IT YOURSELF

B.3 Gas Laws simulation: hold one variable fixed, change another, and watch the molecules and the pressure respond — then check each law against the graphs on this page.

How do the gas laws combine to give $PV/T = \text{constant}$?

For a fixed amount of an ideal gas, the product of pressure and volume divided by the kelvin temperature is constant.

$$\frac{PV}{T} = \text{constant} \quad \Rightarrow \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

This equation is in the data booklet. It follows from the empirical laws by taking the gas from state 1 (P_1, V_1, T_1) to state 2 (P_2, V_2, T_2) in two steps:

1. **Constant temperature** T_1 : change the pressure from P_1 to P_2 . The volume becomes V' . Boyle's law: $P_1 V_1 = P_2 V'$, so $V' = \frac{P_1 V_1}{P_2}$.
2. **Constant pressure** P_2 : change the temperature from T_1 to T_2 . The volume goes from V' to V_2 . Charles's law: $\frac{V'}{T_1} = \frac{V_2}{T_2}$.
3. Substitute for V' : $\frac{P_1 V_1}{P_2 T_1} = \frac{V_2}{T_2} \Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$.

Any two of the three laws are enough; the third then follows. Each single law is just this equation with one variable cancelled.

WORKED EXAMPLE 4 Medium · Show that

The air in a car tyre is at an absolute pressure of 2.3×10^5 Pa at 12°C . After a long drive the air is at 38°C and the tyre volume has increased by 2.0 %. No air escapes.

(a) Show that the pressure is now about 2.5×10^5 Pa. (b) A student calculates $2.3 \times 10^5 \times \frac{38}{12}$. Explain what is wrong.

(a) $T_1 = 285 \text{ K}$, $T_2 = 311 \text{ K}$, $V_2 = 1.020V_1$.

$$P_2 = P_1 \times \frac{T_2}{T_1} \times \frac{V_1}{V_2} = 2.3 \times 10^5 \times \frac{311}{285} \times \frac{1}{1.020} = 2.46 \times 10^5 \text{ Pa} \approx 2.5 \times 10^5 \text{ Pa}$$

(b) The student used Celsius temperatures, giving $7.3 \times 10^5 \text{ Pa}$ — more than three times the original pressure, and about three times the correct answer. The gas laws need kelvin, because only on the kelvin scale is the pressure proportional to temperature. (The student also ignored the 2.0 % increase in volume.)

What is the ideal gas equation?

The equation of state of an ideal gas relates its pressure, volume, kelvin temperature and the amount of gas: $PV = nRT = Nk_B T$.

$$PV = nRT$$

$$PV = Nk_B T$$

- P = pressure (Pa); V = volume (m^3); T = temperature (K)
- n = amount of substance (mol); R = molar gas constant = $8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
- N = number of molecules; k_B = Boltzmann constant = $1.38 \times 10^{-23} \text{ J K}^{-1}$

Both forms, and both constants, are in the data booklet. The constant in $PV/T = \text{constant}$ is proportional to the amount of gas: experiment shows it is nR for any gas that behaves ideally. Since $n = N/N_A$,

$$nR = \frac{N}{N_A} R = Nk_B \quad \text{where} \quad k_B = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}$$

The Boltzmann constant is the gas constant *per molecule*. Notice that PV has the unit of energy: $\text{Pa} \times \text{m}^3 = \text{N m}^{-2} \times \text{m}^3 = \text{N m} = \text{J}$.

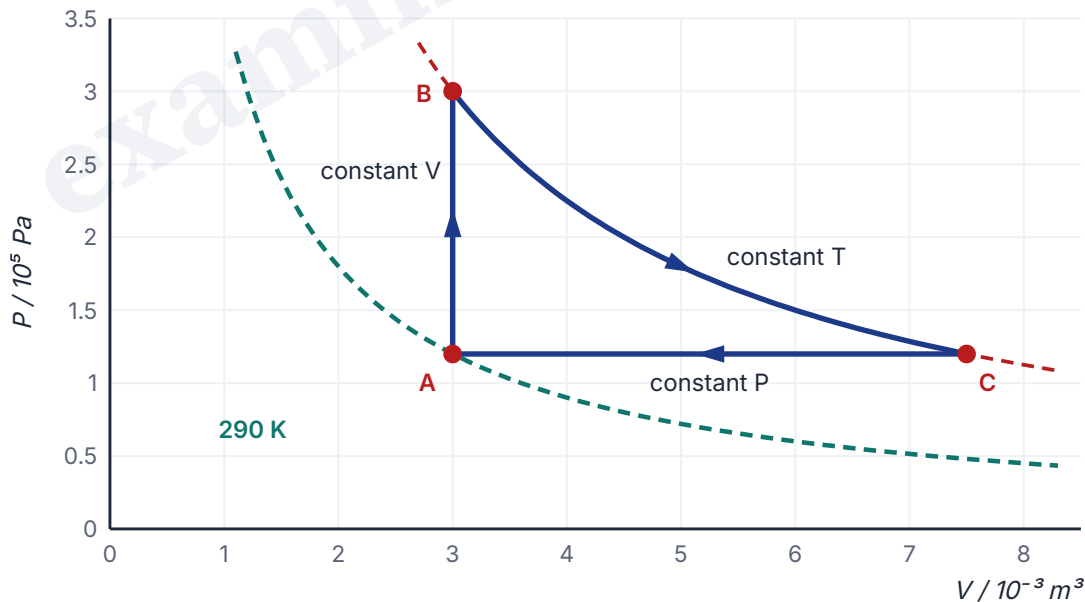
- Equal volumes of different ideal gases at the same temperature and pressure contain **the same number of molecules** — the type of gas does not appear in $PV = Nk_B T$.

- Density form (not in the data booklet — derive it from $PV = nRT$ when needed): $\rho = \frac{nM}{V} = \frac{PM}{RT}$. At constant pressure, doubling the kelvin temperature halves the density.

Showing changes of state on a pressure–volume diagram

Every equilibrium state of a fixed amount of gas is a point on a P – V diagram, and $T = PV/nR$ can be read from that point. A change of state is a line:

Change	Shape on a P–V diagram	What happens to T
constant volume (isovolumetric)	vertical line	$T \propto P$: rises as P rises
constant pressure (isobaric)	horizontal line	$T \propto V$: rises as V increases
constant temperature (isothermal)	curve along an isotherm, $PV = \text{constant}$	unchanged



The cycle in worked example 6: heating at constant volume ($A \rightarrow B$), expansion at constant temperature along the isotherm through B ($B \rightarrow C$), and cooling at constant pressure back to A.

Dashed curves are the isotherms through A (290 K) and through B.

HL students meet the same diagrams again in B.4 Thermodynamics, where the area under the line gives the work done by the gas.

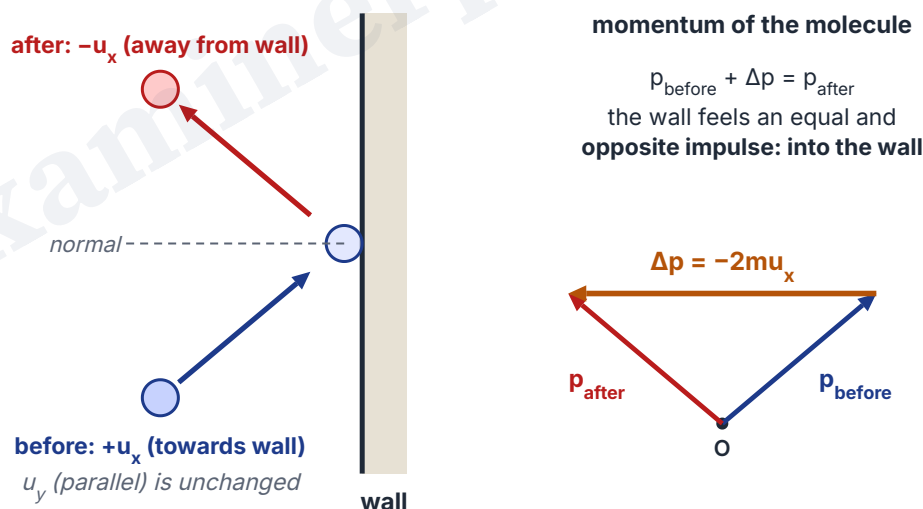
How do molecular collisions produce gas pressure?

Gas pressure is caused by molecules colliding with the walls of the container. Each collision changes a molecule's momentum, so the wall exerts a force on the molecule and, by Newton's third law, the molecule exerts an equal and opposite force on the wall. The total force per unit area is the pressure.

Step 1 — one collision

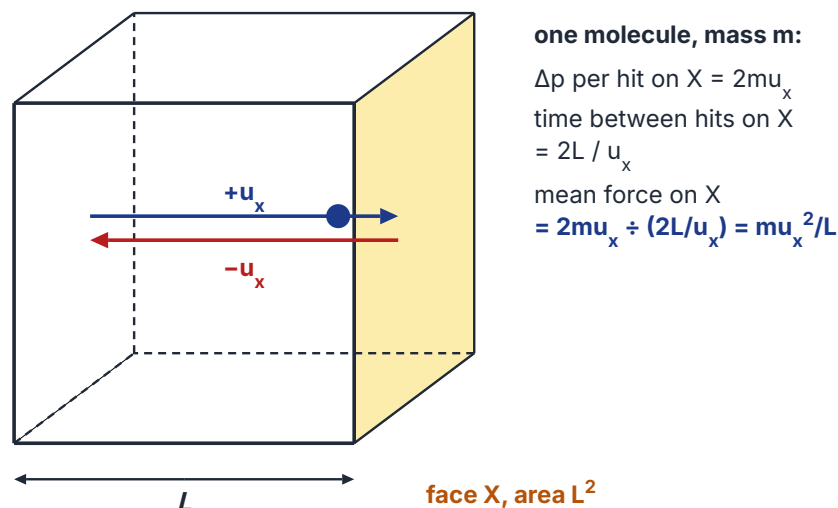
A molecule of mass m hits a wall with velocity component u_x perpendicular to the wall. The collision is elastic, so it rebounds with $-u_x$; the parallel component is unchanged.

$$\Delta p = m(-u_x) - mu_x = -2mu_x$$



Only the component of velocity perpendicular to the wall is reversed. The change in momentum of the molecule, $2mu_x$, points away from the wall; the wall receives an equal impulse into it.

Step 2 — one molecule in a cube



A molecule moving between opposite faces of a cube of side L . It returns to face X after travelling $2L$ in the x -direction.

The molecule hits face X once every $\Delta t = 2L/u_x$. By Newton's second law (A.2), the average force on the face is the rate of change of momentum:

$$F = \frac{2mu_x}{2L/u_x} = \frac{mu_x^2}{L}$$

Step 3 — all N molecules

Adding the forces from all N molecules and dividing by the area L^2 :

$$P = \frac{F_{\text{total}}}{L^2} = \frac{m}{L^3} (u_{x1}^2 + u_{x2}^2 + \dots) = \frac{Nm\overline{u_x^2}}{V}$$

The motion is random, so no direction is special: $\overline{u_x^2} = \overline{u_y^2} = \overline{u_z^2}$. Since $v^2 = u_x^2 + u_y^2 + u_z^2$, $\overline{u_x^2} = \frac{1}{3}\overline{v^2}$. The total mass of gas is Nm , so $Nm/V = \rho$:

$$P = \frac{1}{3}\rho\overline{v^2}$$

- P = pressure (Pa); ρ = density of the gas (kg m^{-3})
- $\overline{v^2}$ = mean of the squares of the molecular speeds, the **mean square speed** ($\text{m}^2 \text{s}^{-2}$)
- $v_{\text{rms}} = \sqrt{\overline{v^2}}$ is the root mean square speed — the measure of average translational speed used with this equation

The equation is in the data booklet (as $P = \frac{1}{3}\rho v^2$, where v^2 is understood as the mean square speed). The derivation itself is not; you should be able to explain its key steps in words: momentum change per collision, rate of collisions, force = rate of change of momentum, pressure = force ÷ area, and the factor $\frac{1}{3}$ from three equivalent directions.

The link to temperature

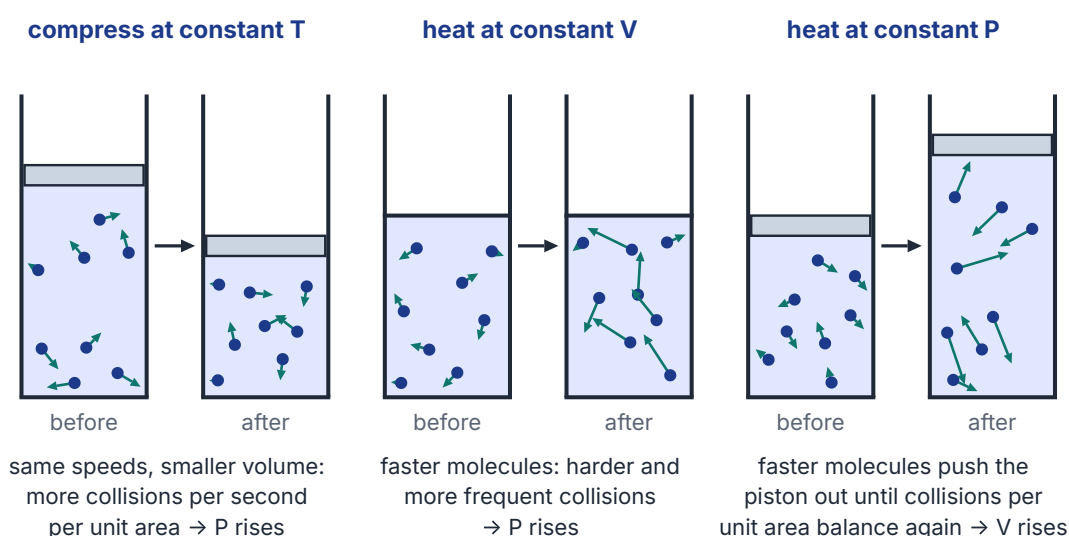
Write $P = \frac{1}{3}\rho \overline{v^2}$ as $PV = \frac{1}{3}Nm\overline{v^2}$ and compare it with $PV = Nk_B T$:

$$\frac{1}{3}Nm\overline{v^2} = Nk_B T \quad \Rightarrow \quad \bar{E}_k = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T$$

This is the B.1 result: the kelvin temperature tells you the mean translational kinetic energy per molecule. At the same temperature, molecules of every gas have the same average kinetic energy, so lighter molecules move faster: $v_{\text{rms}} = \sqrt{3k_B T/m}$ (not in the data booklet — it follows from the line above). For nitrogen at 300 K, $v_{\text{rms}} \approx 520 \text{ m s}^{-1}$.

Explaining the gas laws with molecules

You need a **qualitative** explanation of each law. Every good answer has the same chain: what happens to the molecules → frequency and/or size of the momentum change at the walls → force on the walls → pressure.



Molecular explanations of the three gas laws. Arrow length shows molecular speed: unchanged at constant temperature, longer when the gas is heated.

Change	Molecular explanation
Compress at constant T (Boyle)	Same average speed and kinetic energy. The molecules have less distance to travel between walls, so they strike each unit area of wall more often . Same momentum change per collision, more collisions per second $\rightarrow$ larger force per unit area $\rightarrow$ higher pressure.
Heat at constant V (pressure law)	Higher temperature $\rightarrow$ higher average kinetic energy and speed. Each collision gives a larger momentum change , and collisions are more frequent $\rightarrow$ larger force on the walls $\rightarrow$ higher pressure.
Heat at constant P (Charles)	Faster molecules would increase the pressure, so the gas pushes the piston out. As the volume grows, collisions per unit area per second fall until the force per unit area is back to its original value $\rightarrow$ larger volume at the same pressure.

WORKED EXAMPLE 5 Hard · Determine

A balloon contains 0.020 m^3 of helium (a monatomic gas, molar mass 4.0 g mol^{-1}) at a pressure of $1.10 \times 10^5 \text{ Pa}$ and a temperature of 290 K . Treat the helium as an ideal gas.

Determine (a) the amount of helium and the number of atoms, (b) the density of the helium, (c) the root mean square speed of the atoms, and (d) show that your answer to (c) agrees with $\bar{E}_k = \frac{3}{2}k_B T$.

$$\text{(a) } n = \frac{PV}{RT} = \frac{1.10 \times 10^5 \times 0.020}{8.31 \times 290} = \mathbf{0.913 \text{ mol}}; N = nN_A = 0.913 \times 6.02 \times 10^{23} = \mathbf{5.50 \times 10^{23}}$$

$$\text{(b) Mass} = nM = 0.913 \times 4.0 \times 10^{-3} = 3.65 \times 10^{-3} \text{ kg}; \rho = \frac{3.65 \times 10^{-3}}{0.020} = \mathbf{0.183 \text{ kg m}^{-3}}$$

$$\text{(c) } P = \frac{1}{3}\rho\overline{v^2} \Rightarrow \overline{v^2} = \frac{3P}{\rho} = \frac{3 \times 1.10 \times 10^5}{0.1826} = 1.81 \times 10^6 \text{ m}^2 \text{ s}^{-2} \text{ (keep the unrounded density here; } 0.183 \text{ would give } 1.80 \times 10^6)$$

$$v_{\text{rms}} = \sqrt{1.81 \times 10^6} = \mathbf{1.34 \times 10^3 \text{ m s}^{-1}}$$

$$\text{(d) Mass of one atom } m = \frac{4.0 \times 10^{-3}}{6.02 \times 10^{23}} = 6.64 \times 10^{-27} \text{ kg.}$$

$$\frac{1}{2}m\overline{v^2} = \frac{1}{2} \times 6.64 \times 10^{-27} \times 1.81 \times 10^6 = 6.01 \times 10^{-21} \text{ J}$$

$$\frac{3}{2}k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 290 = 6.00 \times 10^{-21} \text{ J } \checkmark \text{ — the two agree (to within rounding)}$$

What is the internal energy of an ideal gas?

The internal energy of an ideal monatomic gas is the total random translational kinetic energy of its particles: $U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$.

$$U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$$

- U = internal energy (J); N , n , k_B , R , T as before
- Both forms are in the data booklet.

Internal energy is, in general, the total kinetic energy plus intermolecular potential energy of the particles (B.1). For an **ideal** gas:

- there are no intermolecular forces, so the **potential energy is zero**;
- a **monatomic** gas (He, Ne, Ar) has only translational kinetic energy, $\frac{3}{2}k_B T$ per atom on average;
- so $U = N \times \frac{3}{2}k_B T$. Using $Nk_B = nR$ gives the molar form.

Consequences you will be tested on:

- For a fixed amount of ideal gas, U depends **only on temperature** — not on pressure or volume. In an isothermal change $\Delta U = 0$.
- $U \propto T$ in kelvin: doubling T doubles U . Two samples at the same temperature have internal energies in the ratio of their numbers of particles.
- Combining with $PV = nRT$: $U = \frac{3}{2}PV$. The internal energy can be read straight from a point on a P - V diagram.

WORKED EXAMPLE 6 Hard · Determine (P-V diagram)

A monatomic ideal gas is taken round the cycle on the P - V diagram above. At A the pressure is 1.2×10^5 Pa, the volume $3.0 \times 10^{-3} \text{ m}^3$ and the temperature 290 K. The gas is heated at constant volume to B, where the pressure is 3.0×10^5 Pa. It then expands at constant temperature to C, where the pressure is again 1.2×10^5 Pa, and is finally cooled at constant pressure back to A.

Determine (a) the amount of gas, (b) the temperature at B, (c) the volume at C, (d) the change in internal energy from A to B, and (e) the change in internal energy for the complete cycle.

$$\text{(a) } n = \frac{P_A V_A}{RT_A} = \frac{1.2 \times 10^5 \times 3.0 \times 10^{-3}}{8.31 \times 290} = \mathbf{0.149 \text{ mol}}$$

$$\text{(b) Constant volume: } T_B = T_A \frac{P_B}{P_A} = 290 \times \frac{3.0 \times 10^5}{1.2 \times 10^5} = \mathbf{725 \text{ K}}$$

$$\text{(c) Constant temperature: } V_C = \frac{P_B V_B}{P_C} = \frac{3.0 \times 10^5 \times 3.0 \times 10^{-3}}{1.2 \times 10^5} = \mathbf{7.5 \times 10^{-3} \text{ m}^3}$$

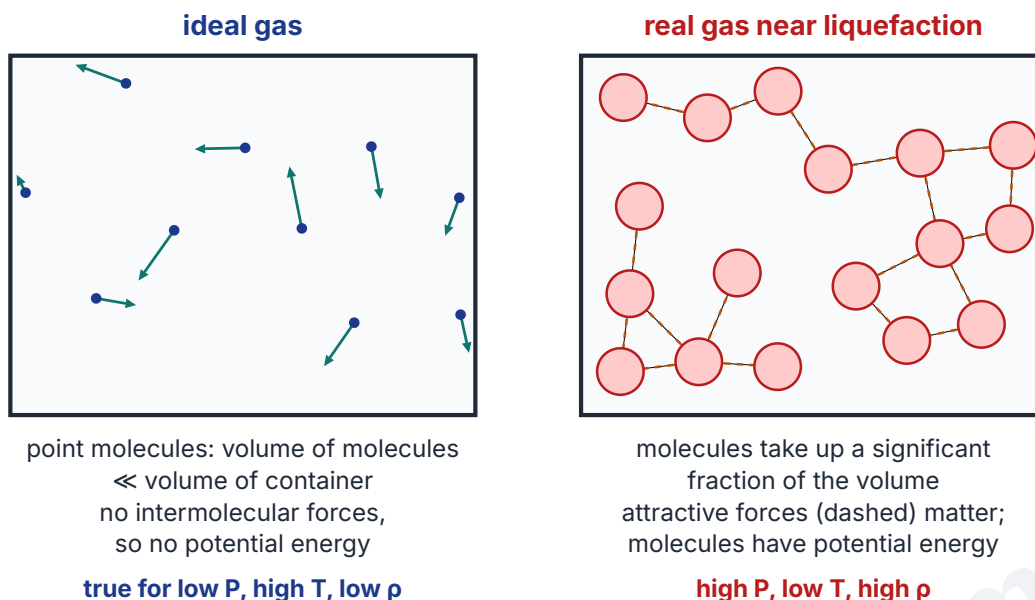
$$\text{(d) } \Delta U = \frac{3}{2} n R \Delta T = \frac{3}{2} \times 0.149 \times 8.31 \times (725 - 290) = \mathbf{810 \text{ J}}$$

Check with $U = \frac{3}{2} PV$: $U_A = \frac{3}{2} (1.2 \times 10^5) (3.0 \times 10^{-3}) = 540 \text{ J}$, $U_B = 1350 \text{ J}$, difference 810 J ✓

(e) The gas returns to the same state A, the same temperature, so $\Delta U = 0$ for the cycle. (B→C is isothermal, so $\Delta U_{BC} = 0$; C→A must therefore have $\Delta U = -810 \text{ J}$.)

When does a real gas behave like an ideal gas?

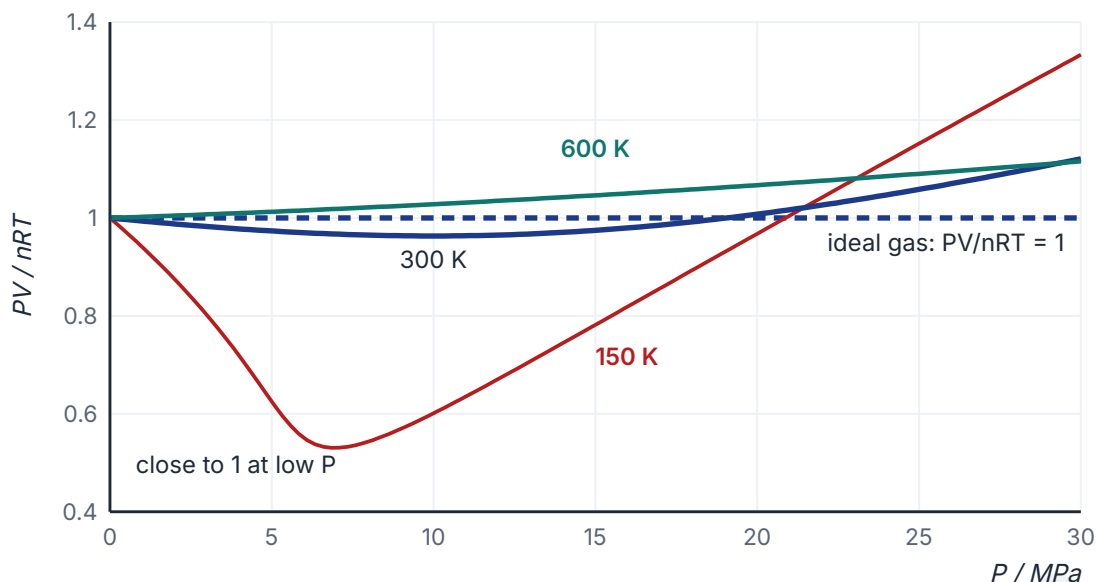
A real gas approximates an ideal gas at low pressure, high temperature and low density — conditions in which its molecules are far apart and moving fast, well away from the conditions where the gas would liquefy.



Left: the ideal-gas picture — negligible molecular volume, no intermolecular forces. Right: a real gas at high pressure and low temperature, where molecules are close together, occupy a noticeable fraction of the space and attract one another.

Condition	Why it makes the gas closer to ideal
Low pressure / low density	Few molecules per unit volume, so they are far apart: the volume of the molecules is negligible compared with the container, and intermolecular forces (which act only over short distances) are negligible.
High temperature	The molecules' kinetic energy is much larger than the potential energy of attraction between them, so the attractions hardly affect their motion. The gas is far above the temperature at which it would condense.

A real gas departs from ideal behaviour when it is compressed to high density or cooled towards liquefaction. Intermolecular attractions then pull molecules back as they approach a wall, reducing the pressure, and the molecules' own volume reduces the space they can move in. Ideal-gas extrapolations such as $V \rightarrow 0$ at 0 K never happen for a real gas: it liquefies first.



PV/nRT against pressure for nitrogen at three temperatures, computed from a simple real-gas model (the model itself is not on the syllabus — only the trend is). An ideal gas would give exactly 1. All three curves approach 1 at low pressure, and the departures are largest at the lowest temperature.

Under everyday conditions (about 10^5 Pa and 300 K), air, nitrogen, oxygen, helium and similar gases obey $PV = nRT$ to within a fraction of a per cent — which is why the ideal-gas model is used throughout this course.

Exam tips for B.3

EXAM TIP 1 — KELVIN, EVERY TIME

Convert every temperature to kelvin before it goes into a gas equation, including ratios such as $P_1/T_1 = P_2/T_2$. Temperature *differences* are the same in $^{\circ}\text{C}$ and K, so ΔT in $\Delta U = \frac{3}{2}nR\Delta T$ may be taken from either scale — but absolute values may not.

EXAM TIP 2 — SI UNITS INSIDE $PV = nRT$

In $PV = nRT$ and $PV = Nk_B T$, volume must be in m^3 and pressure in Pa (not cm^3 , litres or kPa). In $P = \frac{1}{3}\rho\overline{v^2}$, density must be in kg m^{-3} , so

convert a molar mass in g mol^{-1} to kg mol^{-1} . These slips are the commonest reason a correct method loses its final mark.

EXAM TIP 3 — THE FULL CHAIN FOR PRESSURE

"Explain, in terms of molecules, why the pressure increases" needs a complete chain: molecules collide with the **walls** → momentum changes → force = rate of change of momentum → force per unit area is pressure. Then say *what changes*: more frequent collisions (smaller volume) and/or a larger momentum change per collision (higher speed). "The molecules collide more" on its own scores little, and collisions between molecules are not the cause of pressure.

EXAM TIP 4 — STATE WHAT IS HELD CONSTANT

When you use or justify a proportionality, name the constants: "for a fixed mass of gas at constant temperature, $P \propto 1/V$ ". When a question gives a rearranged equation and asks why a graph is a straight line through the origin, show it is of the form $y = kx$ and state that every quantity in k is constant.

EXAM TIP 5 — IDEAL VERSUS REAL: GIVE THE REASON

A question on when a real gas behaves ideally needs the conditions **and** the reason: low pressure / low density / high temperature, so the molecules are far apart (volume negligible, intermolecular forces negligible) or so fast that attractions have no significant effect. For "state one assumption of the kinetic model", quote a precise assumption such as "collisions are elastic" — not "the gas is ideal".

Common misconceptions

Misconception	Why it is wrong	Correct idea
Gas pressure is caused by molecules colliding with each other.	Collisions between molecules exert no net force on the container.	Pressure comes from momentum changes in collisions with the walls .
Heating a gas from 20 °C to 40 °C at constant volume doubles its pressure.	Pressure is proportional to <i>kelvin</i> temperature.	293 K → 313 K multiplies the pressure by only 1.07.
At the same temperature, heavier molecules have more kinetic energy.	$\bar{E}_k = \frac{3}{2}k_B T$ contains no mass.	Same T means same average kinetic energy; heavier molecules simply move more slowly.
All the molecules in a gas move at the same speed.	Collisions constantly redistribute energy at random.	There is a range of speeds; v_{rms} and $\bar{E}_k$ are averages.
Compressing an ideal gas at constant temperature increases its internal energy.	An ideal gas has no intermolecular potential energy.	$U = \frac{3}{2}nRT$ depends only on T : isothermal means $\Delta U = 0$.
A real gas really does shrink to zero volume at -273 °C.	Real molecules have volume and attract each other.	Real gases liquefy (and solidify) first; zero volume at 0 K is an ideal-gas extrapolation.

How this topic connects

Gas pressure as molecular collisions: Newton's laws inside a container

The pressure of a gas is pure A.2 mechanics applied to molecules. Each collision with a wall changes a molecule's momentum by $2mu_x$; Newton's second law turns the rate of change of momentum into a force, and Newton's third law says the wall feels an equal and opposite force. Dividing by area gives pressure. Thermodynamic quantities — pressure, temperature, internal energy — are

therefore averages of mechanical quantities over enormous numbers of particles, and the result $P = \frac{1}{3}\rho\overline{v^2}$ is the bridge between them.

Temperature as molecular kinetic energy: the microscopic story behind the gas laws

Comparing the kinetic-theory result $PV = \frac{1}{3}Nm\overline{v^2}$ with the empirical law $PV = Nk_{\text{B}}T$ shows that $\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_{\text{B}}T$: temperature measures average molecular kinetic energy. Every gas law then has a molecular reading — heating raises molecular speeds, so at constant volume the pressure rises (pressure law) and at constant pressure the gas must expand (Charles's law); at constant temperature the kinetic energy is unchanged, so only the collision rate per unit area changes with volume (Boyle's law).

The ideal gas and other useful simplifications in physics

The ideal gas ignores molecular size and attractions yet predicts real-gas behaviour well at low density. Physics uses the same strategy everywhere: point masses and frictionless surfaces in mechanics (A.1–A.3), the rigid body (A.4, HL), the black body and the simple energy-balance model of the Earth (B.2), the nuclear model of the atom (E.1) and the model of a star as a sphere of hot gas (E.5). A good model keeps only what matters for the question, and its limits — here, high pressure and low temperature — are as important to state as its predictions.

Tools & inquiry — investigating the gas laws

- **Boyle's law:** a sealed syringe and pressure sensor. Change the volume in small steps and wait after each one, so the air returns to room temperature. Plot P against $1/V$ (or $1/P$ against V) to get a straight line.
- **Systematic error from the tubing:** the air inside the connecting tube is not included in the syringe reading, so every measured volume is too small by the same amount. A graph of V against $1/P$ then has a *negative* intercept on the V -axis equal to that hidden volume — an example of how a graph reveals a systematic error that repeated readings cannot.

- **Charles's law and the pressure law:** heat the water bath slowly and stir it, and take each reading only when the temperature is steady, so the gas and the thermometer are at the same temperature. Check that no gas can leak, because the laws need a fixed mass.
- **Extrapolation:** finding absolute zero from data taken between 0 °C and 100 °C means extending a line far beyond the measurements. Small uncertainties in the gradient produce a large uncertainty in the intercept — try drawing the steepest and shallowest acceptable lines.
- **NOS:** the gas laws were empirical relationships discovered by Boyle, Charles and Gay-Lussac; kinetic theory came later and explained them. A theory that reproduces existing laws and makes new predictions (such as v_{rms}) gains strong support.

Key equations

Equation	Use	In data booklet?
$P = \frac{F}{A}$	pressure (F perpendicular to the surface)	yes
$n = \frac{N}{N_A}$	amount of substance	yes
$T/\text{K} = \theta/^{\circ}\text{C} + 273$	converting Celsius to kelvin	yes
$\frac{PV}{T} = \text{constant}$	fixed amount of gas changing state (includes Boyle, Charles and the pressure law)	yes
$PV = nRT = Nk_B T$	ideal gas equation	yes
$k_B = R/N_A$	link between the constants	no — learn it (both constants are given)
$P = \frac{1}{3}\rho\overline{v^2}$	pressure from molecular motion	yes
$\bar{E}_k = \frac{3}{2}k_B T$	average kinetic energy of a molecule (B.1)	yes
$U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$	internal energy of an ideal monatomic gas	yes
$\Delta p = -2mu_x$	momentum change of a molecule bouncing elastically off a wall	no — derive it

Check your understanding

1. A nitrogen molecule of mass 4.65×10^{-26} kg moving at 500 m s^{-1} strikes a wall at 30° to the normal and rebounds elastically. Calculate the magnitude of its change in momentum.

Only the perpendicular component reverses: $u_x = 500 \cos 30^\circ = 433 \text{ m s}^{-1}$.
 $|\Delta p| = 2mu_x = 2 \times 4.65 \times 10^{-26} \times 433 = 4.03 \times 10^{-23} \text{ kg m s}^{-1}$,
 directed perpendicular to the wall, away from it.

2. A sealed can of air is heated. Explain, in terms of molecules, why the pressure inside increases.

The volume is fixed. Heating raises the average kinetic energy, so the molecules move faster. Each collision with the wall gives a larger change in momentum, and collisions with each unit area of wall happen more often. The rate of change of momentum at the walls — the force — increases, so the force per unit area, the pressure, increases.

3. Helium (molar mass 4.0 g mol^{-1}) and neon (20 g mol^{-1}) are at the same temperature. Calculate $v_{\text{rms,He}}/v_{\text{rms,Ne}}$.

Same $T \rightarrow$ same $\frac{1}{2}m\overline{v^2}$, so $v_{\text{rms}} \propto 1/\sqrt{m}$. Ratio = $\sqrt{20/4.0} = \sqrt{5} = 2.24$.
 The helium atoms are about 2.2 times faster.

4. A fixed amount of ideal gas is heated at constant pressure from 300 K to 600 K. What happens to its density?

$V \propto T$ at constant P , so the volume doubles while the mass is unchanged. The density halves (factor 0.50). Equivalently, $\rho = PM/RT \propto 1/T$.

5. A gas in a rigid container is warmed from 20 °C to 40 °C. By what factor does its pressure change?

$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{313}{293} = 1.07$. The pressure rises by about 7 %, not by a factor of 2.

Practise B.3 with exam-style questions

Original Paper 1A, 1B and 2 questions on the gas laws and kinetic theory, with full mark schemes. Tick the questions you want and print them as your own test.

[SL B.3 questions](#)

[HL B.3 questions](#)

Revise and explore

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SL students: B.4 is HL only. Continue with [B.5 Current and circuits](#) — [SL B.5 questions](#).