

THEME C · WAVE BEHAVIOUR

SL & HL

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IB Physics C.1 Simple Harmonic Motion — Notes (SL & HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). Sections 1–6 are for SL and HL. C.1 has additional HL content — phase angle, the equations for x , v and the energies, and quantitative energy problems — which is in the purple **HL** section at the end.

 SL C.1 questions HL C.1 questions Flashcards Simple harmonic motion simulation

What this page covers

- ✓ The conditions an oscillation must meet to be simple harmonic
- ✓ The defining equation $a = -\omega^2 x$, and what its minus sign means
- ✓ Describing an oscillation: its equilibrium position, displacement and amplitude, and its period, frequency and angular frequency
- ✓ Linking period, frequency and angular frequency: $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- ✓ The period of a mass on a spring, $T = 2\pi\sqrt{m/k}$
- ✓ The period of a simple pendulum, $T = 2\pi\sqrt{l/g}$
- ✓ How kinetic, potential and total energy change during one cycle (qualitatively)
- ✓ Using the phase angle to describe where an oscillator is in its cycle (HL)
- ✓ Solving problems with $x = x_0 \sin(\omega t + \phi)$, $v = \omega x_0 \cos(\omega t + \phi)$ and $v = \pm\omega\sqrt{x_0^2 - x^2}$ (HL)
- ✓ Calculating energies with $E_T = \frac{1}{2}m\omega^2 x_0^2$ and $E_p = \frac{1}{2}m\omega^2 x^2$ (HL)

In one minute

- Simple harmonic motion (SHM) happens when the acceleration is **proportional to the displacement** from a fixed equilibrium position and **directed towards it**: $a = -\omega^2 x$.
- ω is the angular frequency: $\omega = 2\pi f = \frac{2\pi}{T}$. The period of SHM does not depend on the amplitude.
- Mass on a spring: $T = 2\pi\sqrt{\frac{m}{k}}$. Simple pendulum (small angles): $T = 2\pi\sqrt{\frac{l}{g}}$ — independent of the mass of the bob.
- Speed is greatest at the equilibrium position; acceleration and restoring force are greatest at the extremes, where the speed is zero.
- Kinetic and potential energy swap back and forth twice per cycle; with no energy dissipated, the total energy stays constant.
- HL: $x = x_0 \sin(\omega t + \phi)$, $v = \pm\omega\sqrt{x_0^2 - x^2}$, $E_T = \frac{1}{2}m\omega^2 x_0^2$. Phase angles are always in radians.

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What conditions lead to simple harmonic motion?

An **oscillation** is a repeated back-and-forth motion about an **equilibrium position** — the position where the resultant force on the body is zero. Many oscillations (a mass on a spring, a pendulum, a ship rolling, an atom vibrating in a molecule) share one special pattern, called simple harmonic motion.

Simple harmonic motion is oscillation in which the acceleration of the body is proportional to its displacement from a fixed equilibrium position and is always directed towards that position.

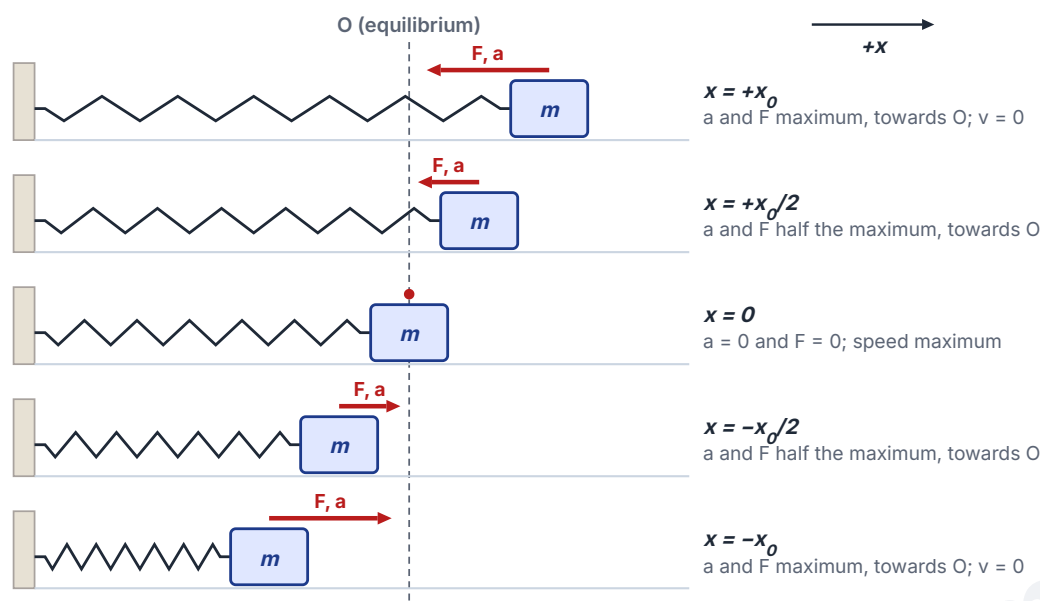
For SHM to happen, the body needs:

- a stable **equilibrium position**;
- a **restoring force** that always acts back towards that position whenever the body is displaced; and
- a restoring force whose size is **proportional to the displacement**: $F \propto -x$.

By Newton's second law, $a = F/m$, so a restoring force proportional to $-x$ gives an acceleration proportional to $-x$. This is written as the **defining equation of SHM**:

$$a = -\omega^2 x$$

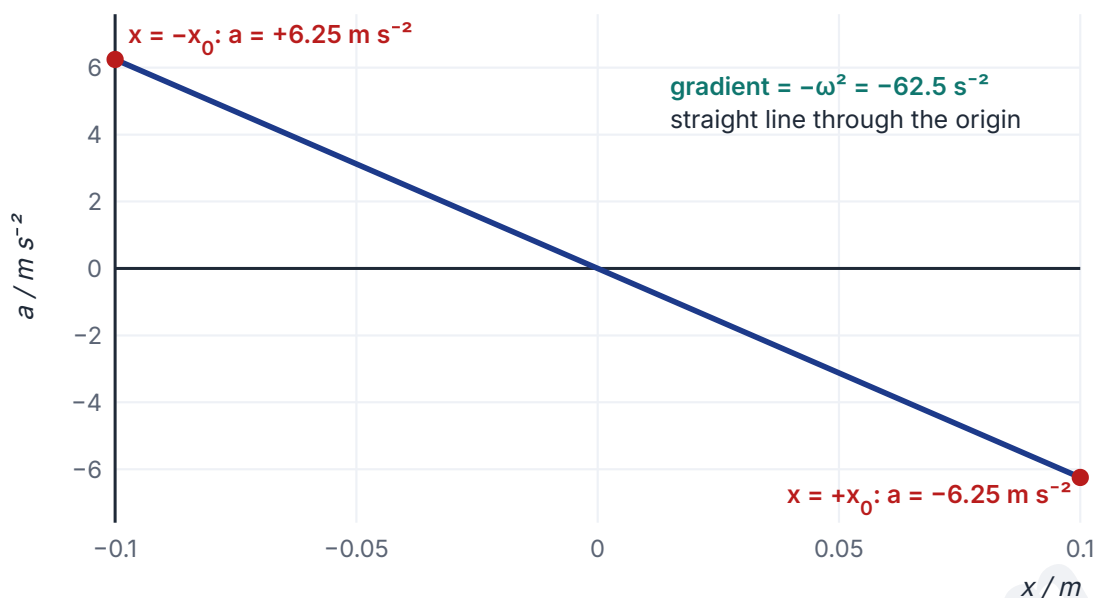
- a = acceleration (m s^{-2}), x = displacement from the equilibrium position (m), ω = angular frequency of the oscillation (rad s^{-1}). In the data booklet.
- ω^2 is a positive constant for a given oscillator, so a is **proportional** to x .
- The **minus sign** shows that the acceleration is always in the **opposite direction to the displacement**, i.e. towards the equilibrium position. Without it the body would accelerate further away and never return.



A mass on a smooth horizontal surface attached to a spring, shown at five displacements. The restoring force F and the acceleration a always point towards O , and their arrows are drawn to scale: twice the displacement gives twice the force.

Why must the defining equation take this form?

- **Proportional** ($a \propto x$): if the restoring force grew faster or slower than x , the time for a cycle would depend on how far the body was displaced. Proportionality is what makes the period the same for every amplitude.
- **Negative**: only a force directed back towards equilibrium produces oscillation. A force in the same direction as x would push the body away.
- The constant is written as a **square**, ω^2 , so that it is always positive and has units of s^{-2} , making ω a frequency in rad s^{-1} . For a particular system it comes from the physics: $\omega^2 = k/m$ for a spring, $\omega^2 = g/l$ for a pendulum.



Acceleration against displacement for an oscillator with $\omega^2 = 62.5 \text{ s}^{-2}$ and amplitude 0.10 m . The graph of a against x for SHM is a straight line through the origin with a negative gradient equal to $-\omega^2$, and it ends at $\pm x_0$.

EXAM TIP — TESTING FOR SHM FROM A GRAPH

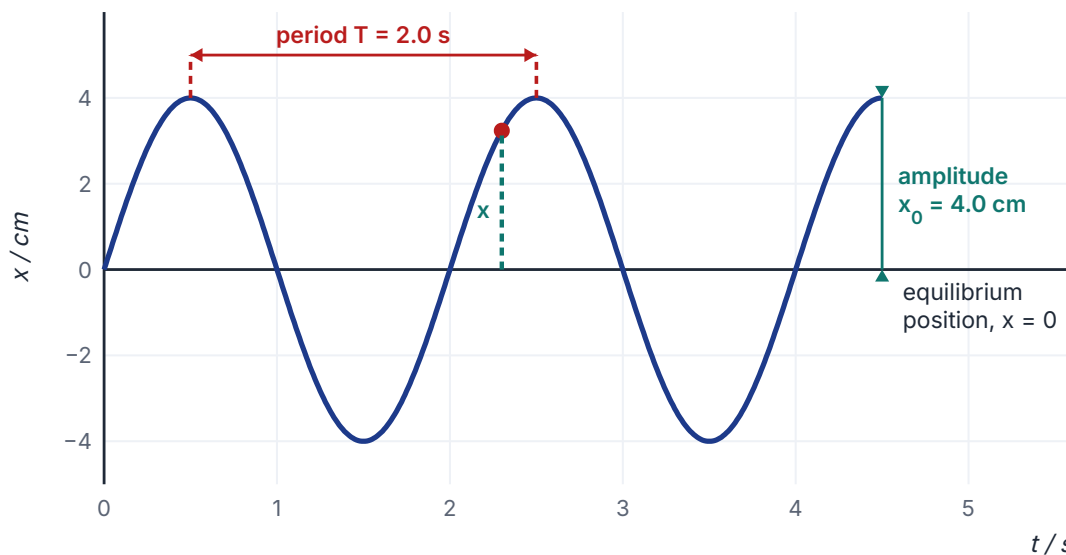
To show that data describe SHM, plot a against x and state **two** features: the line passes through the origin (proportional) and has a negative gradient (opposite direction). The angular frequency is then $\omega = \sqrt{-\text{gradient}}$.

How do we describe an oscillation?

Quantity	Symbol and unit	Meaning
Equilibrium position	$x = 0$	the position where the resultant force on the body is zero; the centre of the oscillation
Displacement	x (m)	distance <i>and direction</i> of the body from the equilibrium position at a given instant; a vector, so it can be negative
Amplitude	x_0 (m)	the maximum displacement from the equilibrium position (not the distance between the two extremes, which is $2x_0$)
Time period	T (s)	the time taken for one complete oscillation
Frequency	f (Hz)	the number of complete oscillations per unit time
Angular frequency	ω (rad s ⁻¹)	2π times the frequency; ω^2 is the constant in $a = -\omega^2 x$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad \Longleftrightarrow \quad \omega = 2\pi f = \frac{2\pi}{T}$$

- Both relationships are in the data booklet.
- One complete oscillation corresponds to 2π radians, which is why $\omega = 2\pi f$. It is called *angular* frequency because of the link with circular motion (see the HL section and How this topic connects).
- For SHM the period is **independent of the amplitude** (the oscillation is *isochronous*). A freely swinging pendulum keeps the same period even as its swing slowly dies away.

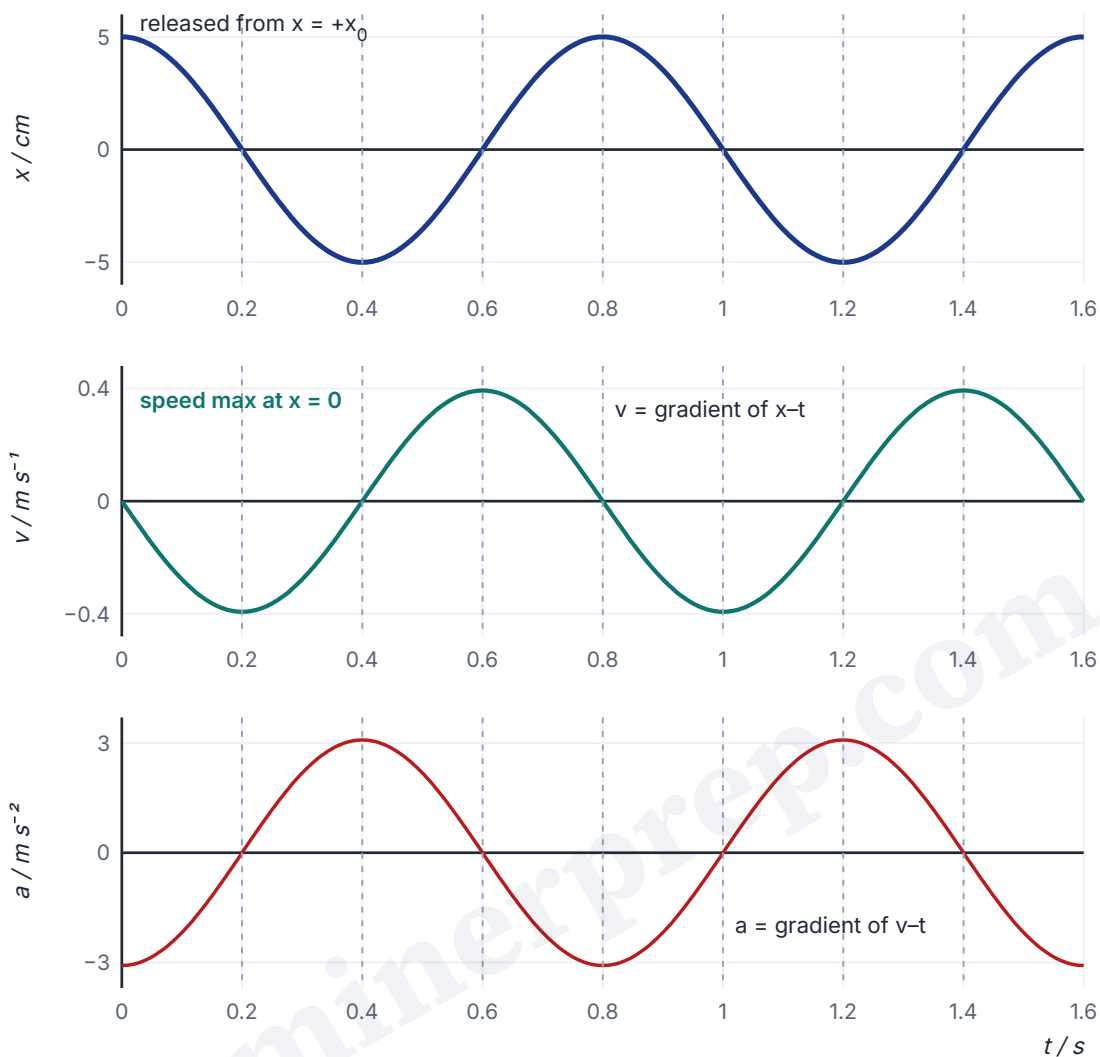


Displacement–time graph for a pendulum bob oscillating with amplitude 4.0 cm and period 2.0 s ($f = 0.50$ Hz, $\omega = \pi \text{ rad s}^{-1}$). The period can be measured between any two points at the same stage of the cycle, e.g. two successive maxima. The displacement x at $t = 2.3$ s is shown in green.

Quick example. A guitar string vibrates at 196 Hz. Its period is $T = 1/196 = 5.10 \times 10^{-3}$ s and its angular frequency is $\omega = 2\pi \times 196 = 1.23 \times 10^3 \text{ rad s}^{-1}$.

How are displacement, velocity and acceleration related?

The motion can be analysed graphically. Below, a mass is pulled to $x = +x_0$ and released from rest at $t = 0$, with $x_0 = 5.0$ cm and $T = 0.80$ s.



Aligned graphs for SHM ($x_0 = 5.0 \text{ cm}$, $T = 0.80 \text{ s}$). Dotted lines mark every quarter period. Velocity is the gradient of the $x-t$ graph, acceleration is the gradient of the $v-t$ graph; maximum speed 0.39 m s^{-1} , maximum acceleration 3.1 m s^{-2} .

- **Velocity is the gradient of the $x-t$ graph (A.1).** The gradient is zero at the extremes ($v = 0$, turning points) and steepest as the body passes through $x = 0$ (maximum speed).
- **Acceleration is the gradient of the $v-t$ graph**, and it is also given by $a = -\omega^2 x$: the $a-t$ graph is the $x-t$ graph turned upside down and scaled by ω^2 .
- All three graphs are sinusoidal with the **same period**. Each is shifted a quarter of a cycle from the next.
- The maximum acceleration is $a_{\text{max}} = \omega^2 x_0$ (from the defining equation). Here $\omega = 2\pi/0.80 = 7.85 \text{ rad s}^{-1}$, so $a_{\text{max}} = 7.85^2 \times 0.050 = 3.1 \text{ m s}^{-2}$.

Position	Displacement	Speed	Acceleration and restoring force
Extreme (turning point)	maximum, $\pm x_0$	zero	maximum, directed towards equilibrium
Equilibrium position	zero	maximum	zero
Moving away from equilibrium	increasing	decreasing	increasing; opposite to the velocity (body slows)
Moving towards equilibrium	decreasing	increasing	decreasing; same direction as the velocity (body speeds up)

TRY IT YOURSELF

C.1 Simple Harmonic Motion simulation: change the mass, spring constant and amplitude, and watch the displacement, velocity and acceleration graphs build up together.

What is the period of a mass–spring system?

For a mass m on a spring of spring constant k , Hooke's law (A.2) gives a restoring force $F = -kx$. Newton's second law then gives $a = -\frac{k}{m}x$.

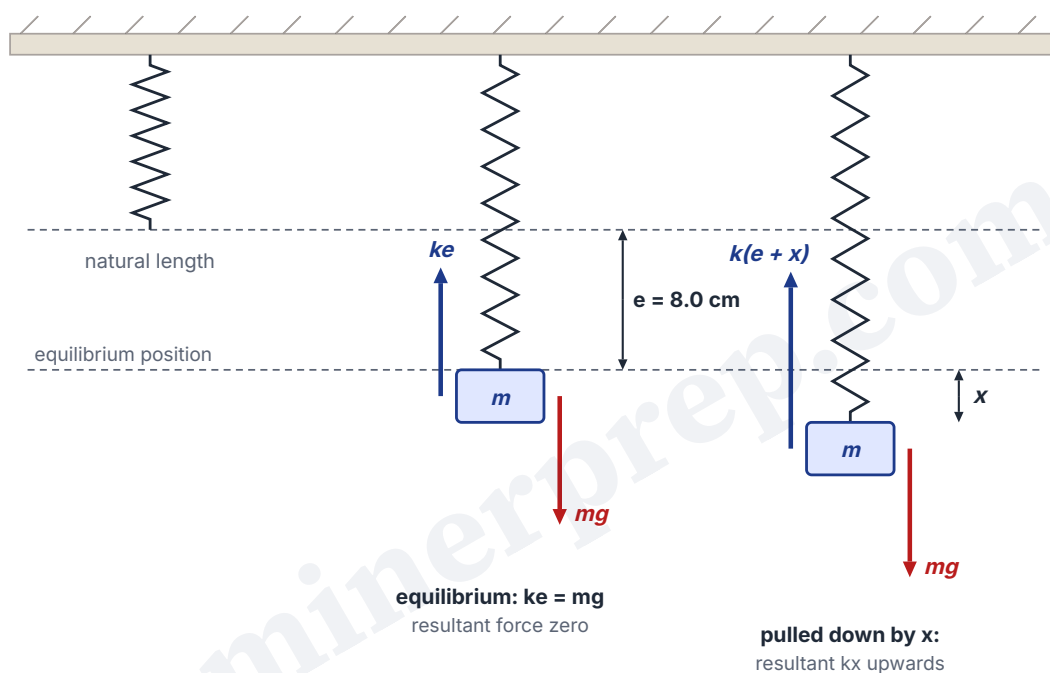
Comparing with $a = -\omega^2 x$:

$$\omega^2 = \frac{k}{m} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k}}$$

- T = period (s), m = oscillating mass (kg), k = spring constant (N m^{-1}). $T = 2\pi\sqrt{m/k}$ is in the data booklet.
- A larger mass (more inertia) gives a longer period; a stiffer spring (larger k) gives a shorter period. $T \propto \sqrt{m}$: four times the mass doubles the period.
- The period does **not** depend on the amplitude or on g . The same spring and mass have the same period horizontally, vertically, or on the Moon.
- The model assumes a light spring that obeys Hooke's law throughout the motion and no energy dissipation.

Vertical springs

When a mass hangs from a spring, the spring stretches by e until $ke = mg$. This new position is the equilibrium position for the oscillation. If the mass is pulled down a further distance x , the upward spring force becomes $k(e + x)$, so the resultant force is $k(e + x) - mg = kx$ upwards — a restoring force proportional to x , exactly as for a horizontal spring. Gravity only shifts the equilibrium position.



A 0.150 kg mass on a spring that stretches 8.0 cm at equilibrium (worked example 2). Force arrows are to scale. At equilibrium $ke = mg$; pulled 3.0 cm lower, the spring force exceeds the weight by kx , giving an upward restoring force.

WORKED EXAMPLE 1 Easy · Calculate

A trolley of mass 0.40 kg is attached to a spring of spring constant 25 N m^{-1} and oscillates on a horizontal track with negligible friction.

- (a) Calculate the period, the frequency and the angular frequency of the oscillations. (b) Determine the mass of trolley needed to double the period.

$$\text{(a)} \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.40}{25}} = \mathbf{0.79 \text{ s}} \text{ (0.795 s)}$$

$$f = \frac{1}{T} = \frac{1}{0.795} = \mathbf{1.3 \text{ Hz}} \text{ (1.26 Hz)}$$

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} = \sqrt{\frac{25}{0.40}} = \mathbf{7.9 \text{ rad s}^{-1}} \text{ (7.91 rad s}^{-1}\text{)}$$

(b) $T \propto \sqrt{m}$, so doubling T needs $4m$: $4 \times 0.40 = \mathbf{1.6 \text{ kg}}$

WORKED EXAMPLE 2 Medium · Show that / Determine / Explain

A mass of 0.150 kg hangs at rest from a light spring, which is extended by 8.0 cm. The mass is pulled down a further 3.0 cm and released from rest.

- (a) Show that the period of the oscillations is about 0.57 s. (b) Determine the magnitude and direction of the acceleration of the mass at the lowest point. (c) Explain why the motion remains simple harmonic throughout.

(a) At equilibrium $ke = mg$, so $\frac{m}{k} = \frac{e}{g}$ and $T = 2\pi\sqrt{\frac{e}{g}} = 2\pi\sqrt{\frac{0.080}{9.8}} =$

0.568 s $\approx 0.57 \text{ s}$

(Alternatively $k = \frac{0.150 \times 9.8}{0.080} = 18.375 \approx 18.4 \text{ N m}^{-1}$, then $T = 2\pi\sqrt{0.150/18.375} = 0.568 \text{ s}$; rounding k to 18.4 first gives 0.567 s.)

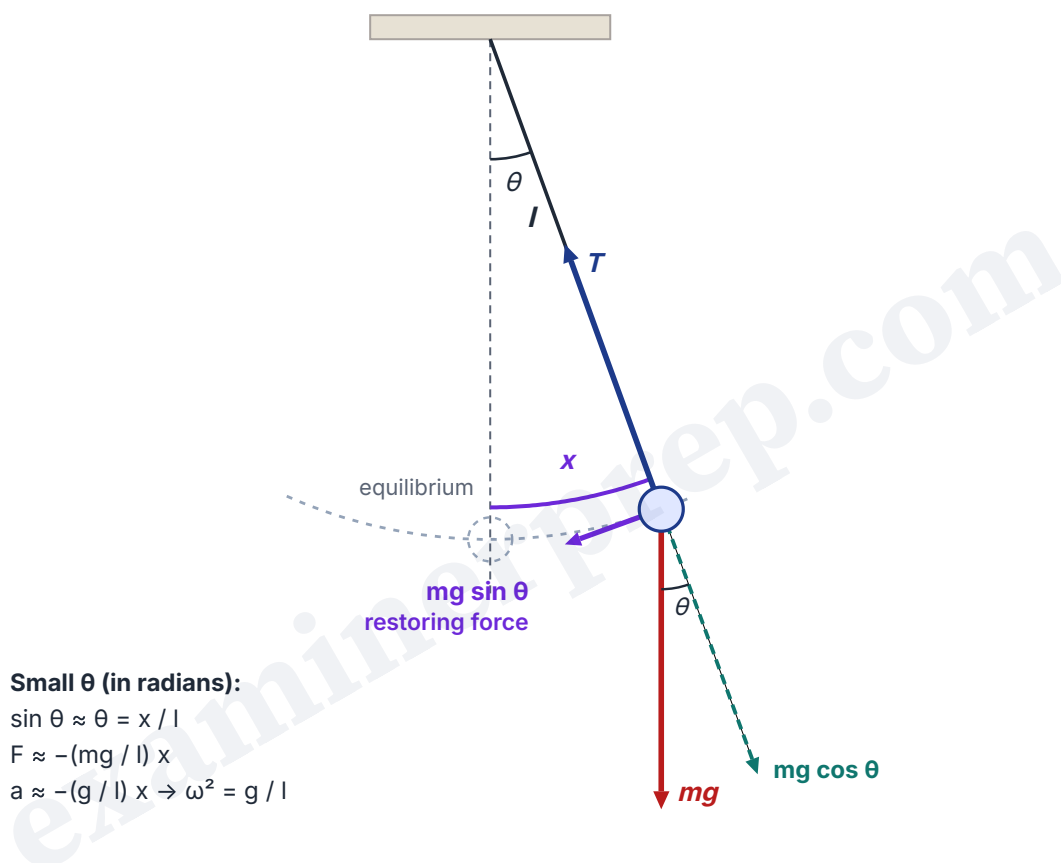
(b) The amplitude is 0.030 m. $\omega = \frac{2\pi}{0.568} = 11.1 \text{ rad s}^{-1}$, so $a_{\max} = \omega^2 x_0 = 11.07^2 \times 0.030 = \mathbf{3.7 \text{ m s}^{-2} \text{ upwards}}$ (3.68 m s⁻²), towards the equilibrium position.

(c) The amplitude (3.0 cm) is less than the equilibrium extension (8.0 cm), so even at the highest point the spring is still stretched by 5.0 cm. The spring therefore always pulls on the mass and obeys Hooke's law throughout, so the resultant force stays proportional to the displacement and directed towards equilibrium.

Check: $a_{\max} = g \times \frac{3.0}{8.0} = 3.68 \text{ m s}^{-2}$, which is less than g , consistent with the spring never going slack.

What is the period of a simple pendulum?

A simple pendulum is a small, dense bob on a light, inextensible string of length l , swinging from a fixed point. When the bob is displaced through an angle θ , the weight has a component $mg \sin \theta$ along the arc, directed back towards equilibrium. This is the restoring force. The tension and the other weight component act along the string and do not restore the bob.



Forces on a pendulum bob at the end of its swing, where $v = 0$ and the tension equals $mg \cos \theta$. The only unbalanced force is the restoring component $mg \sin \theta$, acting along the arc towards equilibrium. The angle is exaggerated for clarity.

For **small angles**, $\sin \theta \approx \theta$ (in radians), and $\theta = x/l$, where x is the displacement along the arc. The restoring force becomes $F \approx -\frac{mg}{l}x$, so $a \approx -\frac{g}{l}x$. This is SHM with $\omega^2 = g/l$:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- l = length of the pendulum, from the point of suspension to the **centre of the bob** (m); g = acceleration of free fall (gravitational field strength) (m s^{-2}). In

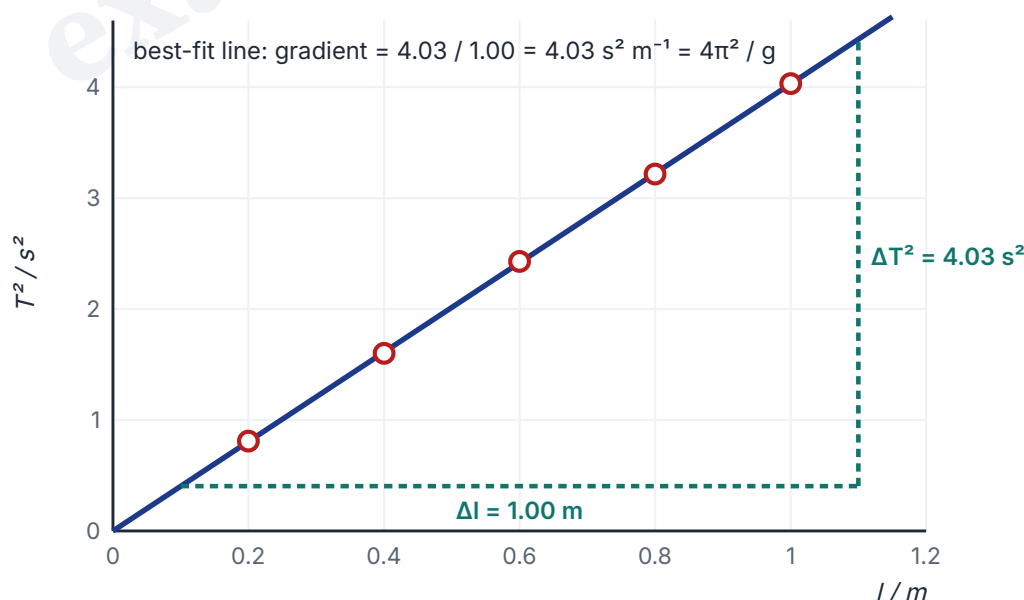
the data booklet.

- The period is **independent of the mass** of the bob: mass appears in both the restoring force and the inertia, and cancels.
- The equation is valid only for **small amplitudes**. At 10° the approximation $\sin \theta \approx \theta$ is in error by only 0.5 %; at 20° by 2.0 %. For large swings the motion is periodic but not simple harmonic.
- A pendulum is a way of measuring g : on the Moon ($g = 1.62 \text{ m s}^{-2}$) the same pendulum has a period $\sqrt{9.8/1.62} = 2.5$ times longer.

Measuring g with a pendulum

Squaring $T = 2\pi\sqrt{l/g}$ gives $T^2 = \frac{4\pi^2}{g}l$. A graph of T^2 against l is a straight line through the origin with gradient $\frac{4\pi^2}{g}$.

l / m	0.200	0.400	0.600	0.800	1.000
T / s	0.900	1.265	1.559	1.794	2.008
T^2 / s^2	0.810	1.60	2.43	3.22	4.03



T^2 against l for the data in the table (worked example 3). The points lie close to a straight line through the origin; the gradient triangle uses points on the line, not data points.

WORKED EXAMPLE 3 Medium · Determine (graph) / Explain

A student uses a light gate and data logger to measure the period of a pendulum for five different lengths and obtains the data in the table above.

(a) Use the graph to determine g . (b) Calculate the length of a pendulum with a period of 2.00 s at this location. (c) Suppose the student had measured each length to the *top* of the bob rather than to its centre. Explain the effect of this on the graph and on the value of g .

(a) Gradient of the best-fit line, from two points on the line far apart, =

$$\frac{4.43 - 0.40}{1.10 - 0.10} = \frac{4.03}{1.00} = 4.03 \text{ s}^2 \text{ m}^{-1}.$$

$$\frac{4\pi^2}{g} = 4.03 \Rightarrow g = \frac{4\pi^2}{4.03} = \mathbf{9.8 \text{ m s}^{-2}} \text{ (9.80 m s}^{-2}\text{)}$$

(b) $l = \frac{gT^2}{4\pi^2} = \frac{T^2}{\text{gradient}} = \frac{2.00^2}{4.03} = \mathbf{0.99 \text{ m}}$ (0.993 m)

(c) Every measured length would be too short by the same amount (the radius r of the bob), a systematic error. The true relationship would be

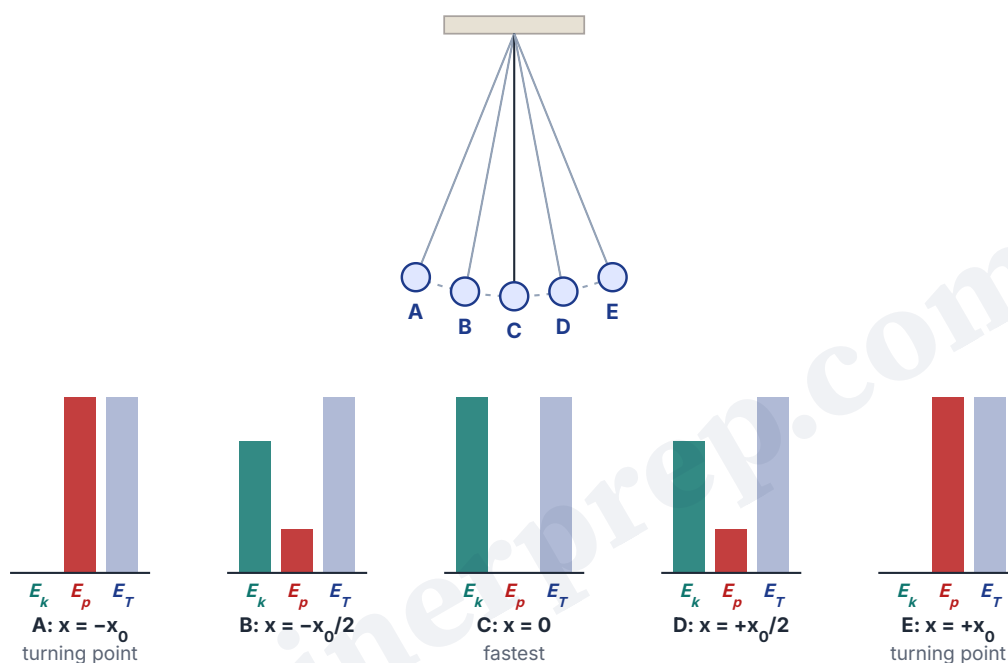
$T^2 = \frac{4\pi^2}{g}(l_{\text{measured}} + r)$, so the line would be shifted: it would no longer pass through the origin but would have a small **positive intercept on the T^2 axis**. The **gradient would be unchanged**, so the value of g found from the gradient would not be affected.

How does energy change during one oscillation?

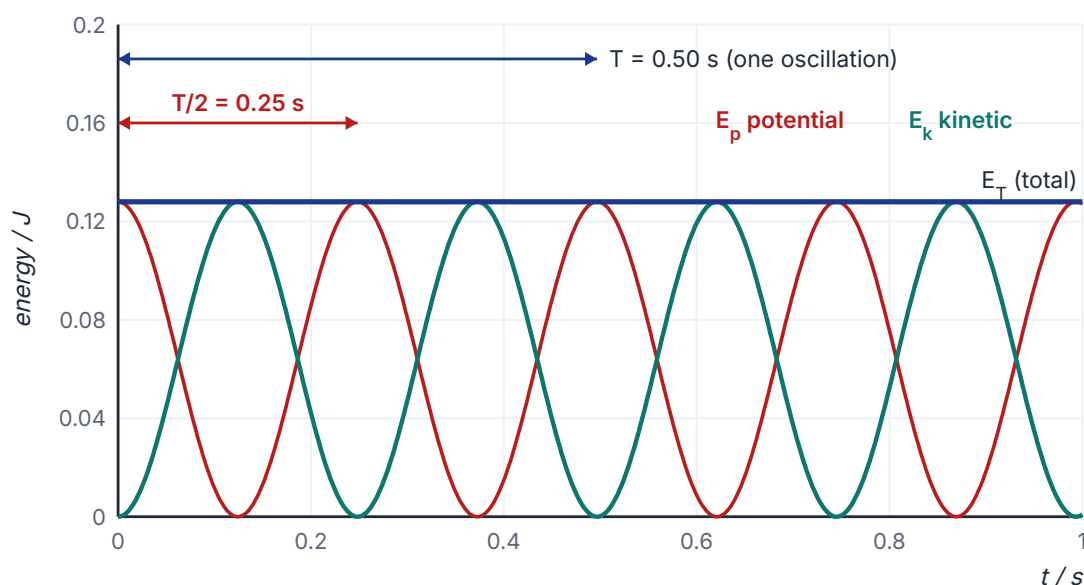
During simple harmonic motion energy is continually transferred between kinetic energy and potential energy. If no energy is dissipated, the total energy of the oscillator stays constant.

- **At the extremes** ($x = \pm x_0$): the body is momentarily at rest, so the kinetic energy is zero and the potential energy is maximum. All the energy is potential.
- **At the equilibrium position** ($x = 0$): the speed is maximum, so the kinetic energy is maximum; the potential energy is minimum (taken as zero). All the energy is kinetic.

- For a pendulum the potential energy is gravitational; for a horizontal spring it is elastic potential energy. For a vertical spring both change, but together they behave as a single potential energy that is zero at the equilibrium position.
- The total energy depends on the amplitude: a larger amplitude means more energy (it is proportional to x_0^2 , HL).



Energy bar charts for a pendulum bob at five positions. $E_k + E_p = E_T$ everywhere. Halfway to the extreme the potential energy is only a quarter of the total, because $E_p \propto x^2$ (shown quantitatively in the HL section).



Kinetic and potential energy against time for an oscillator released from rest at maximum displacement at $t = 0$ (period 0.50 s). Energies are never negative, and each reaches a maximum twice per oscillation, so they vary with period $T/2$ (frequency $2f$). The total energy is constant.

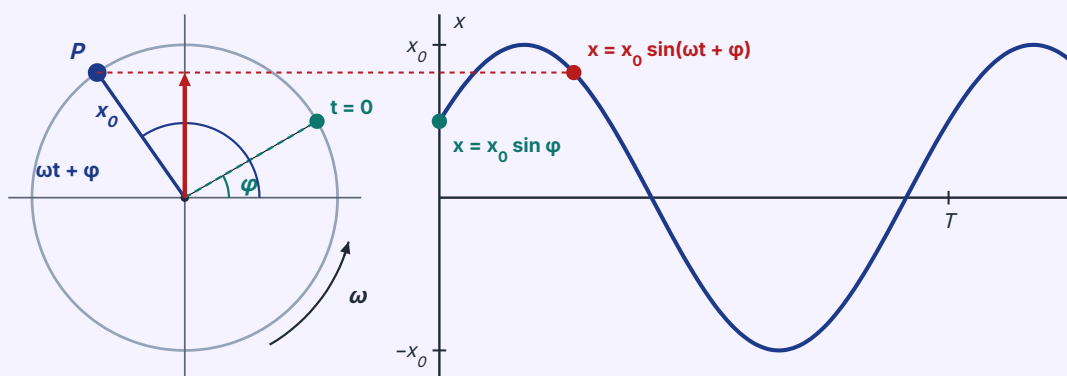
Even without equations, you can sketch these graphs from the physics: E_k is zero whenever the body is at an extreme (twice per cycle) and maximum whenever it passes through equilibrium (also twice per cycle). On an energy–displacement graph, E_p is smallest at $x = 0$ and rises symmetrically towards both extremes, while E_k does the opposite.

HL: How are phase angle and the SHM equations used?

HL Phase angle

The phase angle of an oscillator, $\omega t + \phi$, describes the stage of the cycle it has reached. One complete cycle corresponds to a phase angle of 2π radians. The initial phase ϕ fixes where the oscillation is at $t = 0$.

- Phase angles are always measured in **radians**; set your calculator to radians before evaluating $\sin(\omega t + \phi)$.
- $\phi = 0$: the body starts at equilibrium moving in the positive direction.
 $\phi = \frac{\pi}{2}$: it starts at $x = +x_0$, at rest, and $x = x_0 \cos \omega t$.
- **Phase difference** $\Delta\phi$ compares two oscillations of the same frequency. In phase: $\Delta\phi = 0$ (or 2π); in antiphase: $\Delta\phi = \pi$. A time difference Δt corresponds to $\Delta\phi = \frac{2\pi \Delta t}{T}$.
- In any SHM, the velocity is $\frac{\pi}{2}$ ahead of the displacement, and the acceleration is in antiphase with the displacement ($\Delta\phi = \pi$) — see the aligned graphs in section 3.



Reference circle. A point P moves round a circle of radius x_0 at constant angular velocity ω . Its projection on the vertical diameter moves with SHM, $x = x_0 \sin(\omega t + \phi)$. The phase angle $\omega t + \phi$ is the angle of P ; ϕ is its angle at $t = 0$.

The SHM equations

$$x = x_0 \sin(\omega t + \phi) \qquad v = \omega x_0 \cos(\omega t + \phi) \qquad v = \pm \omega \sqrt{x_0^2 - x^2}$$

- x = displacement at time t (m), x_0 = amplitude (m), v = velocity (m s^{-1}), ω = angular frequency (rad s^{-1}), ϕ = initial phase (rad). All three are in the data booklet.
- The maximum speed is $v_{\text{max}} = \omega x_0$, reached at $x = 0$. The maximum acceleration is $a_{\text{max}} = \omega^2 x_0$, reached at $x = \pm x_0$.
- $v = \pm \omega \sqrt{x_0^2 - x^2}$ gives the speed at a given *position* without needing the time. The $\pm$ is there because the body passes each position twice per cycle, once in each direction.

WORKED EXAMPLE 4 HL · Medium · Determine

A point on a vibrating test platform moves with SHM of amplitude 0.060 m and frequency 1.6 Hz. At $t = 0$ its displacement is +0.030 m and it is moving in the positive direction.

- (a) Determine the initial phase ϕ . (b) Calculate the displacement and the velocity at $t = 0.40$ s. (c) Calculate the speed of the point when its displacement is +0.030 m, and its maximum speed.

$$\omega = 2\pi f = 2\pi \times 1.6 = 10.05 \text{ rad s}^{-1}.$$

(a) $0.030 = 0.060 \sin \phi \Rightarrow \sin \phi = 0.50$, so $\phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. The velocity at $t = 0$ is positive, so $\cos \phi > 0$: $\phi = \pi/6 = \mathbf{0.52 \text{ rad}}$

(b) Phase angle $= \omega t + \phi = 10.05 \times 0.40 + 0.524 = 4.54 \text{ rad}$.
 $x = 0.060 \sin(4.54) = \mathbf{-0.059 \text{ m}}$ (-0.0592 m)

$v = \omega x_0 \cos(4.54) = 10.05 \times 0.060 \times \cos(4.54) = \mathbf{-0.10 \text{ m s}^{-1}}$
 (-0.101 m s^{-1})

The point is close to $-x_0$, still moving in the negative direction and about to turn round ($3\pi/2 = 4.71 \text{ rad}$ is the turning point).

(c) $v = \pm \omega \sqrt{x_0^2 - x^2} = 10.05 \sqrt{0.060^2 - 0.030^2} = \mathbf{0.52 \text{ m s}^{-1}}$
 (0.522 m s^{-1})

$$v_{\max} = \omega x_0 = 10.05 \times 0.060 = \mathbf{0.60 \text{ m s}^{-1}}$$

Check: $v(0) = \omega x_0 \cos(\pi/6) = 0.522 \text{ m s}^{-1}$ — the same as (c), as it must be, because at $t = 0$ the displacement is $+0.030 \text{ m}$.

Energy in SHM (quantitative)

$$E_T = \frac{1}{2}m\omega^2 x_0^2 \qquad E_p = \frac{1}{2}m\omega^2 x^2 \qquad E_k = E_T - E_p = \frac{1}{2}m\omega^2$$

- E_T = total energy (J), E_p = potential energy at displacement x (J), m = mass (kg). E_T and E_p are in the data booklet; the E_k form is not, but follows from $E_k = E_T - E_p$ or from $\frac{1}{2}mv^2$ with $v = \omega \sqrt{x_0^2 - x^2}$.
- $E_T \propto x_0^2$: doubling the amplitude gives four times the energy. For a mass-spring system $m\omega^2 = k$, so $E_T = \frac{1}{2}kx_0^2$.
- $E_k = E_p$ when $x^2 = \frac{1}{2}x_0^2$, i.e. at $x = \pm \frac{x_0}{\sqrt{2}} \approx \pm 0.71x_0$ — not at half the amplitude.

WORKED EXAMPLE 5 HL · Hard · Calculate / Determine

A block of mass 0.25 kg on a spring of spring constant 40 N m^{-1} oscillates on a smooth horizontal surface with amplitude 0.080 m .

- (a) Calculate the total energy of the oscillator. (b) Determine the speed of the block when its displacement is 0.050 m. (c) Determine the displacements at which the kinetic energy equals the potential energy. (d) Determine the frequency at which the kinetic energy of the block varies.

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{40}{0.25}} = 12.6 \text{ rad s}^{-1} \quad (\omega^2 = 160 \text{ s}^{-2}).$$

(a) $E_T = \frac{1}{2}m\omega^2x_0^2 = \frac{1}{2} \times 0.25 \times 160 \times 0.080^2 = \mathbf{0.128 \text{ J}}$

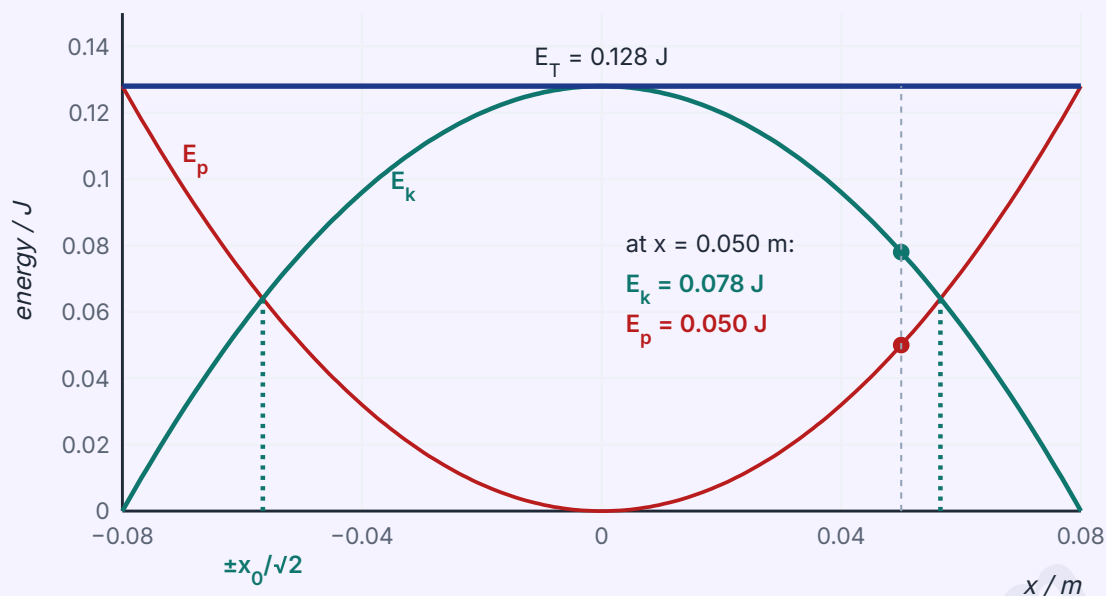
(b) $E_p = \frac{1}{2} \times 0.25 \times 160 \times 0.050^2 = 0.050 \text{ J}$, so $E_k = 0.128 - 0.050 = 0.078 \text{ J}$.

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 0.078}{0.25}} = \mathbf{0.79 \text{ m s}^{-1}} \quad (0.790 \text{ m s}^{-1})$$

Check: $v = \omega \sqrt{x_0^2 - x^2} = 12.65 \sqrt{0.080^2 - 0.050^2} = 0.790 \text{ m s}^{-1}$.

(c) $E_k = E_p \Rightarrow E_p = \frac{1}{2}E_T \Rightarrow x^2 = \frac{1}{2}x_0^2 \Rightarrow x = \pm \frac{0.080}{\sqrt{2}} = \mathbf{\pm 0.057 \text{ m}}$ ($\pm 0.0566 \text{ m}$)

(d) $T = \frac{2\pi}{\omega} = 0.497 \text{ s}$, so the block oscillates at $f = 2.01 \text{ Hz}$. The kinetic energy reaches a maximum each time the block passes through equilibrium — twice per oscillation — so it varies at **4.0 Hz** (4.03 Hz), twice the frequency of the motion.



Energy against displacement for worked example 5 ($m = 0.25 \text{ kg}$, $k = 40 \text{ N m}^{-1}$, $x_0 = 0.080 \text{ m}$). E_p is a parabola, E_k an inverted parabola, and their sum is the constant $E_T = 0.128 \text{ J}$. The curves cross at $x = \pm x_0/\sqrt{2}$.

Exam tips for C.1

EXAM TIP 1 — THE DEFINITION HAS TWO PARTS

When asked to state the conditions for SHM, give both: the acceleration (or restoring force) is **proportional to the displacement from the equilibrium position** and is **directed towards the equilibrium position** (opposite to the displacement). "Proportional to the distance" without the direction, or "towards the centre" without proportionality, earns only part of the credit. Say *displacement from equilibrium*, not just "position".

EXAM TIP 2 — ENERGY GRAPHS REPEAT TWICE PER CYCLE

When sketching E_k or E_p against time, the curve must stay between 0 and E_T , never go negative, and have **two peaks per oscillation period**. Check the starting value too: an oscillator released from rest at an extreme has

$E_k = 0$ at $t = 0$. Drawing an energy curve with the same period as the displacement is a very common error.

EXAM TIP 3 — QUARTER PERIODS

The time from an extreme to the equilibrium position is $T/4$, not $T/2$. A pendulum released from rest first reaches maximum speed a quarter of a period after release, and first returns to its starting point after a full period T .

EXAM TIP 4 — “SHOW THAT” AND RADIANS

In a “Show that” question, write the substituted equation and give the answer to **one more significant figure** than the value quoted (0.568 s, not 0.57 s). In HL calculations with $\sin(\omega t + \phi)$, the calculator must be in **radian** mode — a degree-mode answer scores nothing for that step.

EXAM TIP 5 — WHAT THE PERIOD DEPENDS ON

Learn the dependencies: a pendulum's period depends only on l and g (not on mass or, for small angles, amplitude); a mass-spring period depends only on m and k (not on g or amplitude). Multiple-choice questions often change one of these quantities — use proportionality ($T \propto \sqrt{l}$, $T \propto \sqrt{m}$, $T \propto 1/\sqrt{k}$) rather than recalculating.

Common misconceptions

Misconception	Why it is wrong	Correct idea
A larger amplitude gives a longer period.	The body travels further but also moves faster, because the restoring force is larger.	In SHM the period is independent of the amplitude.
A heavier pendulum bob swings more slowly.	Mass appears in both the restoring force ($mg \sin \theta$) and the inertia, and cancels.	$T = 2\pi\sqrt{l/g}$ does not contain the mass.
The acceleration is zero at the extremes because the body stops there.	Zero velocity does not mean zero acceleration; the restoring force is largest there.	At $x = \pm x_0$, $v = 0$ and $ a $ is maximum, $\omega^2 x_0$.
The acceleration is always in the direction of motion.	The acceleration points towards equilibrium; when the body moves away from equilibrium it is slowing down.	The acceleration is always opposite to the <i>displacement</i> ; it is along the velocity when the body moves towards equilibrium and against it when the body moves away.
Kinetic energy oscillates at the same frequency as the displacement.	E_k is maximum each time the body passes equilibrium, which happens twice per cycle.	Energies vary with frequency $2f$ and period $T/2$.
A mass–spring system oscillates faster on Earth than on the Moon.	g only shifts the equilibrium position of a vertical spring.	$T = 2\pi\sqrt{m/k}$ is independent of g ; only the pendulum period depends on g .

How this topic connects

Shadows of a rotating point: picturing SHM through circular motion

Imagine a point moving round a circle of radius x_0 at constant angular velocity ω (A.2). Its shadow (projection) on a diameter moves back and forth between $-x_0$ and $+x_0$. The centripetal acceleration of the point is $\omega^2 x_0$ towards the centre, and its component along the diameter is $-\omega^2 x$ — exactly the SHM condition. This is why the same ω appears in both topics, why one oscillation corresponds to 2π radians, and why the phase angle is literally an angle on the reference circle (see the HL section).

Energy loss in oscillators: what damping does to amplitude and period

Real oscillators lose energy to their surroundings through friction and air resistance (dissipative forces, A.3). The total energy then decreases with time, and since the energy depends on the square of the amplitude, the amplitude decreases too. For light damping the period stays almost the same — the oscillations simply die away gradually. Heavier damping makes the amplitude fall faster. Light, critical and heavy damping, and the effect of damping on driven oscillators, are developed in C.4.

Oscillating particles, travelling waves: SHM as the building block of the wave model

In a mechanical wave (C.2), each particle of the medium oscillates about its own equilibrium position, to a good approximation with SHM, while the wave carries energy past it. Neighbouring particles oscillate with the same frequency but with a phase difference that grows with distance along the wave. The language of C.1 — amplitude, period, frequency and phase — therefore carries straight over to waves, and a displacement–time graph for one particle in a wave is the SHM graph of section 2.

Tools & inquiry — timing oscillations (NOS: models and approximations)

- **Time many oscillations:** measure the time for 10–20 oscillations and divide. Your reaction-time uncertainty (about 0.2 s) is spread over all of them, so the percentage uncertainty in T is much smaller.
- **Use a fiducial marker** at the equilibrium position and start and stop the timer as the bob passes it: the bob moves fastest there, so the instant of

passing is sharpest.

- **Video analysis:** filming the oscillator next to a ruler and a timer lets you plot its displacement against time frame by frame and see the sinusoidal shape directly.
- **Linearise:** plot T^2 against l (pendulum) or T^2 against m (spring, gradient $4\pi^2/k$) to get a straight line; a non-zero intercept reveals a systematic error.
- **NOS — the small-angle approximation:** $T = 2\pi\sqrt{l/g}$ comes from an idealised model (point mass, light string, $\sin \theta \approx \theta$). Scientists accept such models because they know their range of validity: keep the amplitude below about 10° and the model's error is smaller than typical timing uncertainties.

Key equations

Equation	Use	Level	In data booklet?
$a = -\omega^2 x$	defining equation of SHM	SL & HL	yes
$T = \frac{1}{f} = \frac{2\pi}{\omega}$	period, frequency, angular frequency	SL & HL	yes
$T = 2\pi\sqrt{\frac{m}{k}}$	mass-spring system	SL & HL	yes
$T = 2\pi\sqrt{\frac{l}{g}}$	simple pendulum (small angles)	SL & HL	yes
$a_{\max} = \omega^2 x_0$	maximum acceleration	SL & HL	no — from $a = -\omega^2 x$ at $x = x_0$
$x = x_0 \sin(\omega t + \phi)$	displacement at time t	HL	yes
$v = \omega x_0 \cos(\omega t + \phi)$	velocity at time t	HL	yes
$v = \pm\omega\sqrt{x_0^2 - x^2}$	velocity at displacement x ; $v_{\max} = \omega x_0$	HL	yes ($v_{\max}$ follows from it)
$E_T = \frac{1}{2}m\omega^2 x_0^2$	total energy	HL	yes
$E_p = \frac{1}{2}m\omega^2 x^2$	potential energy at displacement x	HL	yes
$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$	kinetic energy at displacement x	HL	no — use $E_T - E_p$

Check your understanding

1. A pendulum has a period of 1.50 s on the Earth. Calculate its period on the Moon, where $g = 1.62 \text{ m s}^{-2}$.

$T \propto \frac{1}{\sqrt{g}}$, so $T_{\text{Moon}} = 1.50 \sqrt{\frac{9.8}{1.62}} = 3.7 \text{ s}$ (3.69 s). The length is unchanged and the mass is irrelevant.

2. The mass on a spring is replaced by one four times as large. State and explain the effect on the period and on the frequency.

$T = 2\pi \sqrt{m/k}$, so $T \propto \sqrt{m}$: the period **doubles** and the frequency **halves**. The larger inertia means the same restoring force produces a smaller acceleration.

3. A mass on a spring is released from rest at maximum displacement at $t = 0$. Its period is 0.80 s. State the times during the first oscillation at which (a) its speed is greatest, (b) the magnitude of its acceleration is greatest.

(a) At equilibrium: $t = T/4 = 0.20 \text{ s}$ and $t = 3T/4 = 0.60 \text{ s}$. (b) At the extremes: $t = 0$, $t = T/2 = 0.40 \text{ s}$ and $t = T = 0.80 \text{ s}$.

4. A graph of acceleration against displacement for an oscillator is a straight line through the origin with gradient -36 s^{-2} . Determine the frequency of the oscillation.

Gradient $= -\omega^2 \Rightarrow \omega = \sqrt{36} = 6.0 \text{ rad s}^{-1}$. $f = \frac{\omega}{2\pi} = \frac{6.0}{2\pi} = 0.95 \text{ Hz}$ (0.955 Hz).

5. (HL) An oscillator has amplitude 5.0 cm and angular frequency 8.0 rad s^{-1} . Calculate its maximum speed and its speed when the displacement is 3.0 cm.

$v_{\text{max}} = \omega x_0 = 8.0 \times 0.050 = 0.40 \text{ m s}^{-1}$. At $x = 0.030 \text{ m}$: $v = \omega \sqrt{x_0^2 - x^2} = 8.0 \sqrt{0.050^2 - 0.030^2} = 8.0 \times 0.040 = 0.32 \text{ m s}^{-1}$.

Practise C.1 with exam-style questions

Original Paper 1A, 1B and 2 questions on simple harmonic motion, with full mark schemes. Tick the questions you want and print them as your own test.

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