

THEME D · FIELDS

SL &amp; HL

≈ 35 min read

# IB Physics D.1 Gravitational Fields — Notes (SL & HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). Sections 1–5 are for SL and HL. D.1 has a large block of additional HL content — gravitational potential energy and potential, equipotentials, escape and orbital speed, orbital energy and the effect of drag — which is in the three purple **HL** sections that follow.

 SL D.1 questions HL D.1 questions Flashcards Orbital motion simulation Escape velocity simulation

## What this page covers

- ✓ Kepler's three laws of planetary (orbital) motion
- ✓ Newton's universal law of gravitation,  $F = G \frac{m_1 m_2}{r^2}$ , for bodies treated as point masses
- ✓ When an extended body, such as a planet, can be treated as a point mass
- ✓ Gravitational field strength as force per unit mass,  $g = \frac{F}{m} = G \frac{M}{r^2}$ , including the resultant field on a line joining two bodies
- ✓ Drawing and interpreting gravitational field lines, for radial fields and for the uniform field near a surface
- ✓ Circular orbits and Kepler's third law from Newton's law
- ✓ Gravitational potential energy as the work done to assemble a system from infinite separation,  $E_p = -G \frac{m_1 m_2}{r}$  (HL)
- ✓ Gravitational potential,  $V_g = -G \frac{M}{r}$ , and the work done  $W = m \Delta V_g$  (HL)

- ✓ Field strength as potential gradient,  $g = -\frac{\Delta V_g}{\Delta r}$ ; equipotential surfaces and how they relate to field lines (HL)
- ✓ Escape speed  $v_{esc} = \sqrt{\frac{2GM}{r}}$  and orbital speed  $v_{orbital} = \sqrt{\frac{GM}{r}}$ ; energy changes when satellites are launched, change orbit or escape (HL)
- ✓ What a small drag force from the upper atmosphere does to a satellite's orbital height and speed (HL)

### In one minute

- Planets move in ellipses with the Sun at one focus, sweep out equal areas in equal times, and obey  $T^2 \propto r^3$  (Kepler).
- Every pair of point masses attracts with  $F = G\frac{m_1m_2}{r^2}$  — an inverse-square law. A uniform sphere acts as if all its mass were at its centre, so  $r$  is always measured **between centres**.
- Gravitational field strength is force per unit mass:  $g = \frac{F}{m} = \frac{GM}{r^2}$  (N kg<sup>-1</sup>). Field lines show the direction of the force on a test mass; closer lines mean a stronger field.
- In a circular orbit gravity provides the centripetal force:  $\frac{GMm}{r^2} = \frac{mv^2}{r}$ , which gives  $T^2 = \frac{4\pi^2}{GM}r^3$ .
- HL:  $E_p = -\frac{GMm}{r}$ ,  $V_g = -\frac{GM}{r}$  (zero at infinity, negative everywhere else),  $g = -\frac{\Delta V_g}{\Delta r}$ ,  $W = m\Delta V_g$ .
- HL: in orbit  $E_k = \frac{GMm}{2r}$ ,  $E_T = -\frac{GMm}{2r}$ ; to escape,  $E_k + E_p \geq 0$ , giving  $v_{esc} = \sqrt{2GM/r}$ . Drag makes a satellite fall to a lower orbit and **speed up**.

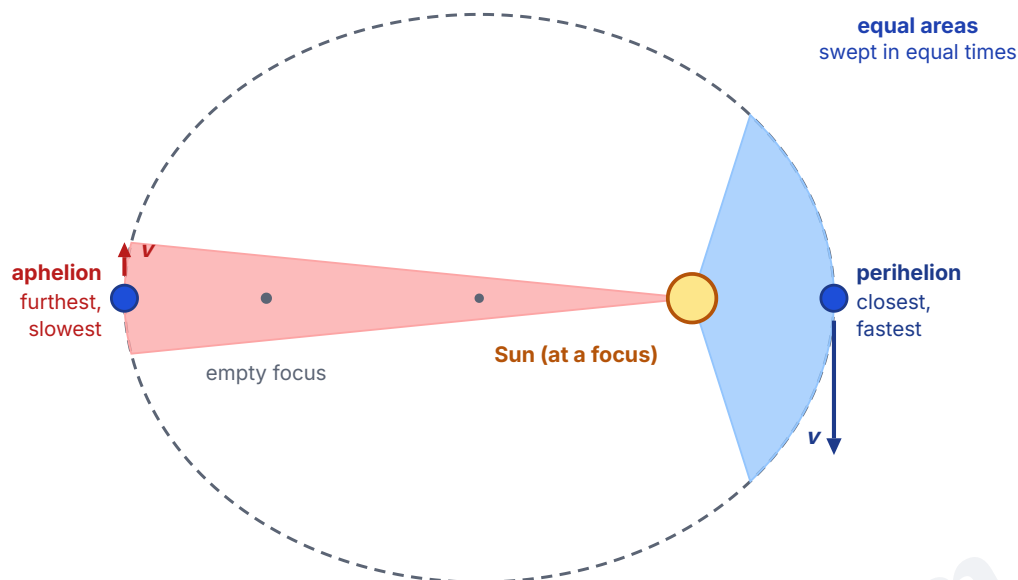
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## What are Kepler's three laws of orbital motion?

Johannes Kepler analysed decades of careful naked-eye observations of the planets and found three empirical rules. They describe *how* planets move; Newton's law of gravitation later explained *why*.

| Law                  | Statement                                                                                                            | What it tells you                                                                                                          |
|----------------------|----------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------|
| 1st — law of orbits  | Each planet moves in an <b>ellipse</b> with the Sun at one <b>focus</b> .                                            | The planet–Sun distance changes during the orbit. A circle is the special case where both foci coincide at the centre.     |
| 2nd — law of areas   | The line joining a planet to the Sun <b>sweeps out equal areas in equal times</b> .                                  | A planet moves fastest when closest to the Sun (perihelion) and slowest when furthest away (aphelion).                     |
| 3rd — law of periods | The <b>square of the orbital period is proportional to the cube of the mean orbital radius</b> : $T^2 \propto r^3$ . | Outer planets take much longer to orbit. The constant $T^2/r^3$ is the same for every body orbiting the same central mass. |



*Shaded areas are equal (computed); eccentricity exaggerated — real planetary orbits are nearly circular*

*Kepler's first and second laws. The Sun is at one focus of the elliptical orbit. The two shaded sectors are swept out in the same time (one-tenth of the period each) and have equal areas, so the planet covers a longer arc, and therefore moves faster, near perihelion.*

- For a **circular** orbit, the second law simply means the speed is constant, and  $r$  in the third law is the radius of the circle. All D.1 calculations use circular orbits.
- The third law applies to any set of bodies orbiting the **same** central mass — the planets round the Sun, or the moons round Jupiter — but the constant is different for each central body.
- Using ratios: if one moon's orbital radius is 4 times another's, its period is  $4^{3/2} = 8$  times longer.

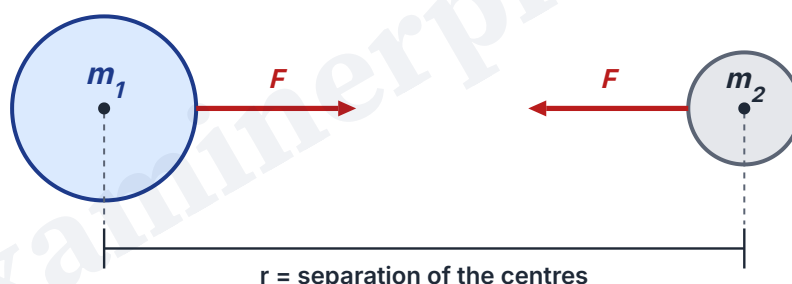
## What does Newton's universal law of gravitation state?

Every point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.

$$F = G \frac{m_1 m_2}{r^2}$$

- $F$  = magnitude of the gravitational force on each mass (N);  $m_1, m_2$  = the masses (kg);  $r$  = separation of the (centres of the) masses (m);  $G$  = gravitational constant,  $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ . Equation and constant are in the data booklet.
- The force is always **attractive** and acts along the line joining the two masses.
- The forces on the two masses are a Newton's third-law pair: **equal in size and opposite in direction**, even if one mass is enormous and the other tiny. The Earth pulls you with the same size of force that you pull the Earth.
- It is an **inverse-square law**: doubling  $r$  gives  $\frac{1}{4}$  of the force; tripling  $r$  gives  $\frac{1}{9}$ .
- $G$  is tiny, so gravity between everyday objects is negligible: two 1.0 kg spheres 10 cm apart attract with only  $6.67 \times 10^{-9} \text{ N}$ . Gravity dominates only when at least one mass is astronomically large.

*equal in size, opposite in direction, attractive (Newton's third-law pair)*



*Two uniform spheres attract with forces of equal magnitude in opposite directions. The separation  $r$  in Newton's law is measured between the centres, not between the surfaces.*

### When can an extended body be treated as a point mass?

- **Spheres:** a sphere of uniform density (or one made of uniform concentric shells, like a planet) attracts any mass *outside* it exactly as if all its mass were concentrated at its **centre**.
- **Distant bodies:** any body, whatever its shape, can be treated as a point mass when its separation from the other body is **much larger than its own size**. A satellite of size 10 m orbiting at 7000 km is a point mass to excellent accuracy.
- This is why, for a person of mass 70 kg on the Earth's surface (Earth mass  $5.97 \times 10^{24} \text{ kg}$ , radius  $6.37 \times 10^6 \text{ m}$ ),  $F =$

$$\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 70}{(6.37 \times 10^6)^2} = 687 \text{ N}$$
 — the same as  $mg$ , with  $r$  equal to the Earth's radius.

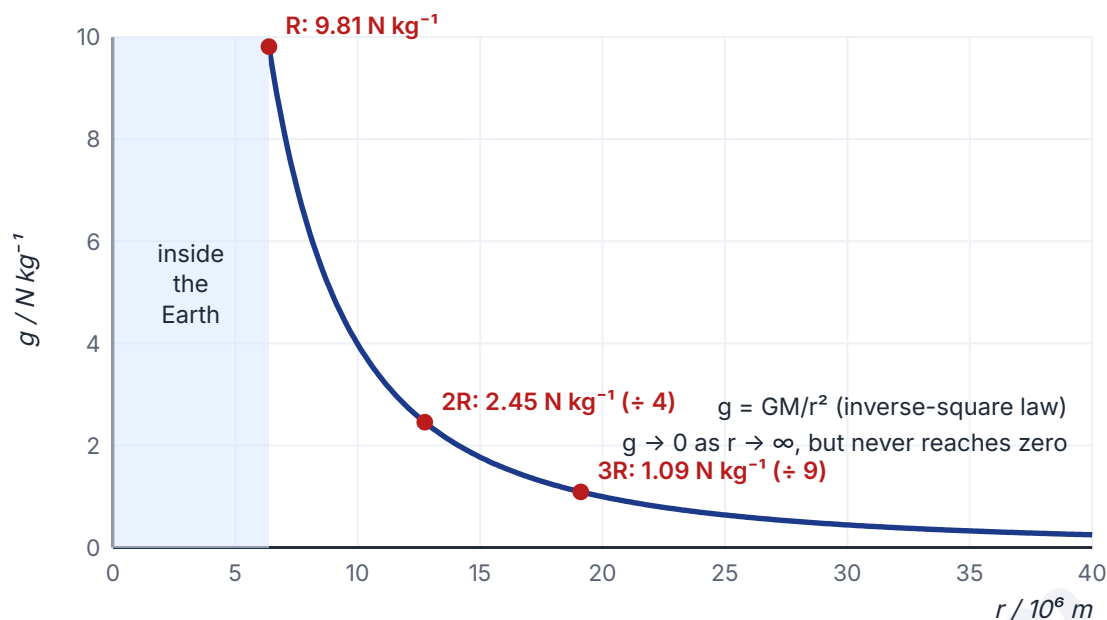
## What is gravitational field strength?

A **field** is a region in which a body experiences a force. Instead of thinking of the planet reaching out to pull each object, we say the planet creates a gravitational field around it, and any mass placed in that field feels a force.

The gravitational field strength at a point is the gravitational force per unit mass experienced by a small point mass placed at that point.

$$g = \frac{F}{m} = G \frac{M}{r^2}$$

- $g$  = gravitational field strength ( $\text{N kg}^{-1}$ , equivalent to  $\text{m s}^{-2}$ );  $F$  = force on the test mass (N);  $m$  = small test mass (kg);  $M$  = mass creating the field (kg);  $r$  = distance from the centre of  $M$  (m). Both forms are in the data booklet.
- The test mass must be **small** so that its own field does not disturb the field being measured.
- $g$  is a **vector**: it points in the direction of the force on a mass, i.e. towards the centre of  $M$ .
- $g$  does not depend on the test mass. Near the Earth's surface,  $g = 9.8 \text{ N kg}^{-1}$  for a feather or a car; a falling body in this field (with no air resistance) accelerates at  $9.8 \text{ m s}^{-2}$ .



Gravitational field strength against distance from the centre of the Earth ( $M = 5.97 \times 10^{24} \text{ kg}$ ,  $R = 6.37 \times 10^6 \text{ m}$ ). Outside the Earth  $g$  follows an inverse-square law: at  $2R$  it is a quarter of the surface value, at  $3R$  a ninth. Only the region outside the surface is required.

### WORKED EXAMPLE 1 Easy · Calculate

A planet has mass  $4.87 \times 10^{24} \text{ kg}$  and radius  $6.05 \times 10^6 \text{ m}$ . A space probe lander of mass  $1200 \text{ kg}$  is on its surface.

(a) Calculate the gravitational field strength at the surface. (b) Calculate the gravitational force on the lander. (c) Calculate the field strength at a height above the surface equal to the planet's radius.

$$\text{(a) } g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 4.87 \times 10^{24}}{(6.05 \times 10^6)^2} = 8.87 \text{ N kg}^{-1}$$

$$\text{(b) } F = mg = 1200 \times 8.87 = 1.06 \times 10^4 \text{ N (the same as } GMm/R^2)$$

$$\text{(c) The distance from the centre is } R + R = 2R, \text{ so } g \text{ falls by a factor of } 2^2 = 4: g = \frac{8.87}{4} = 2.22 \text{ N kg}^{-1}$$

## Resultant field of two bodies

Field strengths add as vectors. On the line joining two bodies (the only case you need), the two fields point in opposite directions between them, so their magnitudes subtract. At one point — the **null point** or **neutral point** — they

cancel. Because the fields depend on  $M/r^2$ , the null point is always **closer to the smaller mass**.

**WORKED EXAMPLE 2** Medium · Determine / Explain

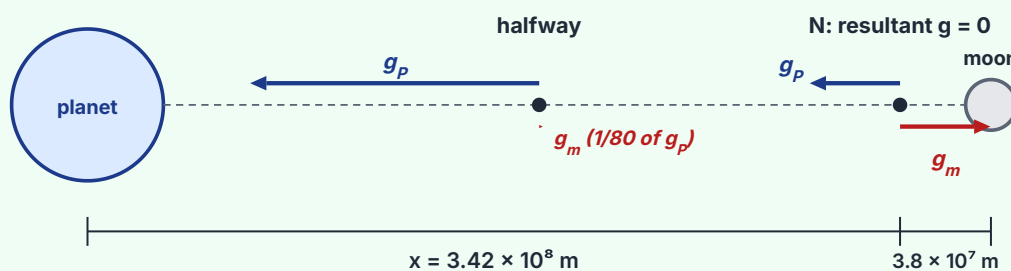
A planet of mass  $6.0 \times 10^{24}$  kg has a moon of mass  $7.5 \times 10^{22}$  kg. The distance between their centres is  $3.8 \times 10^8$  m.

(a) Determine the distance from the centre of the planet to the point on the line between them where the resultant gravitational field strength is zero. (b) Calculate the resultant field strength halfway between the centres and state its direction.

**(a)** Let the point be a distance  $x$  from the planet. At this point  $\frac{GM_P}{x^2} = \frac{GM_m}{(d-x)^2}$ , so  $\frac{x}{d-x} = \sqrt{\frac{M_P}{M_m}} = \sqrt{\frac{6.0 \times 10^{24}}{7.5 \times 10^{22}}} = \sqrt{80} = 8.94$ .  
 $x = 8.94(3.8 \times 10^8 - x) \Rightarrow x = \frac{8.94 \times 3.8 \times 10^8}{9.94} = 3.4 \times 10^8 \text{ m}$  ( $3.42 \times 10^8 \text{ m}$ ), i.e.  $3.8 \times 10^7 \text{ m}$  from the moon's centre.

**(b)** At  $r = 1.9 \times 10^8 \text{ m}$  from each:  $g_P = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(1.9 \times 10^8)^2} = 1.11 \times 10^{-2} \text{ N kg}^{-1}$  and  $g_m = \frac{6.67 \times 10^{-11} \times 7.5 \times 10^{22}}{(1.9 \times 10^8)^2} = 1.39 \times 10^{-4} \text{ N kg}^{-1}$ .

Resultant  $= 1.11 \times 10^{-2} - 1.39 \times 10^{-4} = 1.1 \times 10^{-2} \text{ N kg}^{-1}$  **towards the planet** ( $1.09 \times 10^{-2} \text{ N kg}^{-1}$ )



Not to scale in size; separations to scale. Arrows at N equal length; at the halfway point the planet's field dominates.

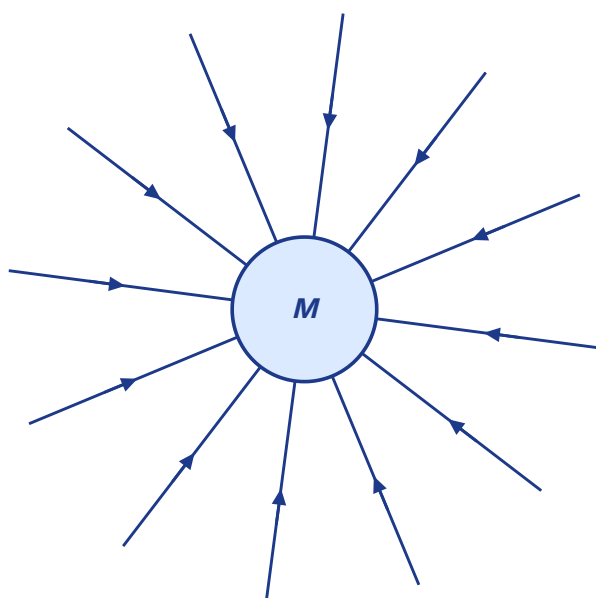
The answers to worked example 2. At N the two field strengths are equal and opposite, so the resultant is zero. At the halfway point the planet's field is 80 times the moon's, so the resultant points towards the planet.

Check: the cancellation point is only 10 % of the way from the moon, as expected for a mass ratio of 80. The square root, not the mass ratio itself, sets the distance ratio.

## How are gravitational field lines drawn and interpreted?

A gravitational field line shows the direction of the force on a small test mass placed in the field; the closer together the lines, the stronger the field.

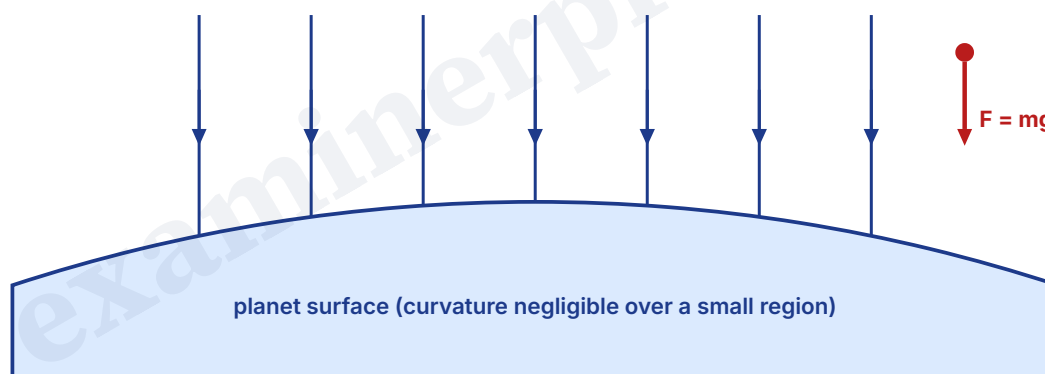
- **Arrows** on field lines always point **towards** the mass (gravity only attracts).
- Lines **never cross**: the field has only one direction at each point.
- Lines meet the surface of a spherical body at **right angles** and are **radial** — they point to the centre.
- The **spacing** shows the strength: radial lines spread out with distance, matching  $g \propto 1/r^2$ .



- lines point towards the mass  
(direction of force on a test mass)
- radial: they meet the surface at right angles
- lines spread out with distance:  
wider spacing = weaker field  
( $g \propto 1/r^2$ )
- lines never cross

*The radial field around a spherical (or point) mass. All lines point towards the centre and spread out with distance, showing that the field weakens.*

*parallel, equally spaced, perpendicular to the surface: uniform field, the same  $g$  everywhere in the region*



*Close to a planet's surface — over a region small compared with the planet's radius — the field lines are almost parallel and equally spaced. The field is then treated as uniform: the same  $g$ , in size and direction, everywhere in the region.*

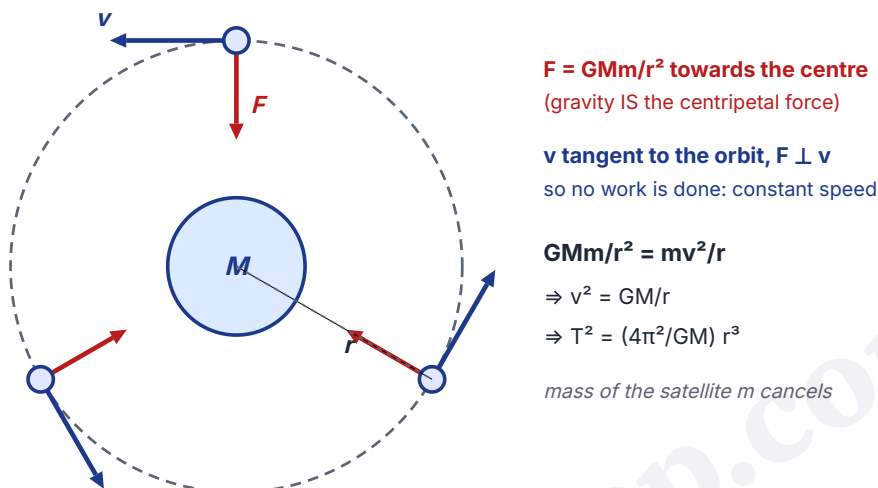
The uniform-field model is what you used in Theme A (projectiles,  $E_p = mg\Delta h$ ). It is an approximation: it works when the height changes are much smaller than the radius of the planet. For satellites, rockets and planets the radial model is needed.

## How does gravity keep a satellite in a circular orbit?

A satellite (natural, like a moon, or artificial) in a circular orbit moves at constant speed but its direction keeps changing, so it accelerates towards the centre

(A.2). The **only** force on it is gravity, so gravity provides the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r} = \frac{4\pi^2mr}{T^2}$$



A satellite in a circular orbit of radius  $r$ . The gravitational force (red) always points to the centre and is perpendicular to the velocity (blue), so it changes the direction of motion but not the speed.

Rearranging the right-hand equality gives Kepler's third law from Newton's law:

$$T^2 = \frac{4\pi^2}{GM} r^3$$

- $T$  = orbital period (s);  $r$  = orbital radius, measured from the **centre** of the central body ( $m$ ) = radius of the planet + height above the surface;  $M$  = mass of the **central** body (kg).
- This equation is **not in the data booklet**: derive it from  $F = G \frac{Mm}{r^2}$  and  $a = \frac{4\pi^2r}{T^2}$  (both in the data booklet). This derivation is often asked for.
- The satellite's mass  $m$  cancels: every body at the same orbital radius around the same planet has the same period and speed.
- Measuring  $T$  and  $r$  of any orbiting body lets you find the **mass of the central body** — this is how the masses of the Sun and of planets with moons are known.

**TRY IT YOURSELF**

**D.1 Orbital motion simulation:** change the orbital radius and launch speed of a satellite and watch how the period, speed and shape of the orbit respond.

|                             |       |       |       |       |       |
|-----------------------------|-------|-------|-------|-------|-------|
| $r / 10^8 \text{ m}$        | 1.5   | 2.4   | 3.3   | 4.2   | 5.3   |
| $T / 10^5 \text{ s}$        | 0.594 | 1.195 | 1.938 | 2.768 | 3.936 |
| $r^3 / 10^{24} \text{ m}^3$ | 3.38  | 13.8  | 35.9  | 74.1  | 149   |
| $T^2 / 10^{10} \text{ s}^2$ | 0.353 | 1.43  | 3.76  | 7.66  | 15.5  |

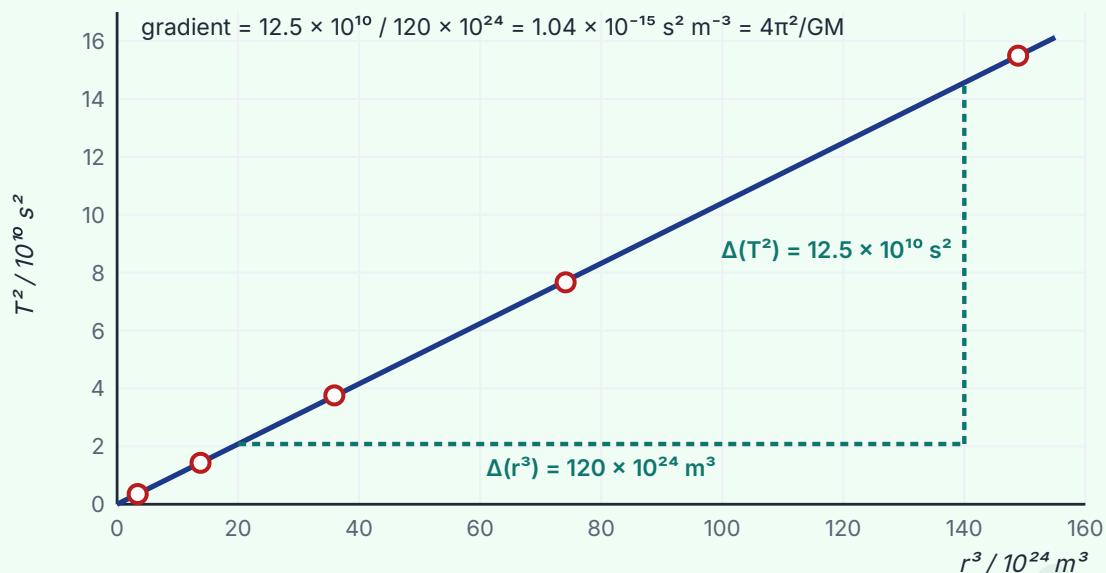
**WORKED EXAMPLE 3** Medium · Show that (graph) / Calculate

The table above gives the orbital radii and periods of five moons of a gas-giant planet. The moons move in circular orbits.

(a) Show that the mass of the planet is about  $5.7 \times 10^{26} \text{ kg}$ . (b) A sixth moon orbits the planet at a radius of  $7.0 \times 10^8 \text{ m}$ . Calculate its orbital period in days.

**(a)** From  $T^2 = \frac{4\pi^2}{GM}r^3$ , the gradient of the  $T^2$ - $r^3$  graph is  $\frac{4\pi^2}{GM}$ .

Plot  $T^2$  against  $r^3$  from the table and draw the line of best fit through the origin:



$T^2$  against  $r^3$  for five moons of the same planet (worked example 3, part (a)). The points lie on a straight line through the origin, confirming Kepler's third law; the gradient equals  $4\pi^2/GM$ .

$$\text{Gradient} = \frac{12.5 \times 10^{10}}{120 \times 10^{24}} = 1.04 \times 10^{-15} \text{ s}^2 \text{ m}^{-3}.$$

$$M = \frac{4\pi^2}{G \times \text{gradient}} = \frac{4\pi^2}{6.67 \times 10^{-11} \times 1.04 \times 10^{-15}} = 5.69 \times 10^{26} \text{ kg} \approx 5.7 \times 10^{26} \text{ kg}$$

$$\text{(b) } T = \sqrt{\text{gradient} \times r^3} = \sqrt{1.04 \times 10^{-15} \times (7.0 \times 10^8)^3} = 5.97 \times 10^5 \text{ s}.$$

$$\text{In days: } \frac{5.97 \times 10^5}{86\,400} = \mathbf{6.9 \text{ days}}$$

Plotting  $T^2$  against  $r^3$  (rather than  $T$  against  $r$ ) turns the relationship into a straight line, so a single gradient uses all the data at once and any anomalous moon stands out.

## HL: What are gravitational potential energy and potential?

### HL Gravitational potential energy of a system

The gravitational potential energy of a system is the work done to assemble the system from infinite separation of its components.

$$E_p = -G \frac{m_1 m_2}{r}$$

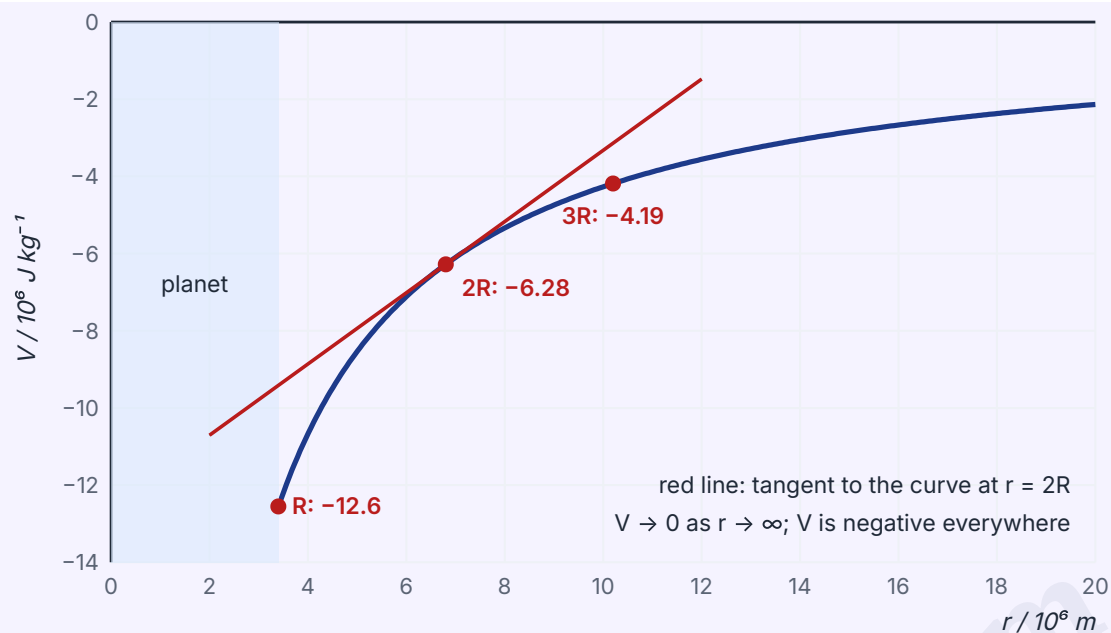
- $E_p$  = gravitational potential energy of the two-body system (J);  $r$  = separation of the **centres of mass** of the two bodies (m). In the data booklet.
- $E_p$  belongs to the **system** of two bodies, not to one of them — although we often loosely say "the satellite's potential energy".
- The zero of  $E_p$  is at **infinite separation**. Bringing the bodies together from infinity, the attractive field does positive work and the external agent does *negative* work, so  $E_p$  is **negative** for every finite separation.
- As  $r$  increases,  $E_p$  increases (becomes less negative) towards zero. Lifting a mass always increases  $E_p$ .
- $\Delta E_p = mg\Delta h$  (A.3) is the special case for a **uniform** field: it is accurate only when  $\Delta h$  is much smaller than the planet's radius.

### Gravitational potential

The gravitational potential at a point is the work done per unit mass in bringing a small mass from infinity to that point.

$$V_g = -G \frac{M}{r} \qquad W = m \Delta V_g$$

- $V_g$  = gravitational potential ( $\text{J kg}^{-1}$ ), a **scalar**;  $M$  = mass creating the field (kg);  $r$  = distance from its centre (m);  $W$  = work done in moving mass  $m$  between two points (J);  $\Delta V_g$  = potential difference between them ( $\text{J kg}^{-1}$ ). Both are in the data booklet.
- Potential is a property of the **point in the field**; potential energy is a property of a mass placed there:  $E_p = mV_g$ .
- $V_g = 0$  at infinity and is negative everywhere else. Because it is a scalar, the potential due to several masses is the simple algebraic **sum** of their potentials — no vector addition needed.
- $W = m\Delta V_g$  depends only on the start and end points, not on the path (the gravitational field is conservative).



Gravitational potential against distance from the centre of the planet in worked example 4 ( $M = 6.4 \times 10^{23} \text{ kg}$ ,  $R = 3.4 \times 10^6 \text{ m}$ ), plotted outside the surface.  $V$  rises towards zero and is proportional to  $-1/r$ . The red line is the tangent to the curve at  $r = 2R = 6.8 \times 10^6 \text{ m}$ ; its gradient gives the field strength there (worked example 4).

## HL: How are field strength, potential gradient and equipotentials related?

### HL Field strength as potential gradient

$$g = -\frac{\Delta V_g}{\Delta r}$$

- The gravitational field strength is the negative of the **potential gradient**. In the data booklet.
- On a  $V_g$ - $r$  graph, the magnitude of  $g$  at a point is the **gradient of the tangent** at that point.
- The minus sign shows that the field points in the direction of **decreasing** potential — towards the mass. A mass released from rest moves to lower potential, just as a ball rolls downhill.

- Check: differentiating  $V_g = -GM/r$  gives a gradient  $GM/r^2$  — the field strength found in section 3.

#### WORKED EXAMPLE 4 HL · Medium · Determine (graph) / Explain

The graph above shows how the gravitational potential varies with distance  $r$  from the centre of a planet of radius  $3.4 \times 10^6$  m.

(a) Use the tangent drawn on the graph to determine the gravitational field strength at  $r = 6.8 \times 10^6$  m. (b) Determine the work needed to move a 250 kg probe from the surface to  $r = 1.02 \times 10^7$  m. (c) A student estimates the same work using  $mg\Delta h$  with the surface value  $g = 3.69 \text{ N kg}^{-1}$ . Explain why this gives a much larger answer.

**(a)** The tangent passes through  $(2.0 \times 10^6 \text{ m}, -10.7 \times 10^6 \text{ J kg}^{-1})$  and  $(12.0 \times 10^6 \text{ m}, -1.48 \times 10^6 \text{ J kg}^{-1})$ .

$$\text{Gradient} = \frac{9.23 \times 10^6}{10.0 \times 10^6} = 0.923 \text{ J kg}^{-1} \text{ m}^{-1}, \text{ so } g = \mathbf{0.92 \text{ N kg}^{-1}}$$

( $0.923 \text{ N kg}^{-1}$ ), directed towards the planet. (Accept 0.88–0.96 from a hand-drawn tangent.)

**(b)** From the graph (or  $V_g = -GM/r$ ):  $V_g(R) = -12.6 \times 10^6 \text{ J kg}^{-1}$  and  $V_g(3R) = -4.19 \times 10^6 \text{ J kg}^{-1}$ , so  $\Delta V_g = +8.37 \times 10^6 \text{ J kg}^{-1}$ .

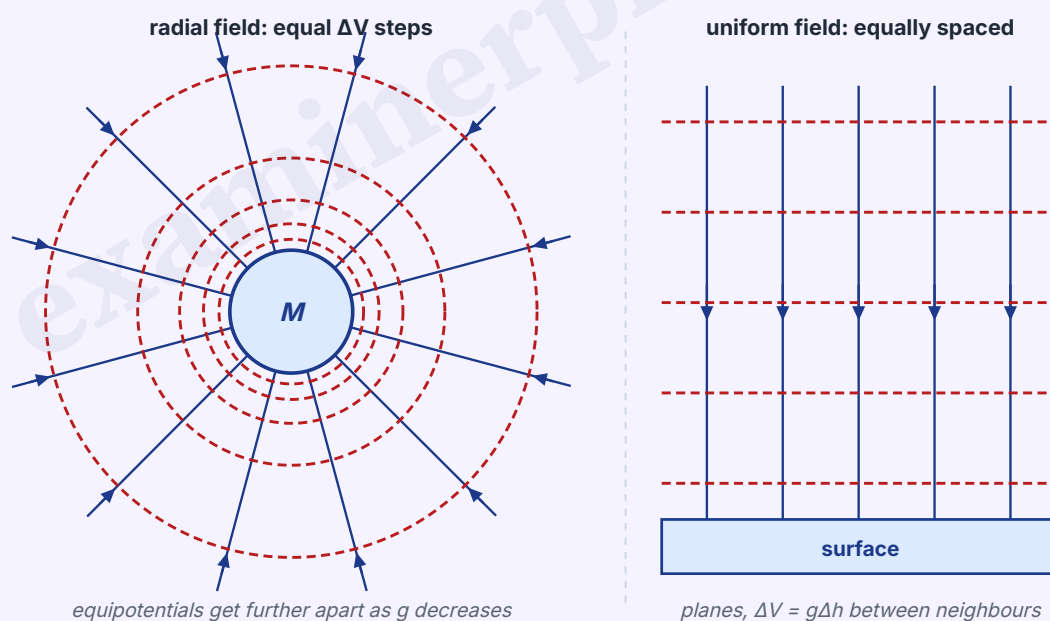
$$W = m\Delta V_g = 250 \times 8.37 \times 10^6 = \mathbf{2.1 \times 10^9 \text{ J}} \text{ (} 2.09 \times 10^9 \text{ J)}$$

**(c)**  $mg\Delta h = 250 \times 3.69 \times 6.8 \times 10^6 = 6.3 \times 10^9 \text{ J}$ , three times too large.  $mg\Delta h$  assumes a **uniform** field, but here the height change (twice the radius) is not small compared with the radius:  $g$  falls from 3.69 to  $0.41 \text{ N kg}^{-1}$  over the journey, so using the surface value throughout greatly overestimates the force and hence the work.

### Equipotential surfaces

An equipotential surface joins points at the same gravitational potential. No work is done in moving a mass along an equipotential surface.

- **Equipotentials are always perpendicular to field lines.** If a field line had a component along an equipotential, moving along it would involve work — contradicting  $W = m\Delta V_g = 0$ .
- **Radial field:** equipotentials are concentric spheres (circles in a 2-D drawing). Drawn at **equal potential steps**, they get **further apart** with distance, because  $g = -\Delta V_g / \Delta r$  is smaller further out — a larger  $\Delta r$  is needed for the same  $\Delta V_g$ .
- **Uniform field:** equipotentials are equally spaced flat planes parallel to the surface (horizontal); the potential difference between neighbours is  $g\Delta h$ .
- **Mapping a field with potential:** given a set of equipotentials, sketch field lines crossing every one at right angles, pointing from higher to lower potential; where the equipotentials are closest, the field is strongest.
- An orbit with constant  $r$  lies on an equipotential, so the gravitational force does no work on a satellite in a circular orbit.



*Equipotentials (red, dashed) and field lines (blue). Left: around a spherical mass, drawn for equal steps in potential — the spacing increases outwards because the field weakens. Right: in the uniform field near a surface the equipotentials are equally spaced and horizontal. In both, field lines cross equipotentials at right angles.*

# HL: What are escape speed, orbital speed and orbital energy?

## HL Escape speed

The escape speed at a point in a gravitational field is the minimum speed a body must have at that point to move to infinity, where it arrives with zero speed (ignoring all forces other than gravity).

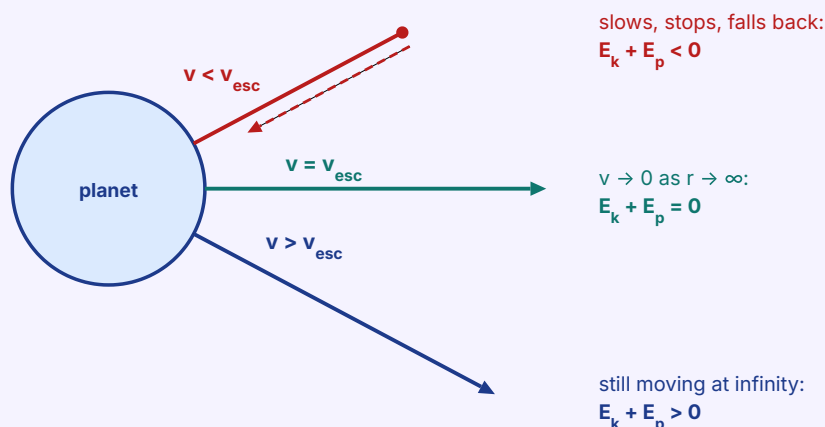
A projected body escapes if its total energy is at least zero:  $E_k + E_p \geq 0$ . At the escape speed,  $\frac{1}{2}mv_{esc}^2 - \frac{GMm}{r} = 0$ , so

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

- $v_{esc}$  = escape speed ( $\text{m s}^{-1}$ );  $r$  = distance from the centre of  $M$  where the body starts (m) — the planet's radius for a launch from the surface. In the data booklet.
- It is independent of the mass of the projected body and, if the atmosphere is ignored, of the direction of launch.
- It is a **speed given at the start**, with no further propulsion. A rocket with engines can leave a planet more slowly, because its engines keep doing work.
- Escape speed decreases with  $r$ : it is easier to escape from an orbit than from the surface.

### Bodies launched radially outwards from the surface (no atmosphere)

$$\frac{1}{2}mv_{\text{esc}}^2 - GMm/R = 0 \Rightarrow v_{\text{esc}} = \sqrt{2GM/R}, \text{ independent of } m$$



Three bodies launched radially outwards from a planet with no atmosphere. Below the escape speed the total energy is negative and the body returns; at exactly  $v_{\text{esc}}$  it just reaches infinity with zero speed.

### TRY IT YOURSELF

**D.1 Escape velocity simulation:** launch a projectile at speeds just below and just above the escape speed and compare what happens to its kinetic and potential energy.

### Orbital speed

For a body orbiting a large mass  $M$  in a circular orbit,  $\frac{GMm}{r^2} = \frac{mv^2}{r}$ , so

$$v_{\text{orbital}} = \sqrt{\frac{GM}{r}}$$

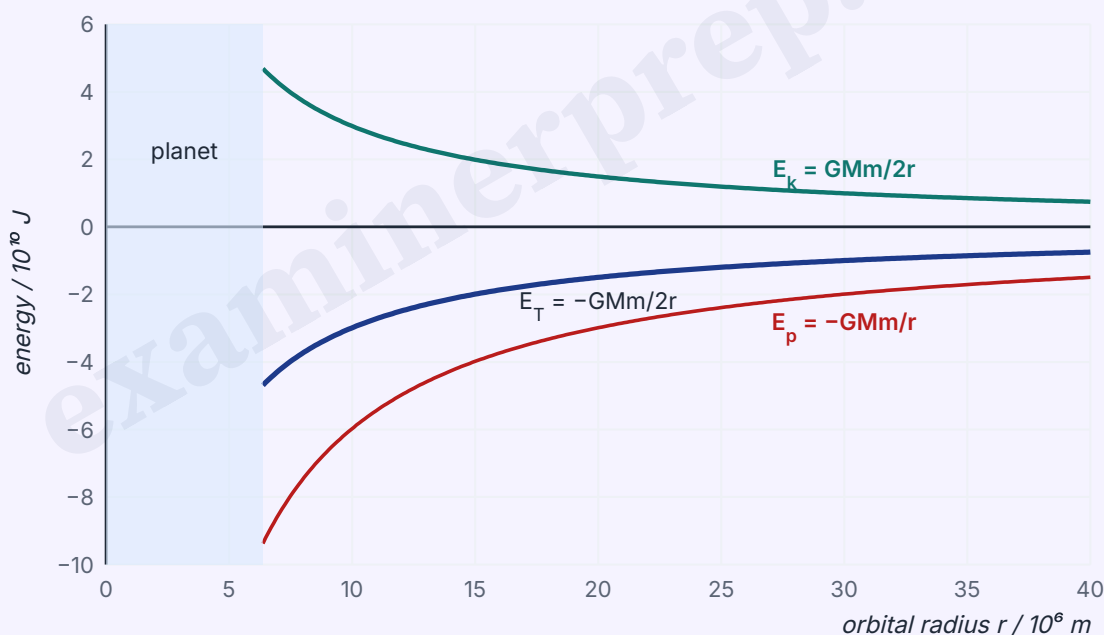
- $v_{\text{orbital}}$  = orbital speed ( $\text{m s}^{-1}$ );  $r$  = orbital radius from the centre of  $M$  (m). In the data booklet.
- Lower orbits are faster. At the same radius,  $v_{\text{esc}} = \sqrt{2} v_{\text{orbital}}$ .

### Energy of a satellite in a circular orbit

$$E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r} \quad E_p = -\frac{GMm}{r} \quad E_T = E_k + E_p = -\frac{GMm}{2r}$$

- The  $E_k$  and  $E_T$  forms are **not in the data booklet**: derive  $E_k$  by substituting  $v^2 = GM/r$  into  $\frac{1}{2}mv^2$ .

- $E_k = -E_T$  and  $E_p = 2E_T$  (see the energy graph below). The total energy is negative: the satellite is **bound** to the planet.
- **Changing orbit:** moving to a higher orbit increases  $E_T$  (less negative) — energy must be supplied, even though the satellite ends up *slower*.  
Energy needed =  $\Delta E_T = \frac{GMm}{2} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$ .
- **Escaping from orbit:** the minimum extra energy is  $|E_T| = \frac{GMm}{2r}$ , which raises the total energy to zero.
- **Launching from rest on the surface of a non-rotating planet:** initial energy =  $-\frac{GMm}{R}$  (potential only), so the energy needed to reach orbit is  $E_T - E_p(R) = GMm \left( \frac{1}{R} - \frac{1}{2r} \right)$ .



Energies of a 1500 kg satellite in circular orbit around a planet ( $M = 5.97 \times 10^{24}$  kg,  $R = 6.37 \times 10^6$  m), against orbital radius.  $E_p$  is always twice as negative as  $E_T$ , and  $E_k = -E_T$ . All three approach zero as  $r \rightarrow \infty$ .

#### WORKED EXAMPLE 5 HL · Hard · Show that / Determine

A planet that does not rotate has mass  $5.97 \times 10^{24}$  kg and radius  $6.37 \times 10^6$  m and no atmosphere. A 1500 kg satellite is to be placed in a circular orbit at a height of  $1.20 \times 10^6$  m above the surface.

(a) Show that the orbital speed is about  $7.3 \text{ km s}^{-1}$ . (b) Calculate the orbital period. (c) Determine the minimum energy needed to place the satellite in this orbit, starting from rest on the surface. (d) Determine the minimum additional energy needed for the satellite to escape from the orbit, and compare the escape speed from the surface with the escape speed from the orbit.

$$r = R + h = 6.37 \times 10^6 + 1.20 \times 10^6 = 7.57 \times 10^6 \text{ m}; GM = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} = 3.98 \times 10^{14} \text{ N m}^2 \text{ kg}^{-1}.$$

$$\text{(a)} \quad v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{3.98 \times 10^{14}}{7.57 \times 10^6}} = 7.25 \times 10^3 \text{ m s}^{-1} \approx 7.3 \text{ km s}^{-1}$$

$$\text{(b)} \quad T = \frac{2\pi r}{v} = \frac{2\pi \times 7.57 \times 10^6}{7253} = 6.56 \times 10^3 \text{ s (109 minutes)}$$

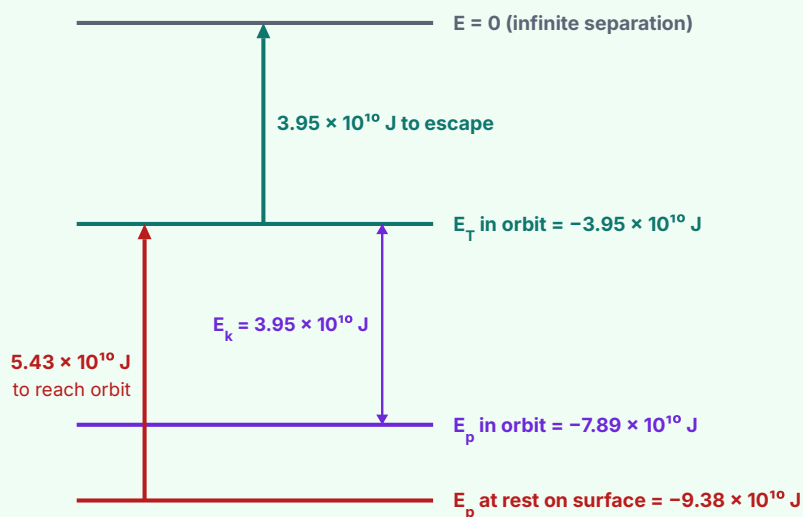
$$\text{(c)} \quad \text{On the surface: } E_p = -\frac{GMm}{R} = -\frac{3.98 \times 10^{14} \times 1500}{6.37 \times 10^6} = -9.38 \times 10^{10} \text{ J and } E_k = 0.$$

$$\text{In orbit: } E_T = -\frac{GMm}{2r} = -\frac{3.98 \times 10^{14} \times 1500}{2 \times 7.57 \times 10^6} = -3.95 \times 10^{10} \text{ J.}$$

$$\text{Energy needed} = -3.95 \times 10^{10} - (-9.38 \times 10^{10}) = 5.4 \times 10^{10} \text{ J} \\ (5.43 \times 10^{10} \text{ J})$$

$$\text{(d)} \quad \text{To escape, } E_T \text{ must rise to zero: extra energy} = 3.9 \times 10^{10} \text{ J (} 3.95 \times 10^{10} \text{ J).}$$

$$v_{esc} \text{ from the surface} = \sqrt{\frac{2GM}{R}} = 1.12 \times 10^4 \text{ m s}^{-1}; \text{ from the orbit} = \sqrt{\frac{2GM}{r}} = 1.03 \times 10^4 \text{ m s}^{-1}. \text{ The satellite already moves at } 7.25 \times 10^3 \text{ m s}^{-1}, \text{ so it needs only an extra } 3.0 \times 10^3 \text{ m s}^{-1} \text{ to escape.}$$



Energies of a 1500 kg satellite (worked example 5), drawn to scale

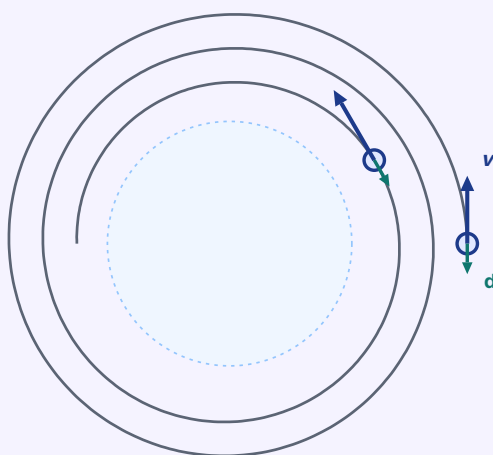
Energy level diagram for the satellite of worked example 5, drawn to scale. The energy needed to put the satellite into orbit is the gap between its potential energy at rest on the surface and its total energy in orbit; the energy to escape from the orbit is the gap from  $E_T$  up to zero.

Check (d):  $\frac{1}{2} \times 1500 \times (1.026 \times 10^4)^2 - \frac{1}{2} \times 1500 \times 7253^2 = 3.95 \times 10^{10} \text{ J}$  — the same as the energy method.

### Effect of atmospheric drag on an orbit

A satellite in a low orbit moves through the very thin upper atmosphere and experiences a small viscous drag force opposite to its velocity.

- Drag does negative work, so the **total energy**  $E_T$  **decreases** (becomes more negative).
- Since  $E_T = -\frac{GMm}{2r}$ , a more negative  $E_T$  means a **smaller**  $r$ : the satellite **loses height**, spiralling slowly inwards.
- Since  $E_k = \frac{GMm}{2r}$ , a smaller  $r$  means **larger kinetic energy**: the satellite **speeds up**. The loss in  $E_p$  is twice the loss in  $E_T$ ; half of it is dissipated by drag and half becomes extra kinetic energy.
- The effect accelerates: a lower orbit is in denser air, so the drag grows. Low satellites need periodic engine burns ("reboosts") to stay in orbit.



small drag force  $\rightarrow$  total energy  $E_T$  decreases

$E_T = -GMm/2r$  becomes more negative

$\Rightarrow$  **orbital radius (height) decreases**

$E_p = -GMm/r$  decreases (by 2  $\times$  the loss in  $E_T$ )

$E_k = GMm/2r$  increases

$\Rightarrow$  **orbital speed increases**

lower orbit  $\rightarrow$  denser air  $\rightarrow$  faster decay

*spiral exaggerated; each loop is almost circular*

A satellite slowly spiralling inwards under a small drag force. The velocity arrows (to scale,  $v \propto 1/\sqrt{r}$ ) show that the satellite moves faster in the lower orbit, even though drag opposes its motion.

## Exam tips for D.1

### EXAM TIP 1 — $r$ IS FROM THE CENTRE

In every D.1 equation,  $r$  is measured from the **centre** of the planet. If a question gives a height above the surface, add the radius first. Using the height alone is one of the most common errors in orbit calculations, and it costs marks in every part that uses that value of  $r$ .

### EXAM TIP 2 — GRAVITY IS THE CENTRIPETAL FORCE

When deriving  $T^2 \propto r^3$  or  $v = \sqrt{GM/r}$ , start with a clear statement: "the gravitational force provides the centripetal force", then equate  $\frac{GMm}{r^2}$  to  $\frac{mv^2}{r}$  or  $\frac{4\pi^2mr}{T^2}$ . On a free-body diagram of a satellite draw **one** force only — never add a separate "centripetal force" arrow.

### EXAM TIP 3 — USE RATIOS IN MULTIPLE CHOICE

Paper 1A questions often change two quantities at once (both masses, or mass and radius). Write the proportionality first —  $F \propto m_1 m_2 / r^2$ ,  $g \propto M / r^2$ ,  $T \propto r^{3/2}$ ,  $v_{\text{orbital}} \propto r^{-1/2}$ , and at HL  $V_g \propto -1/r$  — and multiply the factors. It is faster and avoids rounding errors.

#### EXAM TIP 4 — SIGNS OF POTENTIAL AND ENERGY (HL)

Gravitational potential and potential energy are **negative**; a missing minus sign is penalised. When asked for a *change*, subtract carefully: going from  $-9.38 \times 10^{10} \text{ J}$  to  $-3.95 \times 10^{10} \text{ J}$  is an increase of  $+5.43 \times 10^{10} \text{ J}$ . When explaining why  $E_p$  is negative, refer to the zero at infinity and the fact that work is done **by** the attractive field as the masses come together.

#### EXAM TIP 5 — DRAG ANSWERS NEED THE CHAIN (HL)

For “explain the effect of air resistance on a satellite”, a bare “it slows down and falls” scores nothing. Give the chain: drag does work against the satellite  $\rightarrow$  total energy decreases  $\rightarrow$  since  $E_T = -GMm/2r$ , the radius decreases  $\rightarrow$  since  $v = \sqrt{GM/r}$  (or  $E_k = GMm/2r$ ), the speed **increases**.

## Common misconceptions

| Misconception                                                         | Why it is wrong                                                                                        | Correct idea                                                                                                                      |
|-----------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| There is no gravity in orbit, which is why astronauts float.          | At 400 km above the Earth, $g$ is still about 90 % of its surface value.                               | Astronauts and their spacecraft are in free fall together, with the same acceleration, so there is no contact force between them. |
| The Earth pulls the Moon harder than the Moon pulls the Earth.        | The forces are a Newton's third-law pair.                                                              | The forces are equal in size; the Earth's acceleration is smaller only because its mass is larger.                                |
| A heavier satellite must orbit faster to stay up.                     | The satellite's mass cancels in $GMm/r^2 = mv^2/r$ .                                                   | Orbital speed and period depend only on $M$ and $r$ .                                                                             |
| A satellite needs a forward force to keep moving.                     | No force is needed to maintain speed (Newton's first law); gravity is perpendicular to the velocity.   | The only force is gravity, towards the centre; it changes the direction of motion, not the speed.                                 |
| The null point between a planet and its moon is halfway between them. | The fields depend on $M/r^2$ , so the larger mass needs a larger distance to have the same field.      | The null point is closer to the smaller mass; the distance ratio is $\sqrt{M_1/M_2}$ .                                            |
| (HL) Drag slows a satellite down.                                     | Drag removes total energy, so the satellite drops to a lower orbit, where $v = \sqrt{GM/r}$ is larger. | Drag makes a satellite lose height and speed up.                                                                                  |

## How this topic connects

### Circular orbits versus elliptical ones: strengths and limits of the simple model

The circular-orbit model uses everything from A.2: a constant speed, an acceleration  $v^2/r$  towards the centre, and a single force (gravity) providing it. Real orbits are ellipses (Kepler's first law), so the distance to the central body, the size of the gravitational force and the speed all change around the orbit (Kepler's second law), and the force is no longer perpendicular to the velocity except at the closest and furthest points. Many planetary and satellite orbits are so nearly circular that the circular model gives periods and speeds to within a few per cent, which is why IB calculations assume circular orbits.

### **Fuel for a launch: an energy budget for reaching orbit or escaping**

The minimum energy a launch must supply is the difference between the final total energy and the starting energy. Using the HL expression for  $E_p$ , to reach a circular orbit it is  $GMm \left( \frac{1}{R} - \frac{1}{2r} \right)$ ; to escape it is  $\frac{GMm}{R}$ . Dividing by the energy released per kilogram of fuel (its energy density, A.3) gives a lower limit on the fuel mass. Real launches need far more: the fuel itself must be lifted, energy is dissipated as drag and thermal energy in the exhaust, and the efficiency of the engines is well below 100 %. Launching eastwards near the equator also helps, because the rocket starts with the planet's rotational speed.

### **The role of $G$ and other fundamental constants, and how they are measured**

$G$  turns the proportionality  $F \propto m_1 m_2 / r^2$  into an equation and sets the strength of gravity in SI units. It is “universal”: the same value applies to apples and galaxies, which is itself a claim tested by observation. Because gravity between laboratory masses is so weak,  $G$  was first measured by Cavendish (1798) with a torsion balance, in which large lead spheres produced a tiny measurable twist in a fibre holding small spheres. Even today  $G$  is known to far fewer significant figures than most other fundamental constants. Once  $G$  is known, measuring  $g$  at the Earth's surface gives the mass of the Earth — Cavendish's experiment was described as “weighing the Earth”.

**Tools & inquiry — finding a mass from orbital data (NOS: fields as models)**

- **Linearise Kepler's third law:** plot  $T^2$  against  $r^3$  for several moons of one planet (data from an astronomical database or an orbit simulation). The gradient  $4\pi^2/GM$  gives  $M$ .
- **Log-log alternative:** plotting  $\lg T$  against  $\lg r$  gives a straight line of gradient 1.5, testing the power law itself without assuming it.
- **Units and powers of ten:** astronomical data span many orders of magnitude; label axes such as  $r^3 / 10^{24} \text{ m}^3$  and convert days to seconds before using SI equations.
- **Uncertainties:** because  $T^2$  and  $r^3$  are powers, the percentage uncertainty in  $r^3$  is three times that in  $r$ , and in  $T^2$  twice that in  $T$ .
- **NOS — fields as models:** a gravitational field is a mathematical model that replaces “action at a distance” with a local property of space. Field lines and equipotentials are visual representations of that model, and the same language (field strength, potential, field lines) is used for electric fields in D.2.

## Key equations

| Equation                                           | Use                                          | Level   | In data booklet?                                      |
|----------------------------------------------------|----------------------------------------------|---------|-------------------------------------------------------|
| $F = G \frac{m_1 m_2}{r^2}$                        | force between two point masses               | SL & HL | yes                                                   |
| $g = \frac{F}{m} = G \frac{M}{r^2}$                | gravitational field strength                 | SL & HL | yes                                                   |
| $T^2 = \frac{4\pi^2}{GM} r^3$                      | Kepler's third law for a circular orbit      | SL & HL | no — derive from $F = GMm/r^2$ and $a = 4\pi^2 r/T^2$ |
| $E_p = -G \frac{m_1 m_2}{r}$                       | potential energy of a two-body system        | HL      | yes                                                   |
| $V_g = -G \frac{M}{r}$                             | gravitational potential                      | HL      | yes                                                   |
| $g = -\frac{\Delta V_g}{\Delta r}$                 | field strength as potential gradient         | HL      | yes                                                   |
| $W = m\Delta V_g$                                  | work done moving a mass between two points   | HL      | yes                                                   |
| $v_{esc} = \sqrt{\frac{2GM}{r}}$                   | escape speed                                 | HL      | yes                                                   |
| $v_{orbital} = \sqrt{\frac{GM}{r}}$                | speed in a circular orbit                    | HL      | yes                                                   |
| $E_k = \frac{GMm}{2r},$<br>$E_T = -\frac{GMm}{2r}$ | kinetic and total energy in a circular orbit | HL      | no — derive from $v_{orbital}$ and $E_p$              |

## Check your understanding

1. The distance between two bodies is tripled and the mass of one of them is doubled. State the factor by which the gravitational force between them changes.

$F \propto \frac{m_1 m_2}{r^2}$ , so the factor is  $\frac{2}{3^2} = \frac{2}{9}$  (0.22). The force decreases to two-ninths of its original value.

2. A planet has radius  $6.37 \times 10^6$  m. Determine the height above its surface at which the gravitational field strength is half its surface value.

$g \propto \frac{1}{r^2}$ , so  $g$  halves when  $r = \sqrt{2}R$ . Height  $= (\sqrt{2} - 1)R = 0.414 \times 6.37 \times 10^6 = 2.6 \times 10^6$  m ( $2.64 \times 10^6$  m).

3. Moon B orbits a planet at four times the orbital radius of moon A. Calculate the ratio of their periods,  $T_B/T_A$ .

Kepler's third law:  $T \propto r^{3/2}$ , so  $\frac{T_B}{T_A} = 4^{3/2} = 8$ .

4. (HL) A moon has mass  $7.35 \times 10^{22}$  kg and radius  $1.74 \times 10^6$  m. Calculate the escape speed from its surface.

$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22}}{1.74 \times 10^6}} = 2.37 \times 10^3 \text{ m s}^{-1}$   
(2.4 km s<sup>-1</sup>).

5. (HL) A satellite in a low circular orbit experiences a small drag force from the upper atmosphere. Explain what happens to its total energy, its orbital radius and its speed.

The drag force does work against the satellite, so its **total energy decreases**.

Since  $E_T = -\frac{GMm}{2r}$ , a more negative total energy means a **smaller orbital radius**. Since  $v = \sqrt{\frac{GM}{r}}$ , the smaller radius gives a **higher speed**:

gravitational potential energy falls by twice the loss of total energy, so the kinetic energy increases.

## Practise D.1 with exam-style questions

Original Paper 1A, 1B and 2 questions on gravitational fields, with full mark schemes. Tick the questions you want and print them as your own test.

SL D.1 questions

HL D.1 questions

## Revise and explore

- [Flashcards](#) — choose IB Physics and filter to D.1 Gravitational fields. Free, no sign-up.
- [D.1 Orbital motion simulation](#)
- [D.1 Escape velocity simulation](#)
- [Full IB Physics question bank \(all sub-topics, SL & HL\)](#)

[← Previous: C.5 Doppler effect](#) · [Next: D.2 Electric and magnetic fields →](#)