

THEME D · FIELDS

HL ONLY

≈ 30 min read

IB Physics D.4 Induction — Notes (HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). **D.4 is entirely Additional Higher Level content** — there is no SL material in this sub-topic, and it is examined only on HL papers. It builds on magnetic fields (D.2) and the magnetic force on moving charges and currents (D.3), and explains how almost all of the world's electricity is generated.

[HL D.4 questions](#)[Flashcards](#)[Electromagnetic induction simulation](#)[AC generator simulation](#)

What this page covers

- ✓ Magnetic flux through a surface, $\Phi = BA \cos \theta$, and flux linkage $N\Phi$
- ✓ Faraday's law: a changing magnetic flux induces an emf, $\varepsilon = -N \frac{\Delta \Phi}{\Delta t}$
- ✓ The emf induced in a straight conductor moving at right angles to a uniform magnetic field, $\varepsilon = BvL$
- ✓ Lenz's law for the direction of the induced emf, and why it follows from conservation of energy
- ✓ Induction by time-varying fields, by relative motion of a magnet and a coil, and by rectangular coils moving into and out of a field
- ✓ The sinusoidal emf induced in a coil rotating in a uniform magnetic field
- ✓ How changing the frequency of rotation changes the induced emf
- ✓ Self-induction, qualitatively

In one minute

- **Magnetic flux** $\Phi = BA \cos \theta$ (unit weber, Wb) measures how much field passes through an area; θ is the angle between the field and the *normal* to the area.

- **Faraday's law:** an emf is induced whenever the flux linking a circuit *changes*. Its size is the rate of change of flux linkage, $\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$. A large but steady flux induces nothing.
- A straight conductor of length L moving at speed v at right angles to a field B has an emf $\varepsilon = BvL$ across its ends.
- **Lenz's law:** the induced emf drives a current that *opposes the change* producing it. This is energy conservation: you must do work against the opposition, and that work becomes electrical energy.
- A coil rotating in a uniform field has a **sinusoidal** emf, greatest when the flux through the coil is zero and zero when the flux is greatest. Doubling the frequency of rotation **doubles the peak emf and halves the period**.

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11. Check your understanding

What is magnetic flux?

Induction is about *change* in the amount of magnetic field passing through a circuit. To describe "amount of field through an area" we need a new quantity.

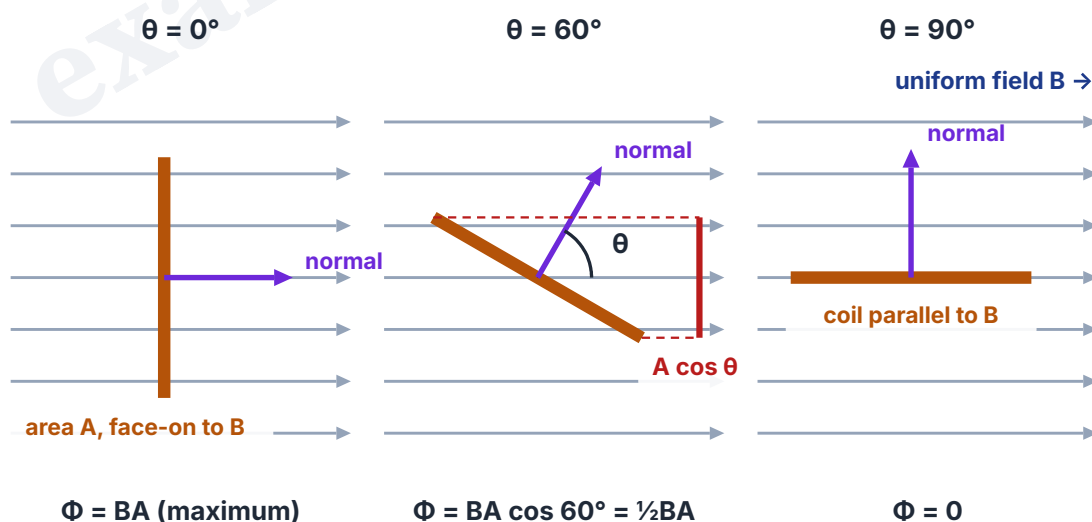
The magnetic flux through a flat area is the product of the magnetic field strength, the area and the cosine of the angle between the field and the normal to the area: $\Phi = BA \cos \theta$.

$$\Phi = BA \cos \theta$$

- Φ = magnetic flux (Wb, weber; $1 \text{ Wb} = 1 \text{ T m}^2$)
- B = magnetic field strength (magnetic flux density) (T)
- A = area enclosed by the loop or coil (m^2)
- θ = angle between the magnetic field and the **normal** (perpendicular) to the area

This equation is in the data booklet. Points to note:

- Flux is greatest, $\Phi = BA$, when the field passes straight through the area (field parallel to the normal, $\theta = 0$).
- Flux is zero when the plane of the coil is **parallel** to the field ($\theta = 90^\circ$): no field lines pass through it.
- $A \cos \theta$ is the area of the coil projected onto a plane perpendicular to the field — the “effective area” the field lines cross.
- Flux is a scalar, but it has a sign: if the coil turns through 180° , the same field passes through it the other way and Φ changes from $+BA$ to $-BA$.
- A useful picture: B is proportional to the density of field lines, so Φ is proportional to the number of field lines passing through the area. This is why B is also called the magnetic flux density.



coils seen edge-on; θ is the angle between the field and the NORMAL to the coil

A flat coil of area A in a uniform field, seen edge-on. The angle θ is measured from the field to the normal to the coil. At $\theta = 60^\circ$ the coil's projected area perpendicular to B is $A \cos 60^\circ = A/2$, so the flux is halved.

Flux linkage

The flux linkage of a coil of N turns is the product of the number of turns and the flux through one turn: $N\Phi$.

- Each turn of the coil has the same flux Φ through it, and an emf is induced in every turn. The turns are in series, so their emfs add — hence the factor N .
- The unit of flux linkage is the weber (some books write Wb turns). Flux linkage itself is not in the data booklet, but it appears inside Faraday's law as $N\Delta\Phi$.

WORKED EXAMPLE 1 Easy · Calculate

A rectangular coil of 200 turns measures 8.0 cm by 5.0 cm. It is placed in a uniform magnetic field of strength 0.30 T so that the **plane of the coil** makes an angle of 40° with the field.

(a) Calculate the magnetic flux through one turn of the coil. (b) Calculate the flux linkage of the coil. (c) The coil is turned in 0.12 s until its plane is parallel to the field. Calculate the average emf induced.

(a) $A = 0.080 \times 0.050 = 4.0 \times 10^{-3} \text{ m}^2$. The plane is at 40° to the field, so the *normal* is at $\theta = 90^\circ - 40^\circ = 50^\circ$:

$$\Phi = BA \cos \theta = 0.30 \times 4.0 \times 10^{-3} \times \cos 50^\circ = \mathbf{7.7 \times 10^{-4} \text{ Wb}} \quad (7.71 \times 10^{-4} \text{ Wb})$$

(b) $N\Phi = 200 \times 7.71 \times 10^{-4} = \mathbf{0.15 \text{ Wb}}$ (0.154 Wb)

(c) With its plane parallel to the field the flux is zero, so $N\Delta\Phi = -0.154 \text{ Wb}$:

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t} = -\frac{(0 - 0.154)}{0.12} = \mathbf{1.3 \text{ V}} \quad (1.29 \text{ V})$$

Using $\cos 40^\circ$ by mistake gives $9.2 \times 10^{-4} \text{ Wb}$ in (a). Always check which angle you have been given: the angle to the plane or the angle to the normal.

What does Faraday's law of induction say?

Faraday's law: the emf induced in a circuit is equal to the rate of change of the magnetic flux linkage of the circuit: $\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$.

$$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$$

- ε = induced emf (V)
- N = number of turns of the coil
- $\Delta\Phi$ = change in magnetic flux through one turn (Wb)
- Δt = time taken for the change (s)
- This is in the data booklet. The **minus sign** expresses Lenz's law (see below): the emf acts to oppose the change. When a question asks only for the size of the emf, you may ignore it.
- It is the **rate of change** of flux that matters. A coil sitting in a strong, steady field has a large flux linkage but no emf.
- $\frac{\Delta\Phi}{\Delta t}$ over a finite interval gives the **average** emf. The **instantaneous** emf is $-N$ times the gradient of the tangent to a graph of Φ against t (or minus the gradient of a graph of $N\Phi$ against t).
- Units check: $1 \text{ Wb s}^{-1} = 1 \text{ T m}^2 \text{ s}^{-1} = 1 \text{ V}$.
- An emf is induced whether or not the circuit is complete. A **current** flows only if there is a closed circuit; its size is then given by the emf and the total resistance, as in B.5.

Three ways to change the flux

Since $\Phi = BA \cos \theta$, the flux linkage $N\Phi$ changes if any of B , A or θ changes. Examples of induction fall into these groups:

What changes	Example	Section
B : a time-varying magnetic field	an electromagnet switched on or off near a stationary coil; a magnet moved towards or away from a coil; a magnet bobbing up and down on a spring over a coil	this section and Lenz's law
A : the area of the circuit inside the field	a rod sliding along rails; a rectangular coil moving into or out of a field region	moving conductor, coils
θ : the orientation of the coil	a coil rotating in a uniform field — a generator	rotating coil

Only *relative* motion matters: moving a magnet towards a stationary coil induces the same emf as moving the coil towards the stationary magnet at the same speed.

WORKED EXAMPLE 2 Medium · Determine · graph-based

A small search coil of 150 turns and cross-sectional area $3.0 \times 10^{-3} \text{ m}^2$ is placed between the poles of an electromagnet, with the field along the axis of the coil. The current in the electromagnet is changed so that the field through the coil rises uniformly from zero to 0.12 T in 0.20 s, stays constant for 0.30 s, and then falls uniformly to zero in 0.10 s.

- (a) Determine the emf induced in the coil during each of the three stages.
 (b) Sketch the variation of the induced emf with time.

Maximum flux through one turn: $\Phi = BA = 0.12 \times 3.0 \times 10^{-3} = 3.6 \times 10^{-4} \text{ Wb}$.

(a) The emf is $-N \times \text{gradient of the } \Phi-t \text{ graph}$ (upper graph below the solution).

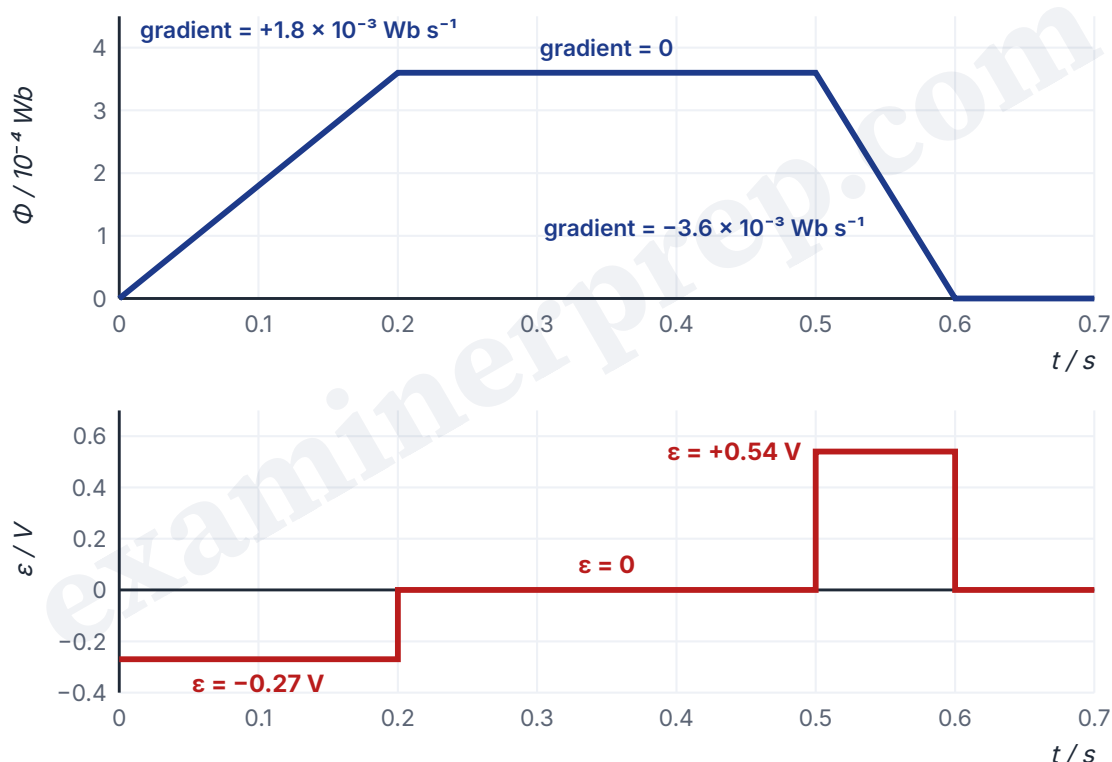
Rising stage: gradient $= \frac{3.6 \times 10^{-4}}{0.20} = 1.8 \times 10^{-3} \text{ Wb s}^{-1}$, so $\varepsilon = -150 \times 1.8 \times 10^{-3} = \mathbf{-0.27 \text{ V}}$

Steady stage: gradient $= 0$, so $\varepsilon = \mathbf{0}$, even though the flux is at its largest.

Falling stage: gradient $= \frac{-3.6 \times 10^{-4}}{0.10} = -3.6 \times 10^{-3} \text{ Wb s}^{-1}$, so $\varepsilon = \mathbf{+0.54 \text{ V}}$

(b) See the lower graph: a constant negative emf, then zero, then a constant positive emf twice as large for half as long.

The two pulses have the same area, $0.27 \times 0.20 = 0.54 \times 0.10 = 0.054 \text{ V s}$, equal in size to the change in flux linkage $N\Delta\Phi = 150 \times 3.6 \times 10^{-4} = 0.054 \text{ Wb}$. The area under an emf-time graph always equals $-N\Delta\Phi$: the same size as the change in flux linkage, with the opposite sign.



Worked example 2. The emf (lower graph) is $-N$ times the gradient of the flux graph (upper graph): it is zero while the flux is constant, and the faster fall gives the larger emf, of opposite sign.

How is an emf induced in a moving straight conductor?

A straight conductor of length L moving with speed v at right angles to a uniform magnetic field B has an emf $\varepsilon = BvL$ induced between its ends.

$$\varepsilon = BvL$$

- ε = induced emf (V)
- B = magnetic field strength (T)
- v = speed of the conductor, perpendicular to the field (m s^{-1})
- L = length of the conductor in the field (m), perpendicular to both B and v

This is in the data booklet. You can derive it in two ways, and both are acceptable.

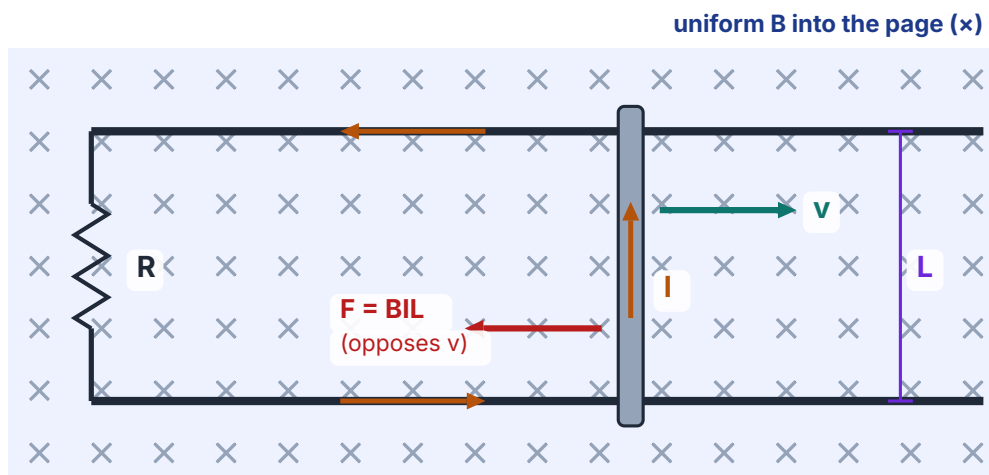
Derivation 1 — from the force on the charges (D.3)

- The free electrons in the rod move with the rod at speed v through the field, so each has a magnetic force evB along the rod ($F = qvB$, D.3).
- Electrons pile up at one end, leaving the other end positive. This separation of charge sets up an electric field E along the rod.
- Charge stops building up when the electric force balances the magnetic force: $eE = evB$, so $E = vB$.
- In a uniform field $E = \frac{\varepsilon}{L}$, so $\varepsilon = EL = BvL$.

Derivation 2 — from Faraday's law

- In a time Δt the rod moves $v\Delta t$ and sweeps out an area $\Delta A = Lv\Delta t$.
- The flux through the circuit changes by $\Delta\Phi = B\Delta A = BLv\Delta t$ (field perpendicular to the area, $\cos\theta = 1$).
- For a single conductor ($N = 1$): $\varepsilon = \frac{\Delta\Phi}{\Delta t} = BvL$. The emf is the rate at which the rod "cuts" flux.

The emf exists whether or not the rod is part of a circuit. If the ends are connected through a resistor — for example by sliding the rod along two conducting rails — a current $I = \frac{\varepsilon}{R}$ flows. Then the rod is a current-carrying conductor in a field, and it feels a force $F = BIL$ (D.3) that **opposes its motion**.



loop area grows $\rightarrow$ flux into page increases $\rightarrow$ induced current anticlockwise
 (its own field points out of the page inside the loop, opposing the increase)

A rod sliding at speed v along conducting rails in a field into the page. The circuit's area and flux increase, the induced current flows anticlockwise, and the force BIL on the rod points against v — so a pulling force is needed to keep it moving.

Direction. In the diagram, positive charges in the rod would be pushed towards the top rail ($F = qvB$ with v to the right and B into the page), so the current flows up through the rod and anticlockwise round the circuit. Lenz's law gives the same answer: the flux into the page is increasing, so the induced current makes its own field *out of* the page inside the loop. Check with the force: a current flowing up the rod in a field into the page feels a force to the left, against the motion — exactly as Lenz's law requires.

WORKED EXAMPLE 3 Medium · Show that / Calculate

A metal rod of length 0.25 m slides along two horizontal rails at a constant speed of 3.0 m s^{-1} , as in the diagram above. The rails are joined by a 1.5Ω resistor; the resistance of the rod and rails is negligible. A uniform vertical magnetic field of 0.40 T acts on the circuit.

- Show that the emf induced in the rod is 0.30 V.
- Calculate the current in the resistor.
- Calculate the force needed to keep the rod moving at constant speed, assuming no friction.
- Show that the rate at which this force does work equals the rate at which energy is dissipated in the resistor.
- Predict what happens to the power if the speed is doubled.

(a) $\mathcal{E} = BvL = 0.40 \times 3.0 \times 0.25 = \mathbf{0.300 \text{ V}}$

$$(b) I = \frac{\varepsilon}{R} = \frac{0.30}{1.5} = \mathbf{0.20\ A}$$

(c) At constant speed the applied force balances the magnetic force on the rod: $F = BIL = 0.40 \times 0.20 \times 0.25 = \mathbf{0.020\ N}$

(d) Mechanical power $= Fv = 0.020 \times 3.0 = 0.060\ \text{W}$. Electrical power $= I^2R = 0.20^2 \times 1.5 = 0.060\ \text{W}$ (and $\varepsilon I = 0.30 \times 0.20 = 0.060\ \text{W}$).

Both are 0.060 W — the work done against the magnetic force becomes internal energy in the resistor.

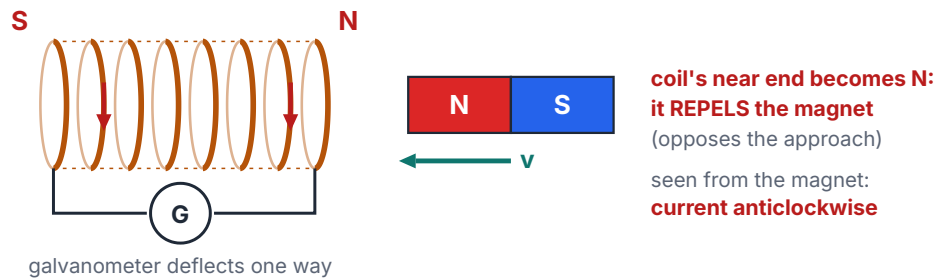
(e) Doubling v doubles ε and I , so it doubles the opposing force BIL to 0.040 N. The power Fv becomes $0.040 \times 6.0 = \mathbf{0.24\ W}$ — **four times as much** (power $\propto v^2$).

What determines the direction of the induced emf?

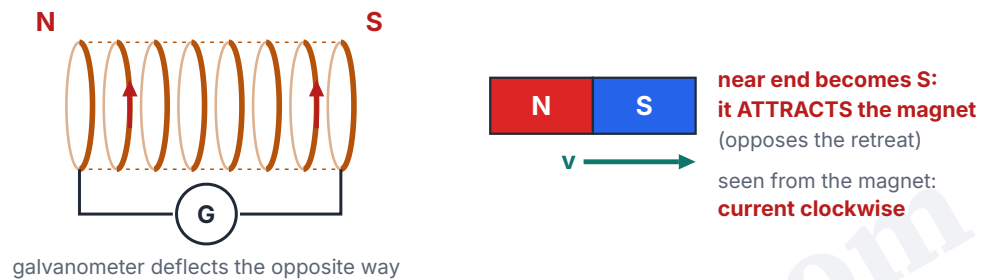
Lenz's law: the direction of the induced emf is such that the current it would drive opposes the change in magnetic flux that produces it.

- The induced current opposes the **change** in flux, not the flux itself. If the flux through a coil is increasing, the induced current's field points the opposite way to the external field inside the coil; if the flux is decreasing, the induced field points the *same* way, trying to keep the flux up.
- In terms of forces: the induced current always produces a force that opposes the relative motion. If the coil is part of a closed circuit, a magnet approaching it is repelled and a magnet moving away is attracted back.
- Lenz's law is the reason for the minus sign in $\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$.

N pole moving towards the coil



N pole moving away from the coil



A bar magnet moving relative to a coil connected to a galvanometer. Approaching, the coil's near end becomes a north pole and repels the magnet; retreating, it becomes a south pole and attracts it. Either way the induced current opposes the motion.

How to find the direction in three steps:

1. Decide whether the flux through the circuit, in the direction of the external field, is increasing or decreasing.
2. The induced field inside the circuit must point *against* the external field if the flux is increasing, and *with* it if the flux is decreasing.
3. Use the right-hand grip rule (D.2) to turn the direction of that induced field into a current direction.

Why Lenz's law is a consequence of energy conservation

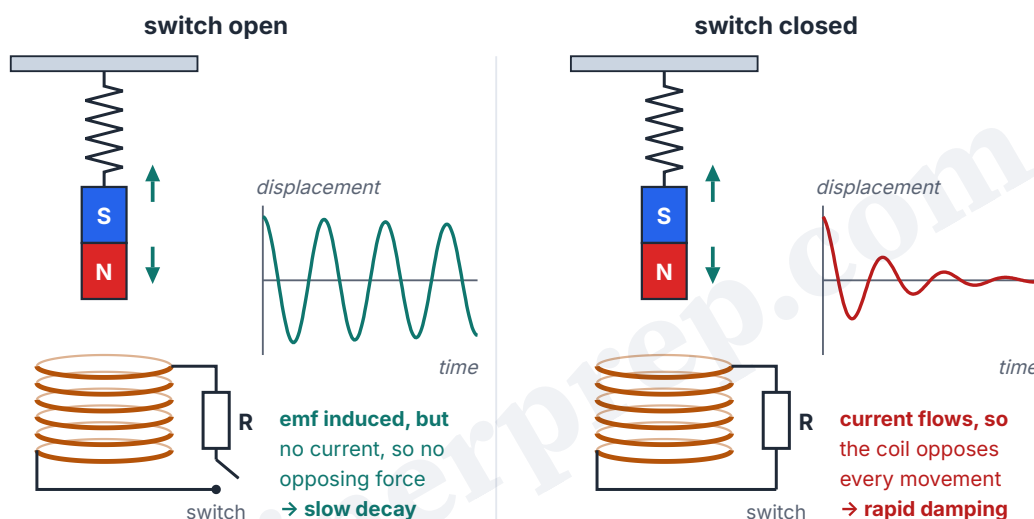
Suppose the induced current *helped* the change instead of opposing it. A magnet pushed gently towards a coil would be pulled in faster, increasing the rate of change of flux, which would increase the current, which would pull harder... The magnet would gain kinetic energy and the circuit would gain electrical energy, both from nothing. That breaks the conservation of energy.

Because the induced current opposes the change, an external agent must **do work** against the opposing force to keep the change going. That work is exactly the electrical energy delivered to the circuit (and dissipated in its resistance). In worked example 3, the 0.060 W supplied by the pulling force is the 0.060 W

dissipated in the resistor. Without the opposition there would be no work done, and no energy to supply the current.

A magnet on a spring oscillating over a coil

This example combines time-varying flux, Lenz's law and energy. A magnet hangs on a spring with one pole just above a coil. As it oscillates, the flux through the coil rises and falls, so an alternating emf is induced.



A magnet oscillating above a coil. With the switch open an emf is induced but no current flows, so the oscillation decays only slowly through air resistance. With the switch closed the induced current opposes every movement of the magnet, and the oscillation is heavily damped.

WORKED EXAMPLE 4 Medium · Explain / Suggest

A magnet oscillates vertically on a spring above a coil, as in the diagram above. The coil is connected to a resistor through a switch.

- (a) Explain why an emf is induced in the coil while the magnet oscillates. (b) State where in the oscillation the induced emf is zero. (c) The switch is closed. Explain, with reference to energy, why the amplitude now decreases more rapidly. (d) Suggest the effect of replacing the resistor with one of smaller resistance.

(a) The magnet's field passes through the coil, so there is a flux linkage. As the magnet moves towards and away from the coil, the field strength at the

coil, and so the flux linkage, changes with time. By Faraday's law, an emf equal to the rate of change of flux linkage is induced.

(b) At the **top and bottom of the oscillation**, where the magnet is momentarily at rest: the flux is not changing at that instant, so the emf is zero.

(c) With the circuit closed, a current flows. By Lenz's law it produces a magnetic field that opposes the motion of the magnet (repelling it as it approaches, attracting it as it moves away). The magnet does work against this force, so energy of the oscillation is transferred to electrical energy and then to internal energy in the resistor and coil. The oscillation loses energy much faster than through air resistance alone, so its **amplitude decreases more quickly**.

(d) For the same emf, a smaller resistance gives a larger current, so a larger opposing force and a larger rate of energy dissipation ($P = \varepsilon^2/R$). The oscillation is **damped more strongly**.

Self-induction (qualitative only)

- A coil carrying a current has a flux linkage produced by *its own* field. If the current in the coil changes, its own flux linkage changes, so an emf is induced in the coil itself. This is **self-induction**.
- By Lenz's law, the self-induced emf opposes the change in current that causes it. When the current is switched on, the self-induced emf opposes the rise, so the current grows gradually rather than instantly. When the current is switched off, it opposes the fall; a very rapid collapse of the flux can induce a large emf, which can cause a spark across the switch contacts.
- The IB requires only this qualitative idea. Inductance and calculations on resistor-inductor circuits are not required.

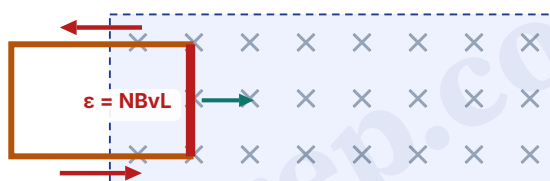
TRY IT YOURSELF

D.4 Electromagnetic Induction simulation: move a magnet into and out of a coil, change its speed and the number of turns, and watch the size and direction of the induced current.

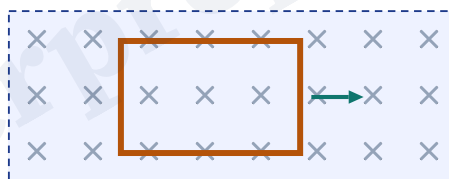
What happens when a coil moves into and out of a field?

A rectangular coil moving at constant speed into a region of uniform field is the IB's standard quantitative example. Treat each stage separately.

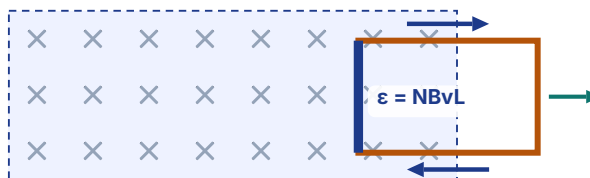
entering
 Φ increasing
 → anticlockwise



fully inside
 Φ constant
 → no current



leaving
 Φ decreasing
 → clockwise



coil $0.080 \text{ m} \times 0.050 \text{ m}$ at 0.40 m s^{-1}

field: B into page, 0.20 m wide

A rectangular coil moving at constant speed through a region of uniform field into the page. While entering, the leading side cuts the field and the current is anticlockwise; fully inside, the flux is constant and there is no current; while leaving, the trailing side cuts the field and the current is clockwise.

- **Entering:** the area of coil inside the field grows at a rate Lv , where L is the side perpendicular to the motion. Only the leading side cuts the field. Each turn has emf BvL , so the coil has $\varepsilon = NBvL$. The flux into the page increases, so the induced current is anticlockwise (seen as drawn).
- **Fully inside:** the flux linkage is constant, so the emf of the coil is zero. (The two sides perpendicular to the motion do each have an emf BvL , but they act in opposite directions around the loop and cancel.) No current, no opposing force.

- **Leaving:** the flux decreases at the same rate, so the emf has the same size and opposite sign; the current is clockwise. Only the trailing side is now in the field.
- While there is a current, the side in the field feels a force $NBIL$ opposing the motion — whether the coil is entering or leaving. To keep the speed constant, an external force must do work, which is dissipated in the coil's resistance.
- If the coil is simply released, this opposing force slows it down on the way in *and* on the way out. This is electromagnetic braking.

WORKED EXAMPLE 5 Hard · Determine / Sketch / Discuss

A rectangular coil of 20 turns measures 0.080 m in the direction of motion and 0.050 m at right angles to it. It is pulled at a constant speed of 0.40 m s⁻¹ through a region 0.20 m wide in which there is a uniform magnetic field of 0.60 T, perpendicular to the plane of the coil (see the diagram above). The coil's resistance is 0.80 Ω. At $t = 0$ the leading side of the coil reaches the field.

(a) Determine the emf induced while the coil enters the field. (b) Determine the times at which the coil is fully inside the field, begins to leave it and has completely left it, and sketch graphs of flux linkage and emf against time. (c) Calculate the force needed to pull the coil while it is entering the field. (d) Determine the total energy dissipated in the coil. (e) Discuss the effect of doubling the speed.

(a) $\varepsilon = NBvL = 20 \times 0.60 \times 0.40 \times 0.050 = \mathbf{0.24\ V}$

Check with Faraday's law: $N\Delta\Phi = 20 \times 0.60 \times (0.080 \times 0.050) = 0.048\ \text{Wb}$ gained in $\frac{0.080}{0.40} = 0.20\ \text{s}$, so $\varepsilon = \frac{0.048}{0.20} = 0.24\ \text{V} \checkmark$

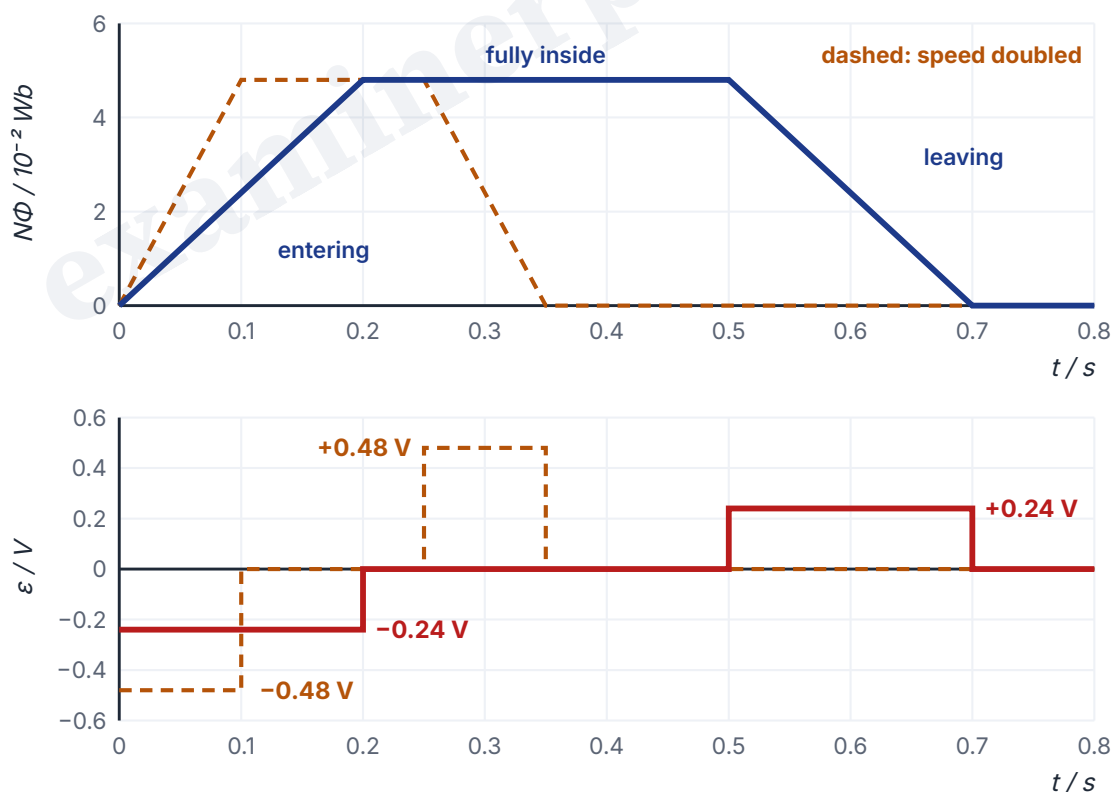
(b) Fully inside at $t = \mathbf{0.20\ s}$. The leading side reaches the far edge after travelling 0.20 m: $t = \mathbf{0.50\ s}$. The coil has completely left after travelling $0.20 + 0.080 = 0.28\ \text{m}$: $t = \mathbf{0.70\ s}$. The graphs are shown below (solid lines): $N\Phi$ rises linearly to 0.048 Wb, stays constant, then falls linearly to zero; the emf is -0.24 V, then 0, then +0.24 V.

(c) $I = \frac{\varepsilon}{R} = \frac{0.24}{0.80} = 0.30 \text{ A}$. The force on the leading side (N turns, each carrying I) is $F = NBIL = 20 \times 0.60 \times 0.30 \times 0.050 = \mathbf{0.18 \text{ N}}$, opposing the motion, so the pulling force is 0.18 N.

(d) A current flows for 0.20 s entering and 0.20 s leaving: $E = \varepsilon It = 0.24 \times 0.30 \times 0.40 = \mathbf{0.029 \text{ J}}$ (0.0288 J).

Check: the pulling force does work $Fd = 0.18 \times (0.080 + 0.080) = 0.0288 \text{ J} \checkmark$

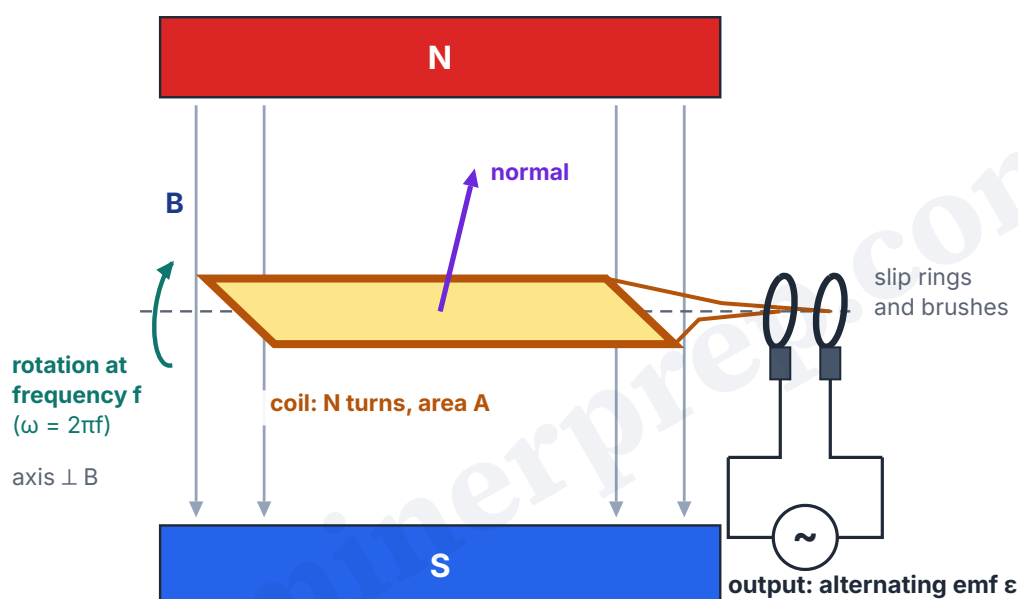
(e) At 0.80 m s^{-1} the emf doubles to 0.48 V but lasts only half as long (0.10 s for each pulse) — dashed lines below. The change in flux linkage is the same, so the *area* of each emf pulse is unchanged (0.048 V s). The current and the opposing force both double, so the energy dissipated, $\frac{\varepsilon^2}{R}t = \frac{0.48^2}{0.80} \times 0.20 = \mathbf{0.058 \text{ J}}$, doubles: moving the coil faster needs more work.



Worked example 5. Solid: flux linkage and emf at 0.40 m s^{-1} . Dashed: the same coil at 0.80 m s^{-1} — the emf doubles and each pulse lasts half as long, so the area of each pulse (the change in flux linkage, 0.048 Wb) is unchanged.

How does a rotating coil produce a sinusoidal emf?

In a simple **generator** (alternator), a rectangular coil of N turns and area A is turned at constant angular velocity ω about an axis perpendicular to a uniform field B . The angle between the field and the normal to the coil changes steadily, so the flux linkage changes all the time and an emf is induced. Slip rings and brushes connect the rotating coil to the external circuit.



A simple generator: a coil rotating about an axis perpendicular to the uniform field between two magnetic poles. Slip rings and brushes connect the rotating coil to the stationary external circuit, which receives an alternating emf.

A coil rotating at constant frequency in a uniform magnetic field has a flux linkage that varies sinusoidally with time, so the induced emf also varies sinusoidally, at the frequency of rotation.

Take $t = 0$ when the normal to the coil is parallel to the field. The angle at time t is $\theta = \omega t$, so

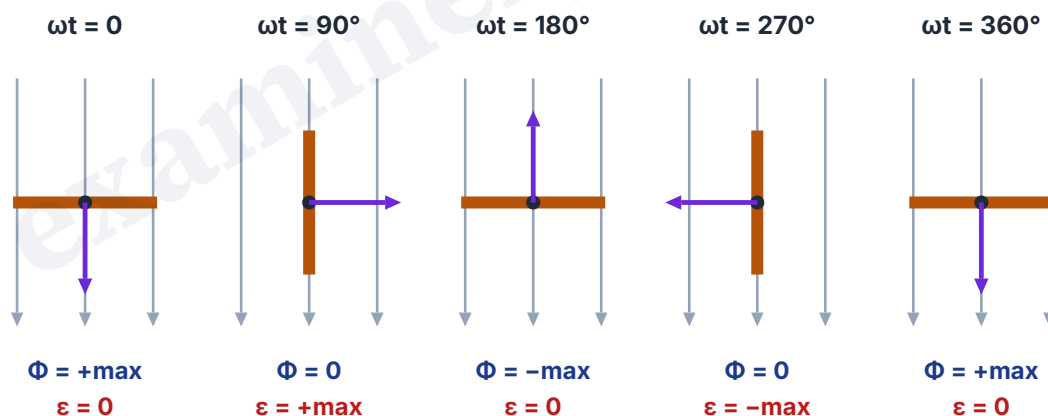
$$N\Phi = NBA \cos \omega t \quad \Rightarrow \quad \varepsilon = NBA\omega \sin \omega t, \quad \varepsilon_0 = NBA\omega = 2\pi fNI$$

- ε = instantaneous emf (V); ε_0 = peak emf (V)
- N = number of turns; A = area of the coil (m^2); B = field strength (T)

- ω = angular velocity of the coil (rad s^{-1}); f = frequency of rotation (Hz), with $\omega = 2\pi f$ and $T = \frac{1}{f}$ (both in the data booklet)

The expressions for ε and ε_0 are **not** in the data booklet. They follow from Faraday's law, because the emf is minus the gradient of the cosine graph of $N\Phi$, which is a sine graph. Without calculus you can derive the peak value from $\varepsilon = BvL$:

- Let the sides parallel to the axis have length L and be a distance $\frac{1}{2}w$ from it, so $A = Lw$. Each moves in a circle at speed $v = \omega \cdot \frac{1}{2}w$.
- Only the component of velocity perpendicular to B cuts the field: $v \sin \omega t$. Each side therefore has an emf $BLv \sin \omega t$.
- The two sides move in opposite directions, so their emfs add round the loop. For N turns: $\varepsilon = 2NBL \left(\frac{1}{2}\omega w\right) \sin \omega t = NBA\omega \sin \omega t$.
- For the two sides perpendicular to the axis, the magnetic force on the charges acts across the wire rather than along it, so they contribute no emf.



end-on view along the axis: field B downwards (grey), normal to the coil in purple

The coil seen end-on along its axis at quarter-turn intervals. When the coil faces the field (normal parallel to B) the flux is greatest and the emf is zero; when the coil's plane is parallel to B the flux is zero but changing fastest, so the emf is greatest.

Position of coil	Angle θ between B and normal	Flux linkage $N\Phi$	Rate of change of $N\Phi$	Emf ε
plane perpendicular to B	0° or 180°	maximum ($\pm NBA$)	zero	zero
plane parallel to B	90° or 270°	zero	greatest	maximum ($\pm NBA\omega$)

So the flux linkage and the emf are a quarter of a cycle ($\frac{\pi}{2}$ rad) out of phase: each is zero when the other is at a peak.

WORKED EXAMPLE 6 Hard · Show that / Determine / Deduce

A rectangular coil of 120 turns measures 6.0 cm by 4.0 cm. It rotates at 25 revolutions per second about an axis perpendicular to a uniform magnetic field of 0.15 T. At $t = 0$ the normal to the coil is parallel to the field.

(a) Show that the peak emf is about 6.8 V. (b) A tangent is drawn to a graph of flux linkage against time at $t = 5.0$ ms. Its gradient is -4.80 Wb s^{-1} . Determine the emf at this instant and confirm it from the emf equation. (c) Determine the average emf as the coil turns from $t = 0$ to $t = 10$ ms, and compare it with the peak emf. (d) Deduce the peak emf and the period of the emf if the coil rotates at 50 revolutions per second.

(a) $A = 0.060 \times 0.040 = 2.4 \times 10^{-3} \text{ m}^2$; $\omega = 2\pi f = 2\pi \times 25 = 157 \text{ rad s}^{-1}$.

$\varepsilon_0 = NBA\omega = 120 \times 0.15 \times 2.4 \times 10^{-3} \times (2\pi \times 25) = \mathbf{6.79 \text{ V} \approx 6.8 \text{ V}}$

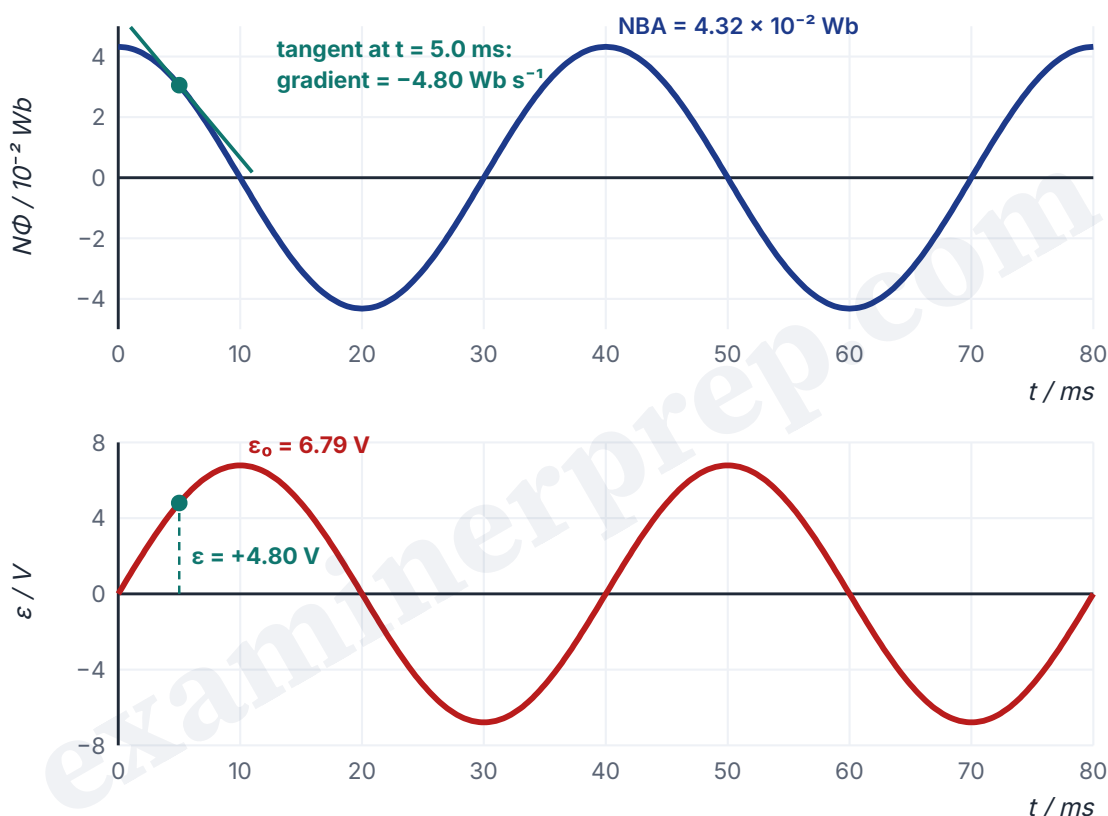
(b) (The graphs and the tangent are drawn below the solution.) $\varepsilon = -$ gradient of the $N\Phi$ - t graph $= -(-4.80) = \mathbf{+4.80 \text{ V}}$.

Check: at $t = 5.0$ ms, $\omega t = 157 \times 0.0050 = 0.785 \text{ rad } (45^\circ)$, so $\varepsilon = 6.79 \sin 45^\circ = 4.80 \text{ V} \checkmark$

(c) 10 ms is a quarter of the period ($T = \frac{1}{25} = 40 \text{ ms}$), so the coil turns from $\theta = 0$ to $\theta = 90^\circ$ and the flux linkage falls from $NBA = 0.0432 \text{ Wb}$ to zero.

$\varepsilon_{\text{av}} = \frac{N\Delta\Phi}{\Delta t} = \frac{0.0432}{0.010} = \mathbf{4.32\text{ V}}$, which is 0.64 of the peak emf — the average over a quarter-cycle is less than the peak because the emf rises from zero.

(d) $\varepsilon_0 = NBA\omega \propto f$, so doubling f doubles the peak: $\varepsilon_0 = 2 \times 6.79 = \mathbf{13.6\text{ V}}$. The period halves: $T = \frac{1}{50}\text{ s} = \mathbf{20\text{ ms}}$.



Worked example 6. Upper: flux linkage of the coil, $NBA \cos \omega t$, with the tangent at $t = 5.0 \text{ ms}$. Lower: the induced emf, $NBA\omega \sin \omega t$. The emf is zero where the flux-linkage graph has a peak or trough (zero gradient) and largest where it crosses zero.

TRY IT YOURSELF

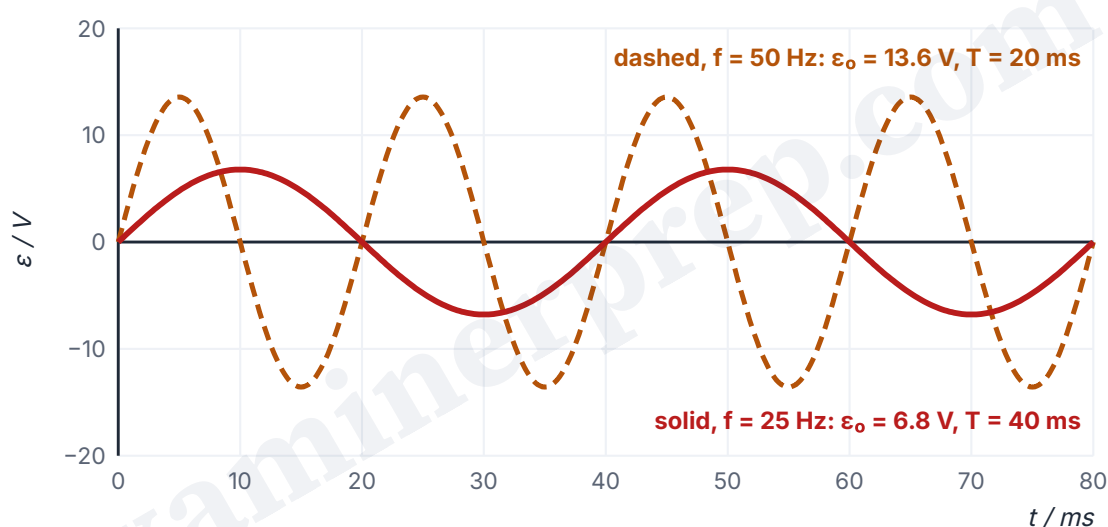
D.4 AC Generator simulation: rotate a coil in a magnetic field, change the frequency, field strength and number of turns, and compare the flux-linkage and emf graphs.

What happens when the frequency of rotation changes?

Increasing the frequency of rotation f has **two** effects on the emf, and you need to state both:

- **The period decreases** ($T = 1/f$): the alternating emf completes more cycles per second, because it has the frequency of rotation.
- **The peak emf increases in proportion** ($\varepsilon_0 = 2\pi f NBA$): the flux linkage swings between the same limits $\pm NBA$, but in less time, so its rate of change is greater at each corresponding point of the cycle.

The peak *flux linkage* does not change — only how quickly it changes. On the emf–time graph, doubling f doubles the height of the curve and halves its period.



The emf of the coil in worked example 6 rotating at 25 Hz (solid) and at 50 Hz (dashed). Doubling the frequency of rotation doubles the peak emf from 6.8 V to 13.6 V and halves the period from 40 ms to 20 ms.

How can the power output of a generator be increased?

The peak emf is $\varepsilon_0 = NBA\omega$, so a generator gives a larger emf, and delivers more power to a given load, if it has:

- more turns N on the coil;
- a stronger magnetic field B (stronger magnets, or an electromagnet; winding the coil on an iron core concentrates the field);
- a larger coil area A ;
- a higher frequency of rotation f .

For a resistive load R , the power at the peak of the cycle is $\frac{\varepsilon_0^2}{R}$, so doubling f multiplies it by four. That energy is not free: the larger current produces a larger opposing torque on the coil (Lenz's law), so whatever turns the generator — a turbine, an engine, or your hand — must do work faster.

Exam tips for D.4

EXAM TIP 1 — TIE INDUCTION TO THE APPARATUS IN THE QUESTION

When you are asked why an emf appears, a bare statement of Faraday's law is not enough. Build the answer from the situation in front of you: say what sets up a flux linkage in *this* coil or circuit, then say what makes that flux linkage *change* (the rotation, the moving magnet, the changing current in the electromagnet), and only then conclude with Faraday's law. Define any symbols you use.

EXAM TIP 2 — θ IS MEASURED FROM THE NORMAL

In $\Phi = BA \cos \theta$, θ is between the field and the **normal** to the coil. If the question gives the angle between the field and the *plane* of the coil, use 90° minus that angle (or sine instead of cosine). For a rotating coil, the emf is greatest at $\theta = 90^\circ$ (or 270°), when the plane is parallel to the field.

EXAM TIP 3 — LENZ: OPPOSE THE *CHANGE*

Write "the induced current opposes the **change** in flux" or "opposes the motion", never "opposes the field" or "opposes the flux". When you are asked to link Lenz's law to energy, say that work must be done against the opposing force, and that this work is transferred to electrical energy — a statement about direction alone does not earn the energy mark.

EXAM TIP 4 — AVERAGE OR INSTANTANEOUS?

$N \frac{\Delta\Phi}{\Delta t}$ over an interval gives the **average** emf: find the flux linkage at the start and the end (use $\cos \theta$ at both angles for a rotating coil) and divide by the time. For an **instantaneous** emf from a graph, draw a tangent and measure its gradient, using a large triangle. Do not divide one reading by one time.

EXAM TIP 5 — THE FREQUENCY QUESTION HAS TWO MARKS

When asked how the output of a generator changes if it turns faster, give both effects: the frequency of the alternating emf increases (period decreases) **and** the peak emf increases in proportion. On a sketch, show both: a taller curve with more cycles in the same time.

Common misconceptions

Misconception	Why it is wrong	Correct idea
A large flux through a coil produces a large emf.	A steady flux, however large, induces nothing.	The emf is set by the <i>rate of change</i> of flux linkage.
No current, so no emf.	The emf is induced by the changing flux, whether or not the circuit is complete.	An emf is always induced; a current needs a closed circuit.
The induced field always points against the magnet's field.	If the flux is decreasing, the induced field points the same way as the external field.	The induced current opposes the <i>change</i> in flux.
A coil moving through a uniform field always has an emf.	Fully inside a uniform field its flux linkage is constant; the emfs in its two sides cancel.	There is an emf only while the coil is entering or leaving.
A rotating coil has its greatest emf when its flux is greatest.	At maximum flux the flux graph is flat — its rate of change is zero.	The emf is greatest when the flux is zero (plane parallel to B).
Turning a generator faster only increases the frequency of the output.	The flux changes between the same limits in less time.	Peak emf and frequency both increase in proportion to the frequency of rotation.

How this topic connects

From fuel to grid: why overall efficiency depends on the energy source

Almost every power station ends in the same device: a turbine turning a generator, which uses electromagnetic induction to transfer kinetic energy of rotation to electrical energy. The generator itself is very efficient — its losses are mainly resistive heating in the coils and friction in the bearings. The big differences come *before* the turbine. Fossil-fuel and nuclear stations are heat engines: they heat water to drive a steam turbine, and the second law of

thermodynamics (B.4) requires a large fraction of the energy to be rejected as thermal energy to the surroundings. Their overall efficiency is therefore limited — typically about a third for older coal and nuclear stations, and up to about 60 % for the best combined-cycle gas stations. Hydroelectric and wind turbines take kinetic energy directly from moving water or air, with no heating stage, so this thermodynamic limit does not apply; hydroelectric stations are among the most efficient of all. A wind turbine cannot extract all the kinetic energy of the wind, because the air must keep moving away from the blades. Photovoltaic cells (not part of D.4) generate electricity with no generator at all.

Faraday's law and the many rates of change in physics

Many laws in physics are statements about how fast something changes: velocity and acceleration are rates of change of displacement and velocity (A.1); force is the rate of change of momentum (A.2); power is the rate of doing work (A.3); torque is the rate of change of angular momentum (A.4); current is the rate of flow of charge (B.5); activity is the rate of radioactive decay (E.3). In each case the same graphical tools apply: the gradient of a graph of the quantity against time gives the rate, a tangent gives the instantaneous value, and the area under the rate–time graph gives the total change — just as the area under an emf–time graph gives the size of the change in flux linkage. Recognising this shared mathematical structure is part of how physicists build general models from particular laws.

Where motors and generators lose energy, and how design reduces it

(This connection also appears in B.4 Thermodynamics.)

A generator transfers mechanical work to electrical energy and a motor does the reverse; both use a coil turning in a magnetic field. In each, some energy is dissipated: resistive heating ($I^2 R$) in the coil windings, and friction in the bearings and brushes. These losses heat the machine, waste energy and limit how much current the coil can safely carry. Engineers raise efficiency by using thick, low-resistance windings, good bearings and strong fields, so that the required emf or torque is reached with a smaller current. Lenz's law sets the background: the electrical energy delivered by a generator can never exceed the work done turning it.

Tools & inquiry — measuring induced emf with a data logger

- Drop a bar magnet through a vertical coil connected to a voltage sensor and record the emf at a high sampling rate (several hundred readings per second or more).
- The emf–time graph shows two pulses of **opposite sign** (flux increasing, then decreasing). The second pulse is taller and narrower because the magnet is moving faster as it leaves the coil.
- The **areas** of the two pulses are equal in size: each equals the change in flux linkage $N\Delta\Phi$, which is the same going in as coming out. Use the data-logging software to find each area — a good test of Faraday's law.
- Repeat from different heights to vary the speed, or use coils with different numbers of turns, and test whether the peak emf is proportional to speed and to N .
- **NOS — evidence and conservation:** Faraday discovered induction experimentally in 1831, looking for a link between electricity and magnetism suggested by Ørsted's earlier discovery. Lenz's law was later recognised as a consequence of the conservation of energy — a general principle constraining the details of a particular phenomenon.

Key equations

Equation	Use	In data booklet?
$\Phi = BA \cos \theta$	magnetic flux through an area; θ from the normal	yes
$N\Phi$	flux linkage of a coil of N turns	no — definition
$\varepsilon = -N \frac{\Delta\Phi}{\Delta t}$	Faraday's law; the minus sign is Lenz's law	yes
$\varepsilon = BvL$	straight conductor moving at right angles to a uniform field	yes
$F = BIL$	opposing force on a current-carrying conductor (D.3)	yes (as $F = BIL \sin \theta$)
$\varepsilon = NBA\omega \sin \omega t$, $\varepsilon_0 = NBA\omega$	coil rotating in a uniform field, normal parallel to B at $t = 0$	no — derived from Faraday's law
$\omega = 2\pi f$, $T = \frac{1}{f}$	frequency of rotation and period of the emf	yes (as $T = \frac{1}{f} = \frac{2\pi}{\omega}$, C.1)
$P = \varepsilon I = I^2 R = \frac{\varepsilon^2}{R}$	power delivered by an induced emf (B.5)	yes (as $P = IV = I^2 R = \frac{V^2}{R}$)

Check your understanding

1. A glider with a wingspan of 15 m flies horizontally at 30 m s^{-1} where the vertical component of the Earth's magnetic field is $4.0 \times 10^{-5} \text{ T}$. Calculate the emf induced between its wingtips.

The wings move horizontally, at right angles to the vertical component of the field: $\varepsilon = BvL = 4.0 \times 10^{-5} \times 30 \times 15 = 0.018 \text{ V}$ (18 mV). The horizontal component of the field pushes the charges vertically — across the wing, not along it — so it does not contribute.

2. A coil of 400 turns and area $5.0 \times 10^{-4} \text{ m}^2$ lies with its plane perpendicular to a field of 0.25 T. It is pulled completely out of the field in 0.050 s. Calculate the average emf induced.

$$N\Delta\Phi = 400 \times 0.25 \times 5.0 \times 10^{-4} = 0.050 \text{ Wb, so } \varepsilon = \frac{0.050}{0.050} = 1.0 \text{ V.}$$

3. A generator produces an alternating emf of peak value 12 V when its coil rotates at 40 Hz. State the peak emf and the period of the emf when it rotates at 10 Hz.

The peak emf is proportional to the frequency of rotation: $12 \times \frac{10}{40} = 3.0 \text{ V.}$

The period is $\frac{1}{10} = 0.10 \text{ s}$ (compared with 25 ms at 40 Hz).

4. A rod of length 0.50 m moves at right angles to a uniform field of 0.080 T. The emf between its ends is 0.12 V. Calculate its speed, and state whether a current flows in the rod.

$v = \frac{\varepsilon}{BL} = \frac{0.12}{0.080 \times 0.50} = 3.0 \text{ m s}^{-1}.$ No current flows unless the ends are connected by a conducting path outside the field to form a closed circuit; charge separates until the electric force on the electrons balances the magnetic force.

5. A student turns the handle of a small hand-cranked generator. Explain why the handle becomes harder to turn when a lamp is connected to the generator.

With the lamp connected the circuit is complete, so the induced emf drives a current through the coil. By Lenz's law the current flows in the direction that opposes the change producing it: the magnetic force on the current-carrying coil produces a torque opposing the rotation. The student must do extra work against this torque, and that work is the electrical energy transferred to the lamp. With no lamp, there is an emf but no current and no opposing torque.

Practise D.4 with exam-style questions

Original Paper 1A, 1B and 2 questions on electromagnetic induction, with full mark schemes. D.4 is HL only, so there is no SL bank for this sub-topic. Tick the questions you want and print them as your own test.

HL D.4 questions

Full question bank

Revise and explore

- [Flashcards](#) — choose IB Physics and filter to D.4 Induction. Free, no sign-up.
- [D.4 Electromagnetic Induction simulation](#)
- [D.4 AC Generator simulation](#)
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