

THEME B · THE PARTICULATE NATURE OF MATTER

HL ONLY

≈ 35 min read

IB Physics B.4 Thermodynamics — Notes (HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). **B.4 is entirely Additional Higher Level content** — there is no SL material in this sub-topic, and it is examined only on HL papers. It builds directly on B.1 (internal energy and temperature) and B.3 (the ideal gas and its P–V diagrams), and in quantitative problems the system is always a monatomic ideal gas.



HL B.4 questions



Flashcards



Heat engines & Carnot simulation

What this page covers

- ✓ The first law of thermodynamics, $Q = \Delta U + W$, as energy conservation for a closed system, with the Clausius sign convention
- ✓ Work done by or on a gas when its volume changes, $W = P\Delta V$, and as the area under a P–V graph
- ✓ The change in internal energy of a monatomic ideal gas, $\Delta U = \frac{3}{2}Nk_B\Delta T = \frac{3}{2}nR\Delta T$
- ✓ Entropy as a measure of the disorder of the particles of a system
- ✓ Entropy from macroscopic quantities, $\Delta S = \frac{\Delta Q}{T}$, and from microstates, $S = k_B \ln \Omega$
- ✓ The second law: the entropy of an isolated system, and the limits it places on what processes can happen
- ✓ Why real processes are irreversible, so the entropy of a real isolated system increases
- ✓ How entropy can fall locally in a non-isolated system while the surroundings gain at least as much
- ✓ Isovolumetric, isobaric, isothermal and adiabatic processes, and $PV^{5/3} = \text{constant}$ for adiabatic changes

- ✓ Cyclic processes, heat engines and their efficiency $\eta = \frac{\text{useful work}}{\text{input energy}}$
- ✓ The Carnot cycle and the maximum efficiency $\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$

In one minute

- **First law:** $Q = \Delta U + W$. Q is thermal energy supplied to the gas; W is work done by the gas. Expansion: $W > 0$. Compression: $W < 0$.
- For a monatomic ideal gas, U depends only on temperature: $\Delta U = \frac{3}{2}nR\Delta T$. No temperature change, no change in internal energy.
- Work is the **area under** a P–V graph; for a cycle, the net work is the **area enclosed**.
- Four processes, each with one thing fixed: isovolumetric ($W = 0$), isobaric ($W = P\Delta V$), isothermal ($\Delta U = 0$), adiabatic ($Q = 0$, $PV^{5/3} = \text{constant}$).
- **Entropy** measures disorder: $\Delta S = \Delta Q/T$ and $S = k_B \ln \Omega$. The entropy of an isolated system never decreases; real (irreversible) processes increase it.
- A heat engine takes Q_h from a hot reservoir, does work W and must reject Q_c to a cold one. No engine can beat the **Carnot** efficiency $1 - T_c/T_h$.

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| 2. Work done by a gas | 8. The Carnot cycle |
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| 4. The four processes | 10. Common misconceptions |
| 5. Entropy | 11. How this topic connects |
| 6. The second law | 12. Check your understanding |

What is the first law of thermodynamics?

Thermodynamics describes a **system** — here, a sample of gas — and the energy that crosses its boundary. Two kinds of system matter in B.4:

- A **closed system**: no mass can enter or leave, but energy can be transferred in either direction, as thermal energy (heat) or as work. Gas trapped in a cylinder by a piston is a closed system.
- An **isolated system**: neither mass nor energy can enter or leave. Its total energy is fixed.

The first law of thermodynamics states that the thermal energy supplied to a closed system equals the increase in its internal energy plus the work done by the system: $Q = \Delta U + W$. It is the principle of conservation of energy applied to a closed system.

$$Q = \Delta U + W$$

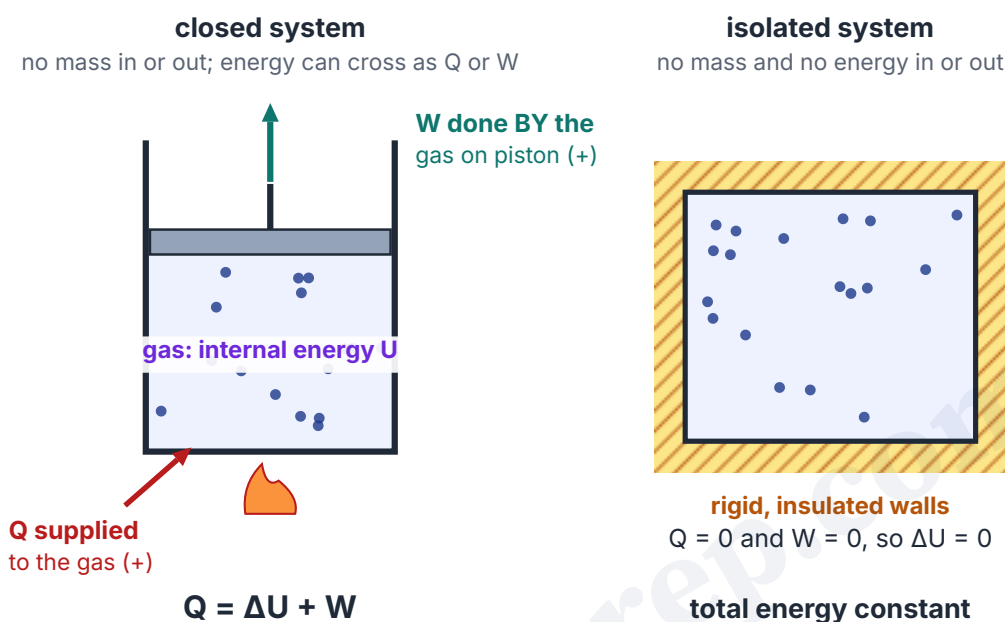
- Q = resultant thermal energy **supplied to** the system (J)
- ΔU = change in internal energy of the system (J)
- W = resultant work done **by** the system (J)

This equation is in the data booklet. IB problems use the **Clausius sign convention**, and every sign error in B.4 comes from forgetting it:

Quantity	Positive (+) when...	Negative (–) when...
Q	thermal energy flows into the gas	thermal energy flows out of the gas
W	the gas does work on the surroundings (it expands)	work is done on the gas (it is compressed)
ΔU	internal energy increases (temperature rises)	internal energy decreases (temperature falls)

Rearranged, $\Delta U = Q - W$: the internal energy goes up by whatever thermal energy comes in, minus whatever the gas spends doing work. Recall from B.1 that

internal energy is the total kinetic and potential energy of the particles; for an ideal gas there are no intermolecular forces, so U is entirely the random kinetic energy of the particles.



A closed system can exchange energy with its surroundings as heat Q and work W ; an isolated system cannot exchange energy at all. Arrows show the positive directions in the Clausius convention.

WORKED EXAMPLE 1 Easy · Calculate

0.20 mol of a monatomic ideal gas is held in a cylinder. During a process, 350 J of thermal energy is supplied to the gas while the piston does 120 J of work compressing it.

(a) State the values of Q and W in the first law. (b) Calculate the change in internal energy of the gas. (c) Determine the change in temperature of the gas.

(a) Thermal energy is supplied to the gas: $Q = +350$ J. Work is done *on* the gas, so the work done *by* the gas is negative: $Q = +350$ J, $W = -120$ J

(b) $\Delta U = Q - W = 350 - (-120) = +470$ J

(c) $\Delta U = \frac{3}{2}nR\Delta T \Rightarrow \Delta T = \frac{2\Delta U}{3nR} = \frac{2 \times 470}{3 \times 0.20 \times 8.31} = 190$ K (188.5 K)
— a rise, since $\Delta U > 0$.

Both energy inputs raise the internal energy. The most common wrong answer, 230 J, comes from writing $W = +120 \text{ J}$.

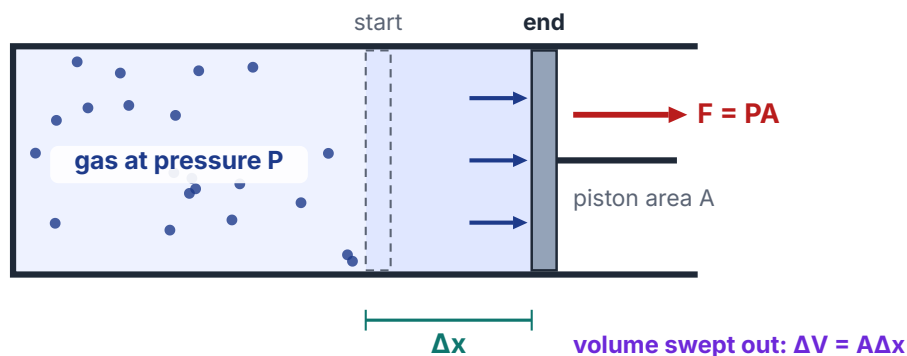
How do you calculate the work done by a gas?

A gas at pressure P pushes on a piston of area A with a force $F = PA$. If the piston moves out through a small distance Δx while the pressure stays constant, the work done by the gas is $W = F\Delta x = PA\Delta x$. Since $A\Delta x$ is the change in volume:

When the boundary of a closed system moves, the work done by the system is the product of its pressure and its change in volume: $W = P\Delta V$ (for constant pressure).

$$W = P\Delta V$$

- W = work done by the gas (J)
- P = pressure of the gas (Pa), constant during the change
- ΔV = change in volume, final – initial (m^3)
- This equation is in the data booklet. The sign comes out automatically: expansion ($\Delta V > 0$) gives $W > 0$; compression gives $W < 0$.
- Convert volumes to m^3 : $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$, $1 \text{ dm}^3 = 1 \text{ litre} = 10^{-3} \text{ m}^3$. Then $\text{Pa} \times \text{m}^3 = \text{N m}^{-2} \times \text{m}^3 = \text{N m} = \text{J}$.
- **When the pressure is not constant**, the work done is the **area under the P–V graph** between the initial and final volumes. Count squares, or split the area into rectangles and triangles. (You are not expected to integrate.)
- Work depends on the **path** between two states, not just on the start and end points — different paths on a P–V diagram enclose different areas.



$$W = F\Delta x = PA\Delta x = P\Delta V$$

expansion: $\Delta V > 0$, $W > 0$ (work done by the gas)

compression: $\Delta V < 0$, $W < 0$ (work done on the gas)

A gas pushing a piston of area A through a distance Δx sweeps out a volume $\Delta V = A\Delta x$ and does work $W = P\Delta V$ on the surroundings.

How is internal energy related to temperature?

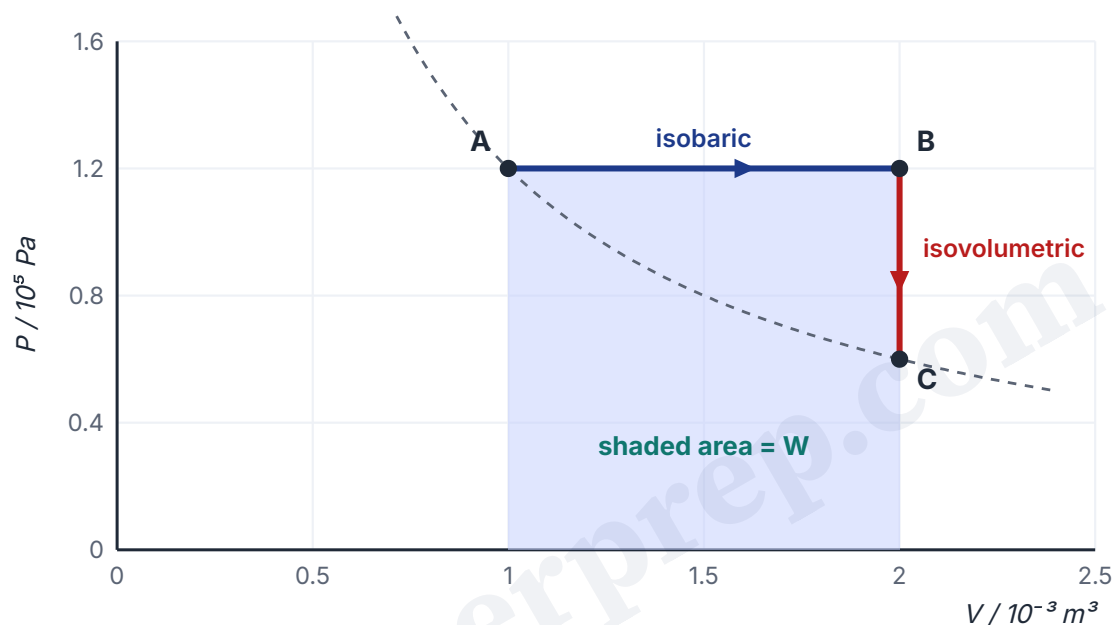
In B.3 you met $U = \frac{3}{2}Nk_B T = \frac{3}{2}nRT$ for a monatomic ideal gas. Its particles have no potential energy, and the mean kinetic energy of a particle is proportional to the absolute temperature, so the internal energy changes only when the temperature changes.

The change in internal energy of a monatomic ideal gas is proportional to its change in absolute temperature: $\Delta U = \frac{3}{2}Nk_B \Delta T = \frac{3}{2}nR\Delta T$.

$$\Delta U = \frac{3}{2}Nk_B \Delta T = \frac{3}{2}nR\Delta T$$

- N = number of particles; k_B = Boltzmann constant, $1.38 \times 10^{-23} \text{ J K}^{-1}$
- n = amount of substance (mol); R = gas constant, $8.31 \text{ J K}^{-1} \text{ mol}^{-1}$
- ΔT = change in temperature (K; a change of 1 K equals a change of 1 °C)
- This equation is in the data booklet. It holds for **any** process of a monatomic ideal gas, whatever the pressure or volume does.
- Combining with $PV = nRT$ gives a useful shortcut (not in the data booklet): $\Delta U = \frac{3}{2}\Delta(PV)$. You can find ΔU straight from a P–V graph without knowing n or T .

- U is a **function of state**: it depends only on where the gas is on the P–V diagram, not on how it got there. Q and W are not — they depend on the path. Over a complete cycle, $\Delta U = 0$.
- Any two states on the same **isotherm** (curve of constant T , where PV is constant) have the same internal energy.



Isobaric expansion $A \rightarrow B$ followed by isovolumetric cooling $B \rightarrow C$. The shaded area under $A \rightarrow B$ is the work done by the gas. The dashed curve is the isotherm through A.

WORKED EXAMPLE 2 Medium · Determine (graph)

0.048 mol of a monatomic ideal gas is taken from state A to state B at a constant pressure of 1.2×10^5 Pa, and then at constant volume from B to state C, where the pressure is 0.60×10^5 Pa, as shown in the P–V diagram above.

- (a) Calculate the temperatures at A and B. (b) For the change $A \rightarrow B$, determine the work done by the gas, the change in internal energy and the thermal energy supplied. (c) For $B \rightarrow C$, determine Q , W and ΔU . (d) Comment on the change in internal energy from A to C.

(a) $T_A = \frac{P_A V_A}{nR} = \frac{1.2 \times 10^5 \times 1.0 \times 10^{-3}}{0.048 \times 8.31} = \mathbf{300\text{ K}}$ (301 K). The volume doubles at constant pressure, so $T_B = 2T_A = \mathbf{600\text{ K}}$ (602 K).

(b) $W = P\Delta V = 1.2 \times 10^5 \times (2.0 - 1.0) \times 10^{-3} = \mathbf{120 \text{ J}}$ (the shaded area).

$\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2} \times 0.048 \times 8.31 \times 301 = \mathbf{180 \text{ J}}$ (check: $\frac{3}{2}P\Delta V = \frac{3}{2} \times 120 = 180 \text{ J}$).

$Q = \Delta U + W = 180 + 120 = \mathbf{300 \text{ J}}$

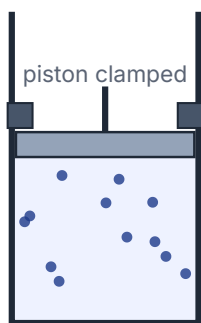
(c) No volume change, so $W = 0$. $\Delta U = \frac{3}{2}V\Delta P = \frac{3}{2} \times 2.0 \times 10^{-3} \times (0.60 - 1.2) \times 10^5 = \mathbf{-180 \text{ J}}$, so $Q = \Delta U + W = \mathbf{-180 \text{ J}}$: 180 J of thermal energy leaves the gas.

(d) $\Delta U_{AC} = +180 - 180 = 0$. $P_A V_A = P_C V_C = 120 \text{ J}$, so A and C are at the same temperature (300 K). Yet the gas absorbed a net 120 J of thermal energy, all of which was used to do the 120 J of work.

At constant pressure only $\frac{3}{5}$ of the thermal energy supplied raises the internal energy; the other $\frac{2}{5}$ is spent pushing the piston out.

What are the four thermodynamic processes?

Each standard process keeps one quantity fixed. Apply the first law to each one by asking which term is zero.

isovolumetric (V constant)

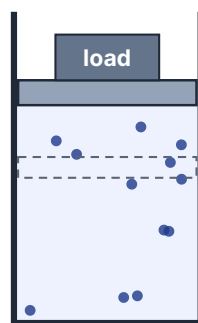
$$\Delta V = 0, \text{ so } W = 0$$

$$Q = \Delta U$$

P and T rise



Q in

isobaric (P constant)

moves up

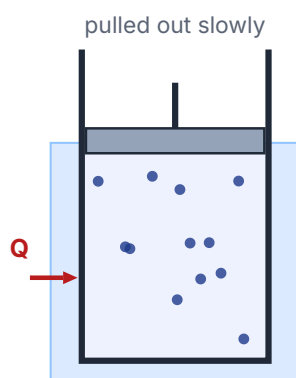
$$W = P\Delta V$$

$$Q = \Delta U + P\Delta V$$

V and T rise



Q in

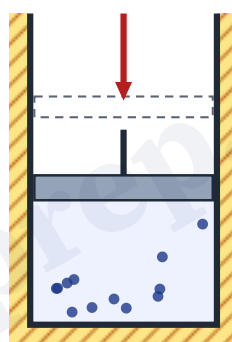
isothermal (T constant)

$$\Delta T = 0, \text{ so } \Delta U = 0$$

$$Q = W$$

$$PV = \text{constant}$$

large water bath at T

adiabatic (Q = 0)

pushed in fast

$$Q = 0, \text{ so } \Delta U = -W$$

$$PV^{5/3} = \text{constant}$$

compressed: T rises

insulated walls

How each process is achieved in practice: clamp the piston (isovolumetric), let a fixed load ride on it (isobaric), keep the cylinder in a large bath and move the piston slowly (isothermal), or insulate it and move the piston quickly (adiabatic).

Process	Fixed	Work done by gas W	ΔU	First law becomes	P-V graph
Isovolumetric (isochoric)	V	0	$\frac{3}{2}nR\Delta T$	$Q = \Delta U$	vertical line
Isobaric	P	$P\Delta V$	$\frac{3}{2}nR\Delta T = \frac{3}{2}P\Delta V$	$Q = \Delta U + P\Delta V$	horizontal line
Isothermal	T	area under curve	0	$Q = W$	$PV = \text{constant}$ (hyperbola)
Adiabatic	no thermal energy transfer, $Q = 0$	area under curve = $-\Delta U$	$\frac{3}{2}nR\Delta T$	$\Delta U = -W$	$PV^{5/3} = \text{constant}$ (steeper)

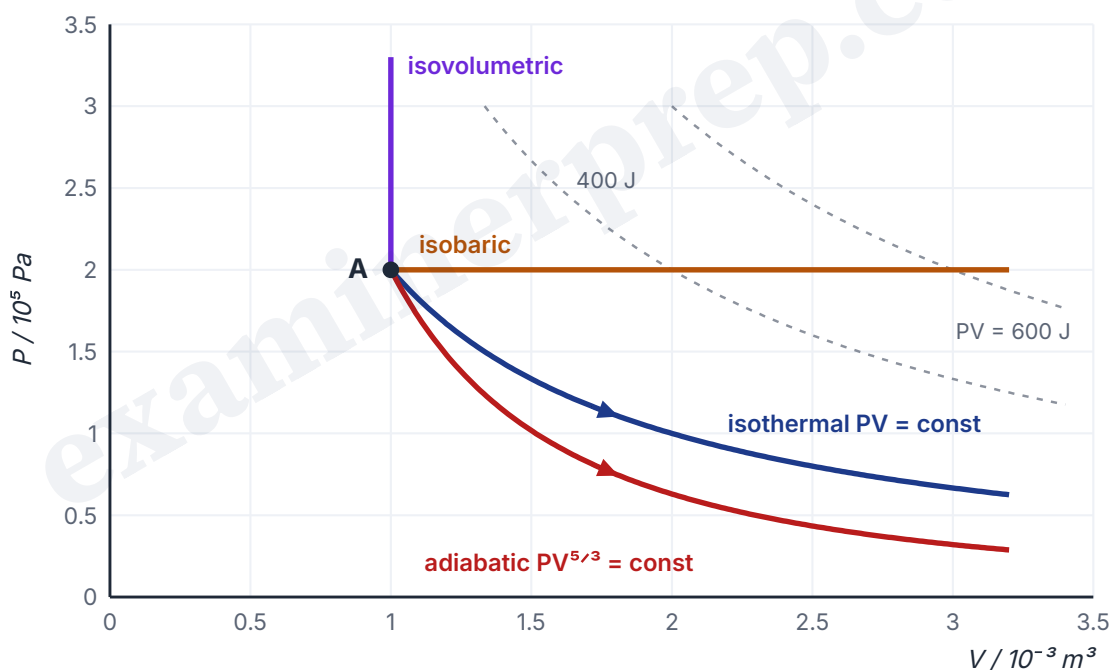
- **Isothermal:** all the thermal energy absorbed is turned into work (expansion), or all the work done on the gas leaves as thermal energy (compression). The change must be *slow*, with the gas in good thermal contact with its surroundings, so that its temperature never moves away from that of the surroundings.
- **Adiabatic:** no thermal energy crosses the boundary — the gas is insulated, or the change is so *fast* that there is no time for heat transfer. An adiabatic **expansion cools** the gas (it does work using its own internal energy); an adiabatic **compression warms** it.
- "Adiabatic" does **not** mean "constant temperature", and "isothermal" does **not** mean "no heat". They are opposite cases.

The adiabatic equation

For an adiabatic change of a monatomic ideal gas, the product $PV^{5/3}$ is constant.

$$PV^{\frac{5}{3}} = \text{constant} \quad \Rightarrow \quad P_1 V_1^{\frac{5}{3}} = P_2 V_2^{\frac{5}{3}}$$

- This is in the data booklet. It applies **only** to monatomic ideal gases and only to adiabatic changes. P and V can be in any units, as long as you are consistent on both sides.
- Combined with $PV = nRT$ (which still holds at every instant) it gives $TV^{2/3} = \text{constant}$ — not in the data booklet, but a quick route to the new temperature. Alternatively find P_2 first and then use $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$.
- From any starting point, the adiabatic curve is **steeper** than the isotherm through that point (by a factor of $\frac{5}{3}$). On an adiabatic expansion, the pressure falls for two reasons: the volume increases *and* the temperature falls.



All four processes starting from the same state A (P - V diagram to scale). The dashed lines are isotherms. Expanding adiabatically, the gas crosses to lower isotherms — it cools — so its curve falls more steeply than the isotherm.

WORKED EXAMPLE 3 Medium · Show that / Calculate / Explain

A cylinder contains a monatomic ideal gas at 1.0×10^5 Pa, 300 K and volume 6.0×10^{-4} m³. The gas is compressed very rapidly to a volume of 1.5×10^{-4} m³, so that the change may be treated as adiabatic.

(a) Show that the final pressure is about 1.0×10^6 Pa. (b) Calculate the final temperature. (c) Determine the work done by the gas. (d) Explain, in terms of the first law, why the temperature of the gas rises.

$$\text{(a)} P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{5/3} = 1.0 \times 10^5 \times 4.0^{5/3} = 1.0 \times 10^5 \times 10.08 = \mathbf{1.01 \times 10^6 \text{ Pa}} \approx 1.0 \times 10^6 \text{ Pa}$$

$$\text{(b)} T_2 = T_1 \frac{P_2 V_2}{P_1 V_1} = 300 \times \frac{1.008 \times 10^6 \times 1.5 \times 10^{-4}}{1.0 \times 10^5 \times 6.0 \times 10^{-4}} = 300 \times 2.52 = \mathbf{760 \text{ K}} \text{ (756 K)}$$

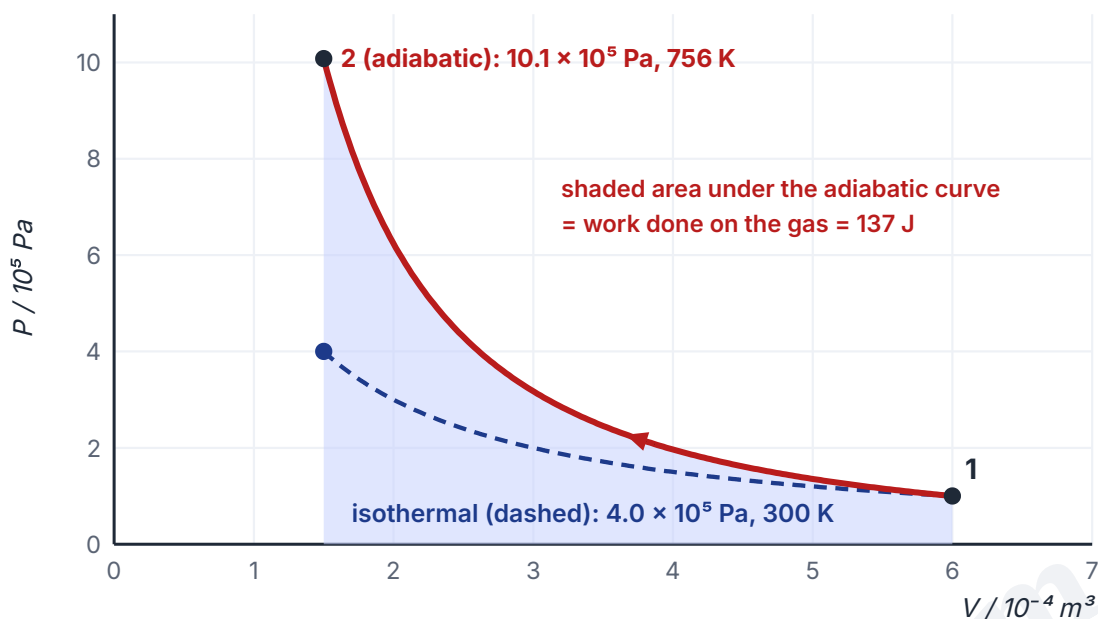
(c) $Q = 0$, so $W = -\Delta U$. Using $\Delta U = \frac{3}{2} \Delta(PV)$:

$$\Delta U = \frac{3}{2} (1.008 \times 10^6 \times 1.5 \times 10^{-4} - 1.0 \times 10^5 \times 6.0 \times 10^{-4}) = \frac{3}{2} (151.2 - 60.0) = \mathbf{137 \text{ J}}$$

Work done by the gas $W = \mathbf{-137 \text{ J}}$ (137 J of work is done *on* the gas).

Check with $\Delta U = \frac{3}{2} n R \Delta T$: $n = \frac{P_1 V_1}{R T_1} = 0.0241 \text{ mol}$, and $\frac{3}{2} \times 0.0241 \times 8.31 \times 456 = 137 \text{ J} \checkmark$

(d) Work is done on the gas ($W < 0$) and no thermal energy leaves it ($Q = 0$), so $\Delta U = Q - W = -W > 0$. The internal energy — the total random kinetic energy of the particles — increases, so the mean kinetic energy of the particles and hence the temperature increase.



Worked example 3 drawn to scale. Compressed adiabatically to a quarter of its volume, the gas ends at about ten times its initial pressure; compressed isothermally to the same volume, the pressure would only quadruple.

What is entropy?

Entropy S is a thermodynamic property of a system that is a measure of the degree of disorder of its particles. Its unit is J K^{-1} .

Entropy can be found in two equivalent ways — from measurable (macroscopic) quantities, or by counting the arrangements of the particles (microscopic).

Macroscopic: thermal energy and temperature

$$\Delta S = \frac{\Delta Q}{T}$$

- ΔS = change in entropy of the system (J K^{-1})
- ΔQ = thermal energy transferred to the system (J); negative if thermal energy leaves
- T = absolute temperature at which the transfer happens (K)
- This is in the data booklet. Use it when the temperature stays constant during the transfer: a phase change (melting, boiling), a large reservoir whose

temperature barely changes, or an isothermal process of a gas.

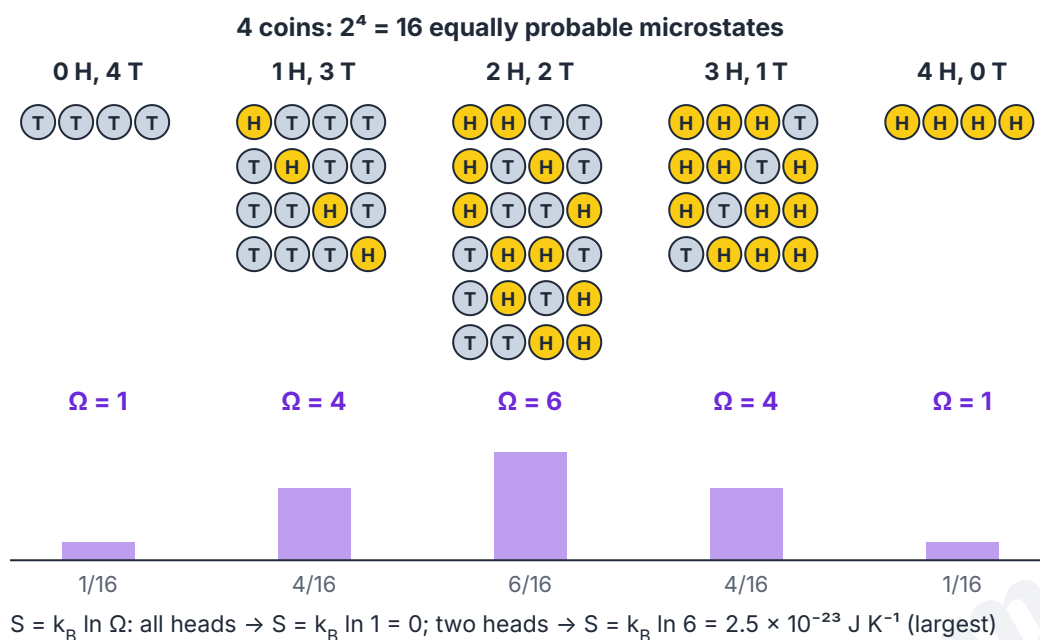
- The same ΔQ produces a **larger** entropy change at a **lower** temperature. This is why thermal energy flowing from hot to cold always raises the total entropy.

Microscopic: counting microstates

A **macrostate** is described by bulk quantities (for coins: how many heads; for a gas: P, V, T). A **microstate** is one particular arrangement of every individual particle consistent with that macrostate. All microstates are equally probable.

$$S = k_B \ln \Omega$$

- S = entropy of a macrostate (J K^{-1})
- k_B = Boltzmann constant (J K^{-1})
- Ω = number of microstates that give that macrostate (no unit)
- This is in the data booklet. $\ln$ is the natural logarithm. A macrostate with only one microstate (perfect order) has $S = k_B \ln 1 = 0$.
- Because every microstate is equally likely, the macrostate with the **most** microstates is the most probable. Disordered macrostates have vastly more microstates than ordered ones, so systems drift towards disorder simply because it is overwhelmingly likely.
- The IB model uses coins (or particles in two halves of a box): with N two-sided objects there are 2^N microstates in total.



Four coins have 16 equally likely microstates. The macrostate "2 heads, 2 tails" can happen in 6 ways, so it has the highest entropy and is the most probable; "all heads" can happen in only one way.

WORKED EXAMPLE 4 Medium · Calculate / Determine

(a) Five coins are tossed. Calculate the entropy of the macrostate "three heads and two tails", and determine the probability of obtaining it.

(b) An ice cube of mass 0.250 kg at 0 °C (273 K) melts completely in a room whose temperature stays at 295 K. The specific latent heat of fusion of ice is $3.34 \times 10^5 \text{ J kg}^{-1}$. Determine the entropy change of the ice, of the room, and of the ice and room together.

(a) $\Omega = 10$: the number of ways of choosing which 3 of the 5 coins show heads (HHHTT, HHTHT, HHTTH, HTHHT, HTHTH, HTTHH, THHHT, THHTH, THTHH, TTHHH).

$$S = k_B \ln \Omega = 1.38 \times 10^{-23} \times \ln 10 = \mathbf{3.2 \times 10^{-23} \text{ J K}^{-1}} \quad (3.18 \times 10^{-23})$$

$$\text{Probability} = \frac{10}{2^5} = \frac{10}{32} = \mathbf{0.31} \quad (0.3125)$$

(b) Thermal energy absorbed by the ice: $Q = mL = 0.250 \times 3.34 \times 10^5 = 8.35 \times 10^4 \text{ J}$.

$$\text{Ice (at constant 273 K): } \Delta S = \frac{+8.35 \times 10^4}{273} = \mathbf{+306 \text{ J K}^{-1}}$$

Room (loses the same energy at 295 K): $\Delta S = \frac{-8.35 \times 10^4}{295} = -283 \text{ J K}^{-1}$

Together: $306 - 283 = +23 \text{ J K}^{-1}$ (22.8 J K⁻¹)

The room's entropy falls — allowed, because the room is not isolated — but the ice gains more, so the total for the isolated ice-plus-room system rises, as the second law requires.

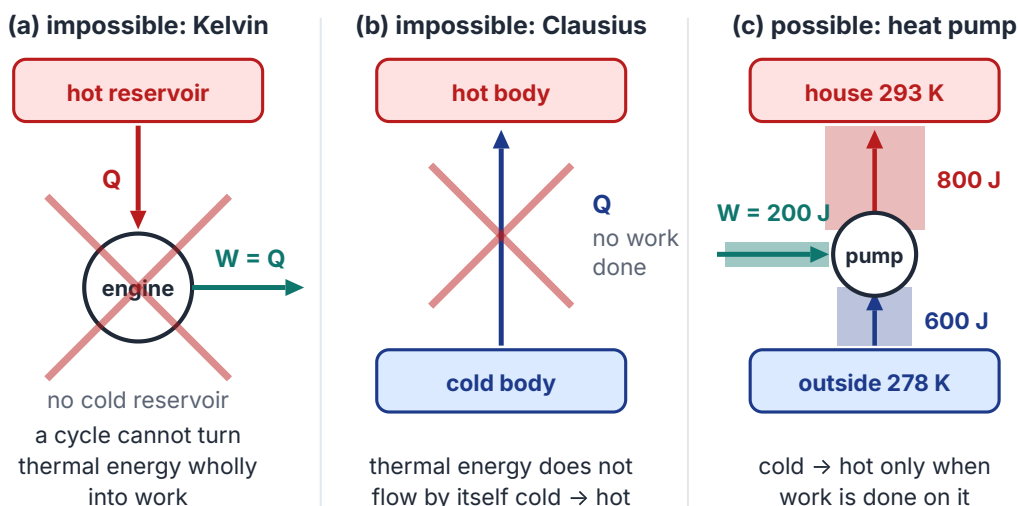
What does the second law of thermodynamics say?

The first law says energy is conserved, but it does not forbid a cold drink from spontaneously cooling further while warming the room. The second law says which processes actually happen. You must be able to state it in three equivalent forms.

Entropy form: the entropy of an isolated system never decreases. It stays constant in a reversible process and increases in an irreversible process.

Clausius form: thermal energy cannot spontaneously transfer from a colder body to a hotter body. (Moving thermal energy from a colder body to a hotter one requires work to be done.)

Kelvin form: no cyclic process can extract thermal energy from a single reservoir and convert it completely into work. (Some energy must always be rejected to a colder reservoir.)

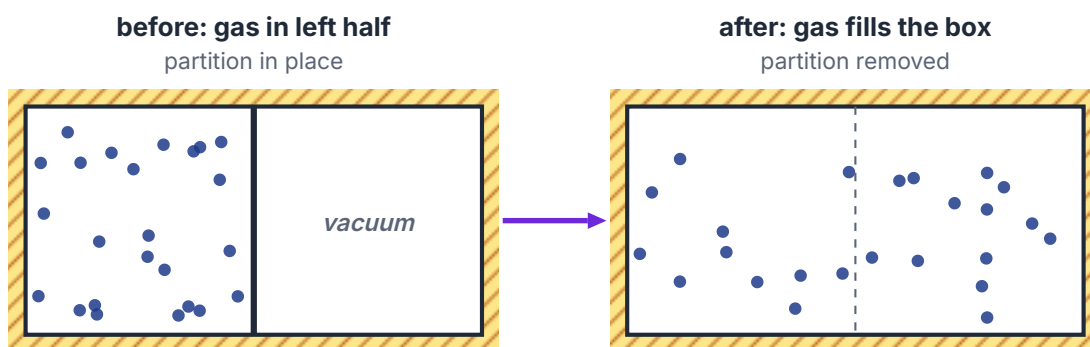


(a) A "perfect" engine that turns all the heat it absorbs into work breaks the Kelvin form. (b) Spontaneous flow from cold to hot breaks the Clausius form. (c) A heat pump is allowed: it moves 600 J from the cold outside into the warm house only because 200 J of work is done on it (band widths to scale).

- The heat pump in (c) obeys both laws. First law: $600 \text{ J} + 200 \text{ J} = 800 \text{ J}$ delivered. Second law: entropy change $= -\frac{600}{278} + \frac{800}{293} = -2.16 + 2.73 = +0.57 \text{ J K}^{-1} > 0$.
- The three forms are equivalent: if you could build a device that broke one, you could combine it with an ordinary engine or heat pump to break the others.

Reversible and irreversible processes

- A **reversible** process is an idealisation: it happens infinitely slowly, with no friction or turbulence, and with thermal energy only ever flowing across a vanishingly small temperature difference. It could be run backwards, restoring the system *and* its surroundings exactly. For a reversible process, the total entropy of system + surroundings is unchanged.
- Real processes are **irreversible**: friction, thermal energy flowing across a finite temperature difference, mixing, a gas expanding freely into a vacuum. Real processes in isolated systems are almost always irreversible, so the entropy of a real isolated system **increases**.

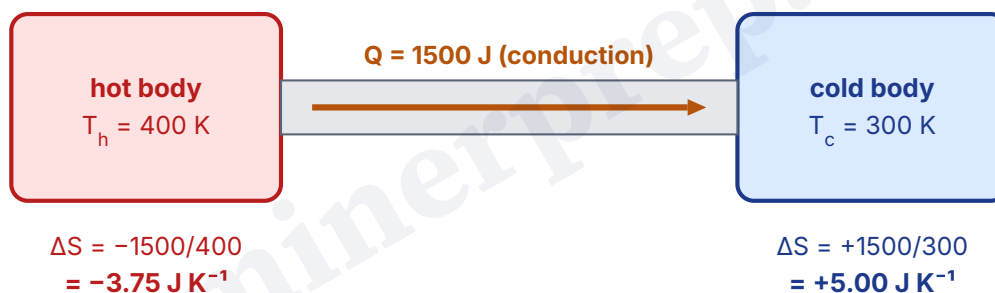


isolated system: $Q = 0$, $W = 0$, $\Delta U = 0$ (T unchanged for an ideal gas)

many more microstates after, so S increases; the reverse never happens by itself

Free expansion: when the partition is removed, the gas spreads into the vacuum. No energy enters or leaves, but the number of available microstates rises enormously. The particles will never all return to the left half by themselves — the process is irreversible.

two bodies isolated from everything else; temperatures treated as constant



total $\Delta S = +1.25 \text{ J K}^{-1} > 0$

the cold body gains more entropy than the hot body loses

Thermal energy flowing across a temperature difference is irreversible. Because the cold body is at a lower temperature, it gains more entropy than the hot body loses, so the total entropy increases.

Can entropy ever decrease?

A system that is not isolated can lose entropy, but only if the entropy of its surroundings increases by an equal or greater amount.

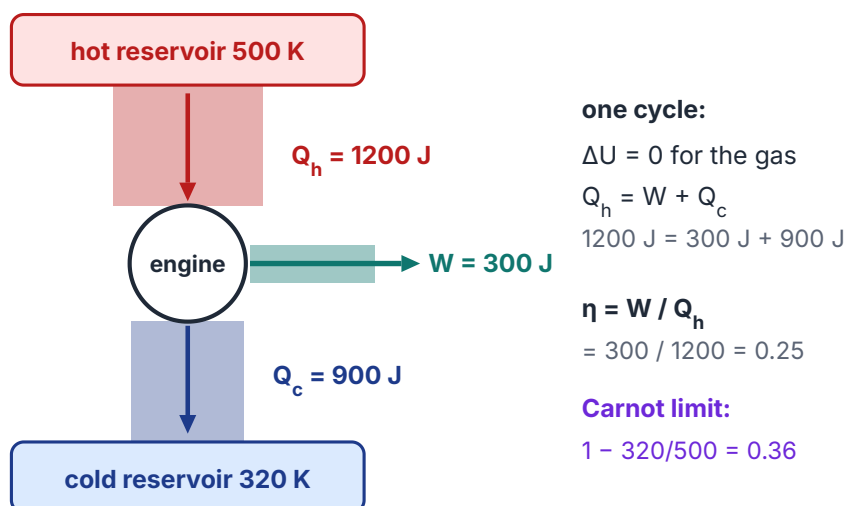
- Water freezing in a freezer: the water's entropy falls (ordered crystal), but the freezer releases even more thermal energy into the kitchen, whose entropy rises by more.
- A living organism builds ordered molecules, lowering its own entropy, while releasing thermal energy and disordered waste that raise the entropy of its surroundings by more.

- The total entropy (system + surroundings, treated together as an isolated system) never decreases.

How do heat engines use cycles?

A single expansion can do work only once — the piston runs out of cylinder. A **heat engine** repeats a **cycle** of processes that returns the gas (the working substance) to its starting state, so it can run continuously.

- Over one complete cycle the gas is back where it started, so $\Delta U = 0$ and the first law gives **net $Q = \text{net } W$** .
- On a P–V diagram, the **net work done by the gas per cycle is the area enclosed by the loop**. A loop traced **clockwise** gives positive net work (an engine); anticlockwise gives net work done *on* the gas (a heat pump or refrigerator).
- For a positive net work, the gas must be compressed when it is cooler (low pressure, little work needed) and expand when it is hotter (high pressure, more work done).
- Every heat engine absorbs thermal energy Q_h from a hot reservoir at T_h , does useful work W , and — by the Kelvin form of the second law — must reject thermal energy Q_c to a cold reservoir at T_c : $Q_h = W + Q_c$.
- An engine can be designed around different cycles (a petrol engine, for example, is modelled by a cycle of two adiabatic and two isovolumetric processes). Each cycle gives a different efficiency between the same temperatures.

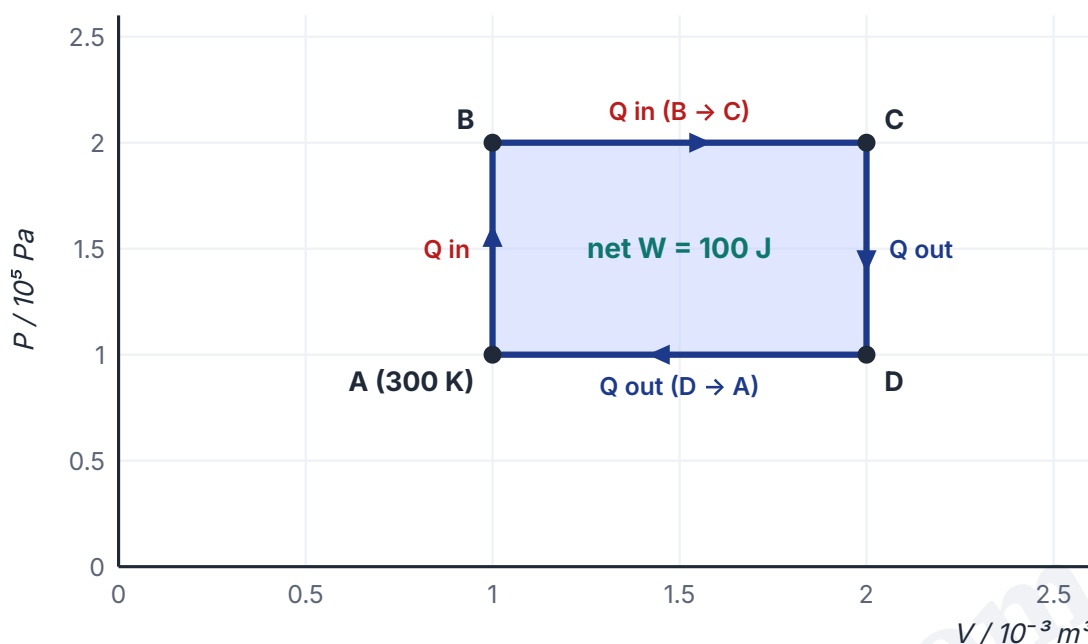


Energy-flow (Sankey) diagram for a heat engine, with band widths to scale. Per cycle the engine absorbs 1200 J, does 300 J of work and rejects 900 J — an efficiency of 0.25, below the Carnot limit of 0.36 for these reservoirs.

The efficiency of a heat engine is the ratio of the useful work it does to the energy supplied to it: $\eta = \frac{\text{useful work}}{\text{input energy}}$.

$$\eta = \frac{\text{useful work}}{\text{input energy}} = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

- The first form is in the data booklet; the other two follow from $Q_h = W + Q_c$ and are not. Efficiency has no unit and may be given as a decimal or a percentage.
- The “input energy” is the thermal energy absorbed in **all** stages where $Q > 0$ — not the net thermal energy, which equals the work.
- Power = work per cycle $\times$ number of cycles per second.



A clockwise rectangular cycle. Thermal energy is absorbed in $A \rightarrow B$ and $B \rightarrow C$ and rejected in $C \rightarrow D$ and $D \rightarrow A$; the shaded enclosed area is the net work done by the gas per cycle.

WORKED EXAMPLE 5 Hard · Show that / Determine / Explain

A monatomic ideal gas is taken round the cycle $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$ shown above. The temperature at A is 300 K.

(a) Show that the net work done by the gas per cycle is 100 J. (b) Determine the temperatures at B, C and D. (c) Determine the thermal energy absorbed by the gas per cycle, and hence the efficiency of the cycle. (d) Calculate the Carnot efficiency for reservoirs at the highest and lowest temperatures of the cycle, and suggest why the cycle's efficiency is much lower.

(a) Net work = enclosed area = $(2.0 - 1.0) \times 10^5 \times (2.0 - 1.0) \times 10^{-3} = 100 \text{ J}$

(b) For a fixed amount of gas $T \propto PV$. PV at A is 100 J. At B (200 J) and D (200 J): $T_B = T_D = 600 \text{ K}$. At C (400 J): $T_C = 1200 \text{ K}$

(c) Use $\Delta U = \frac{3}{2}\Delta(PV)$ and $Q = \Delta U + W$ for each stage:

Stage	W / J	$\Delta U / \text{J}$	Q / J
A $\rightarrow$ B (isovolumetric)	0	$\frac{3}{2}(200 - 100) = 150$	+150
B $\rightarrow$ C (isobaric)	$2.0 \times 10^5 \times 1.0 \times 10^{-3} = 200$	$\frac{3}{2}(400 - 200) = 300$	+500
C $\rightarrow$ D (isovolumetric)	0	-300	-300
D $\rightarrow$ A (isobaric)	-100	-150	-250
Cycle	100	0	net +100

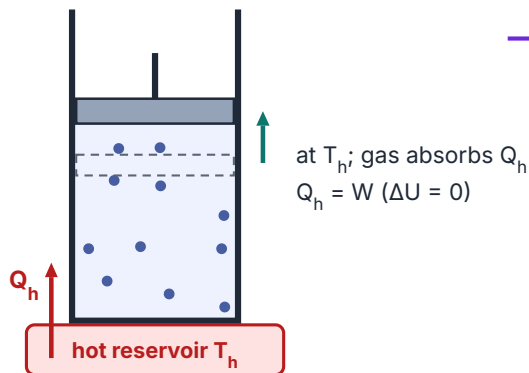
Thermal energy absorbed = $150 + 500 = \mathbf{650 \text{ J}}$. $\eta = \frac{W}{Q_h} = \frac{100}{650} = \mathbf{0.15}$
(15.4 %)

(d) $\eta_{\text{Carnot}} = 1 - \frac{300}{1200} = \mathbf{0.75}$. Much of the thermal energy is absorbed while the gas is far below 1200 K (A $\rightarrow$ B) and rejected while it is far above 300 K (C $\rightarrow$ D). Thermal energy flowing across large temperature differences between reservoirs and gas is irreversible, and generates entropy, so less of the input can become work.

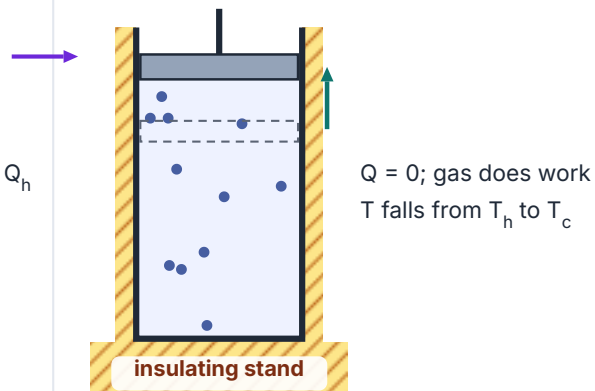
What is the Carnot cycle?

The Carnot cycle is a theoretical, perfectly **reversible** cycle operating between two reservoirs. It consists of two isothermal and two adiabatic processes:

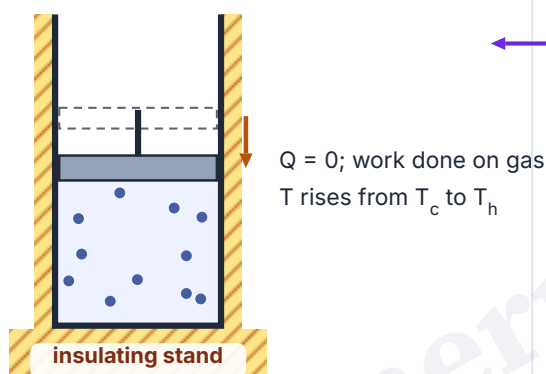
1 A → B isothermal expansion



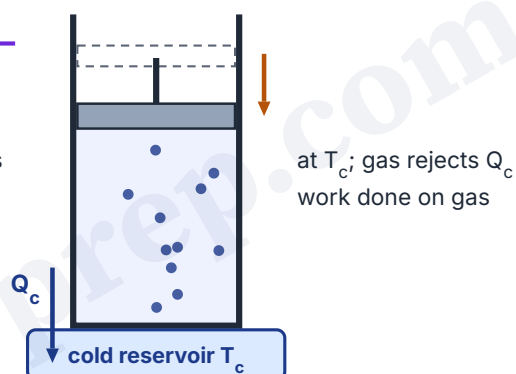
2 B → C adiabatic expansion



4 D → A adiabatic compression

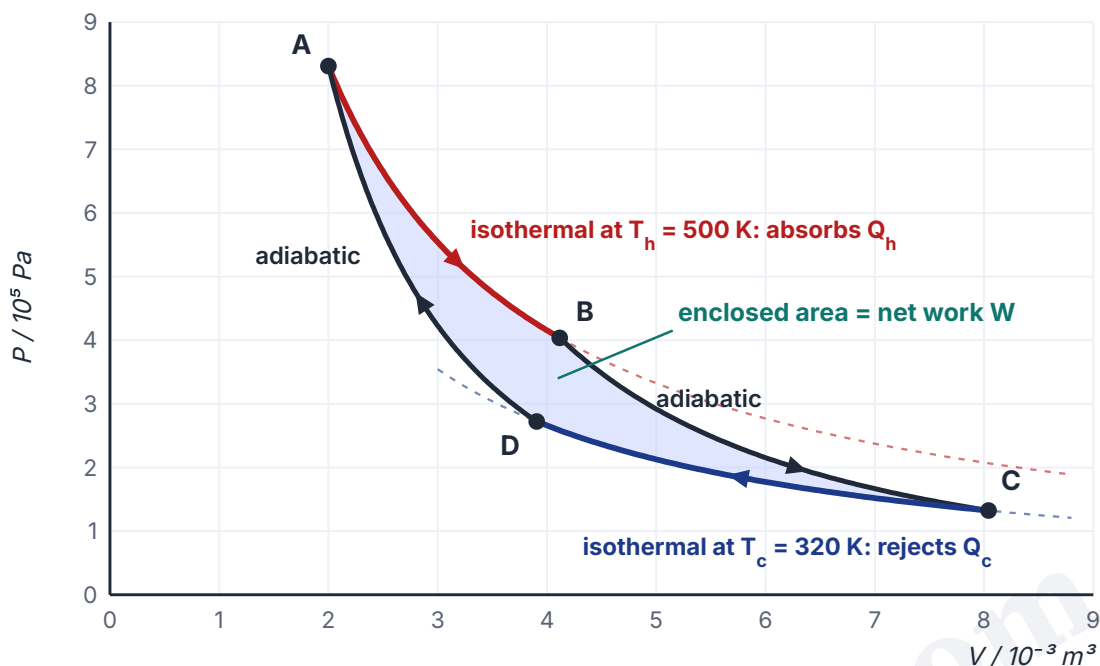


3 C → D isothermal compression



The four stages of a Carnot cycle. Thermal energy enters only at T_h and leaves only at T_c ; the adiabatic stages change the temperature of the gas between the two reservoir temperatures without any heat transfer.

1. **A → B, isothermal expansion at T_h :** the gas absorbs Q_h from the hot reservoir and does an equal amount of work ($\Delta U = 0$).
2. **B → C, adiabatic expansion:** the gas does work using its internal energy and cools from T_h to T_c .
3. **C → D, isothermal compression at T_c :** work is done on the gas and it rejects Q_c to the cold reservoir.
4. **D → A, adiabatic compression:** work done on the gas raises its temperature from T_c back to T_h .



A Carnot cycle drawn to scale for 0.40 mol of a monatomic ideal gas between 500 K and 320 K, absorbing 1200 J per cycle. The two adiabatic curves join the two isotherms; the enclosed area is the net work done by the gas per cycle.

The Carnot cycle sets the maximum possible efficiency of any heat engine operating between reservoirs at temperatures T_h and T_c : $\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$.

$$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$$

- T_h = temperature of the hot reservoir (K); T_c = temperature of the cold reservoir (K)
- This is in the data booklet. Temperatures **must** be in kelvin.
- Why it is the limit: the cycle is reversible, so the total entropy does not change. The hot reservoir loses Q_h/T_h , the cold one gains Q_c/T_c , and these are equal, so $\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$. Any irreversibility makes Q_c larger and the efficiency smaller.
- Even a perfect engine cannot reach $\eta = 1$ unless $T_c = 0$ K. To raise the maximum efficiency, raise T_h or lower T_c .
- Real engines fall well short of η_{Carnot} : they run fast (not quasi-statically), have friction, and exchange heat across finite temperature differences.

TRY IT YOURSELF

B.4 Heat Engines & Carnot simulation: run a gas round a cycle on a P–V diagram, watch the enclosed area (the work) change, and compare the engine's efficiency with the Carnot limit as you change the reservoir temperatures.

WORKED EXAMPLE 6 Hard · Calculate / Show that / Discuss

An engine uses 0.40 mol of a monatomic ideal gas in a Carnot cycle between reservoirs at 500 K and 320 K (see the P–V diagram above). In each cycle the gas absorbs 1200 J from the hot reservoir.

(a) Calculate the efficiency of the engine. (b) Determine the work done by the gas during the adiabatic expansion B → C, and explain why the two adiabatic stages make no net contribution to the work per cycle. (c) Determine the work done per cycle and the thermal energy rejected per cycle. (d) Show that the total entropy change of the two reservoirs per cycle is zero. (e) A real engine working between the same reservoirs also absorbs 1200 J per cycle but has an efficiency of 0.25. Determine the entropy change of the reservoirs per cycle and comment on your answer.

$$\text{(a)} \quad \eta = 1 - \frac{T_c}{T_h} = 1 - \frac{320}{500} = \mathbf{0.36}$$

$$\text{(b)} \quad Q = 0, \text{ so } W = -\Delta U = -\frac{3}{2}nR\Delta T = -\frac{3}{2} \times 0.40 \times 8.31 \times (320 - 500) = \mathbf{+897 \text{ J}} \approx 900 \text{ J}.$$

In D → A the temperature changes by the same amount in the opposite direction (320 K → 500 K), so the work done by the gas is –897 J. The two cancel.

$$\text{(c)} \quad W = \eta Q_h = 0.36 \times 1200 = \mathbf{432 \text{ J}} \text{ (the enclosed area). } Q_c = Q_h - W = 1200 - 432 = \mathbf{768 \text{ J}}$$

$$\text{(d)} \quad \text{Hot reservoir: } \Delta S = -\frac{1200}{500} = -2.40 \text{ J K}^{-1}. \text{ Cold reservoir: } \Delta S = +\frac{768}{320} = +2.40 \text{ J K}^{-1}. \text{ Total} = \mathbf{0} \text{ (the gas itself returns to its initial state, so its entropy change over the cycle is also zero).}$$

(e) $W = 0.25 \times 1200 = 300 \text{ J}$, so $Q_c = 900 \text{ J}$.

$$\Delta S = -\frac{1200}{500} + \frac{900}{320} = -2.40 + 2.81 = +0.41 \text{ J K}^{-1} \text{ per cycle}$$

The total entropy increases, as the second law requires for a real (irreversible) engine. The Carnot engine, being reversible, leaves the total entropy unchanged; an engine with $\eta > 0.36$ would need $Q_c < 768 \text{ J}$, giving a total entropy decrease — which is impossible.

Exam tips for B.4

EXAM TIP 1 — WRITE THE SIGNS BEFORE THE NUMBERS

Start every first-law calculation by writing $Q = \dots$, $W = \dots$, with a sign on each. W is the work done **by** the gas: negative whenever the gas is compressed. Students who carry over the convention used in some other courses (where W is the work done *on* the gas) while still writing $Q = \Delta U + W$ get the wrong value of ΔU and lose the marks.

EXAM TIP 2 — NAME THE TERM THAT IS ZERO

For any process, say which quantity is zero and why: "isothermal, so $\Delta U = 0$ ", "adiabatic, so $Q = 0$ ", "constant volume, so $W = 0$ ". That sentence often earns the first mark, and it tells you which form of the first law to use. In a cycle, also state " $\Delta U = 0$ over a cycle".

EXAM TIP 3 — INTERNAL ENERGY FOLLOWS PV

When asked where on a cycle the gas has the most internal energy, find where PV (and so T) is largest. In a Carnot cycle the whole hot isothermal stage is at the highest temperature, so the answer is a **process** (the whole stage), not a single corner of the diagram.

EXAM TIP 4 — KELVIN, CONSISTENT UNITS, "INPUT" ENERGY

η_{Carnot} , $PV = nRT$ and $\Delta S = \Delta Q/T$ all need kelvin. Convert volumes to m^3 before using $W = P\Delta V$ (a common factor-of-1000 error). For a cycle's efficiency, divide by the thermal energy **absorbed**, not the net heat.

EXAM TIP 5 — SECOND-LAW ANSWERS NEED THE WHOLE SYSTEM

When a system's entropy falls (water freezing, a refrigerator's contents cooling), full marks need both halves: "the entropy of the system decreases, **but** the entropy of the surroundings increases by more, so the total entropy increases". For coin problems, Ω is the number of microstates of the macrostate asked about, not the total 2^N .

Common misconceptions

Misconception	Why it is wrong	Correct idea
In an isothermal process no heat is transferred.	The temperature stays constant <i>because</i> thermal energy flows in or out.	Isothermal: $\Delta U = 0$ and $Q = W$. It is the adiabatic process that has $Q = 0$.
In an adiabatic process the temperature is constant.	With $Q = 0$, any work changes the internal energy.	Adiabatic expansion cools the gas; adiabatic compression warms it.
Heating a gas always raises its temperature.	If the gas does more work than the heat supplied, $\Delta U = Q - W < 0$.	Temperature follows ΔU , not Q . In an isothermal expansion, heat is supplied but T is constant.
Entropy can never decrease.	That applies only to an isolated system.	A non-isolated system's entropy can fall if its surroundings gain at least as much.
A better-engineered engine could reach 100 % efficiency.	The Kelvin form forbids converting all the input heat into work in a cycle.	The upper limit is $1 - T_c/T_h$, even for a perfect, reversible engine.
The net work of a cycle is the area under the top curve.	The work done on the gas on the return path must be subtracted.	Net work = area enclosed by the loop; clockwise = positive.

How this topic connects

The second law and entropy: why no engine can reach 100 % efficiency

Any device that turns thermal energy into work runs in a cycle, and the second law (Kelvin form) forbids converting all the absorbed thermal energy into work: some must be rejected to a cooler reservoir. The minimum that must be rejected is set by entropy. The hot reservoir loses entropy Q_h/T_h , so the cold reservoir must gain at least as much, $Q_c/T_c \geq Q_h/T_h$, which gives $\eta \leq 1 - T_c/T_h$. Power stations, car engines and jet engines are all limited this way; this is why

they run their hot side as hot as materials allow, and why roughly half or more of the input energy in a thermal power station leaves as waste heat.

Energy losses in motors and generators, and why Carnot does not cap them

Motors (D.3) and generators (D.4) convert between electrical and mechanical energy without passing through a heat cycle, so the Carnot limit does not apply to them directly, and well-designed machines can exceed 90 % efficiency. Their losses are dissipative: resistive heating of the coils and friction in the bearings. Both are irreversible processes that turn ordered energy into internal energy and raise the entropy of the surroundings. However, a generator in a fossil-fuel or nuclear power station is driven by a heat engine (a turbine), so the overall efficiency of the station is limited by the Carnot efficiency of its steam cycle.

Entropy on a cosmic scale: the second law and the long-term fate of the universe

The universe can be treated as an isolated system: there is nothing outside it to exchange energy with. Because real processes are irreversible, its total entropy keeps increasing. Energy is conserved, but it is steadily spread out into less useful forms — thermal energy at ever more uniform temperatures, from which less work can be extracted. Local order (stars, planets, living things) can still form, but always at the cost of a larger entropy increase elsewhere. The one-way growth of entropy also gives time a direction: it is how we distinguish past from future.

Tools & inquiry — investigating compression of a gas

- Connect a gas syringe to a pressure sensor and data logger. Compress the trapped air **slowly**, recording P and V ; then repeat **quickly**. Slow compression is close to isothermal; rapid compression is closer to adiabatic and gives a steeper P - V curve.
- Plot $\ln P$ against $\ln V$. The gradient is -1 for an isothermal change. For an adiabatic change of a monatomic gas it would be $-5/3$; air is mainly diatomic, so its adiabatic gradient is smaller in magnitude (about -1.4). A

real fast compression lies between the two because some heat leaks through the walls.

- Systematic errors: the dead volume in the tube connecting the syringe to the sensor (add it to every reading), and leaks past the plunger, which reduce n during the experiment.
- **NOS — paradigm shifts and models:** the steam engine drove the Industrial Revolution before anyone understood why it worked. Sadi Carnot found the maximum efficiency in 1824 using the caloric theory — the idea that heat is a fluid — which was later shown to be wrong. His result survived the change of model, and the attempt to explain it led Clausius and Kelvin to the concept of entropy and the laws of thermodynamics.

Key equations

Equation	Use	In data booklet?
$Q = \Delta U + W$	first law (Clausius convention: W done by the gas)	yes
$W = P\Delta V$	work done by a gas at constant pressure; otherwise area under P–V graph	yes
$\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}Nk_B\Delta T$	change in internal energy, monatomic ideal gas	yes
$\Delta U = \frac{3}{2}\Delta(PV)$	internal energy change read from a P–V graph	no — combine with $PV = nRT$
$PV^{5/3} = \text{constant}$	adiabatic change, monatomic ideal gas	yes
$TV^{2/3} = \text{constant}$	temperature after an adiabatic change	no — derive from the two above
$\Delta S = \frac{\Delta Q}{T}$	entropy change for transfer at constant temperature	yes
$S = k_B \ln \Omega$	entropy from the number of microstates	yes
$\eta = \frac{\text{useful work}}{\text{input energy}}$	efficiency of an engine	yes
$Q_h = W + Q_c, \eta = 1 - \frac{Q_c}{Q_h}$	energy per cycle of an engine	no — follow from the first law
$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}$	maximum efficiency between two reservoirs	yes

Check your understanding

1. A monatomic ideal gas is compressed at a constant pressure of 1.5×10^5 Pa from $4.0 \times 10^{-3} \text{ m}^3$ to $2.5 \times 10^{-3} \text{ m}^3$. Determine W , ΔU and Q .

$W = P\Delta V = 1.5 \times 10^5 \times (2.5 - 4.0) \times 10^{-3} = -225 \text{ J}$ (work done on the gas). $\Delta U = \frac{3}{2}P\Delta V = -338 \text{ J}$ (337.5 J; the gas cools). $Q = \Delta U + W = -338 + (-225) = -563 \text{ J}$: about 560 J of thermal energy leaves the gas.

2. 2400 J of thermal energy is conducted from a large body at 480 K to a large body at 320 K. Calculate the total entropy change and state what it shows.

$\Delta S = -\frac{2400}{480} + \frac{2400}{320} = -5.0 + 7.5 = +2.5 \text{ J K}^{-1}$. The total entropy increases, so the process is irreversible — the energy will not flow back from 320 K to 480 K by itself (Clausius form).

3. Calculate the maximum possible efficiency of an engine working between 650 K and 290 K. Give two reasons why a real engine has a lower efficiency.

$\eta_{\text{Carnot}} = 1 - \frac{290}{650} = 0.55$ (0.554). A real engine runs quickly, so its processes are not reversible; there is friction between moving parts; and thermal energy is exchanged across finite temperature differences. All of these generate entropy and increase the energy rejected.

4. Six coins are tossed. Calculate the entropy of the macrostate “three heads, three tails”, and explain why this macrostate is the most likely.

$\Omega = 20$ (the number of ways of choosing which 3 of the 6 coins are heads), so $S = k_B \ln 20 = 1.38 \times 10^{-23} \times 3.00 = 4.1 \times 10^{-23} \text{ J K}^{-1}$. All $2^6 = 64$ microstates are equally probable, and this macrostate contains more of them (20) than any other, so it is the most probable ($20/64 \approx 0.31$).

5. A bicycle pump is pushed in rapidly with the outlet blocked, and the air inside gets warmer. Explain this using the first law of thermodynamics.

The compression is rapid, so there is no time for thermal energy to leave: the process is approximately adiabatic, $Q = 0$. Work is done on the air, so $W <$

0. Then $\Delta U = Q - W = -W > 0$: the internal energy of the air increases, so the mean kinetic energy of its molecules, and hence its temperature, rises.

Practise B.4 with exam-style questions

Original Paper 1A, 1B and 2 questions on thermodynamics, with full mark schemes. B.4 is HL only, so there is no SL bank for this sub-topic. Tick the questions you want and print them as your own test.

[HL B.4 questions](#)

[Full question bank](#)

Revise and explore

- [Flashcards](#) — choose IB Physics and filter to B.4 Thermodynamics. Free, no sign-up.
- [B.4 Heat Engines & Carnot simulation](#)
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