

THEME E · NUCLEAR AND QUANTUM PHYSICS

HL ONLY

≈ 30 min read

IB Physics E.2 Quantum Physics — Notes (HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). **E.2 is entirely Additional Higher Level content** — there is no SL material in this sub-topic, and it is examined only on HL papers. It asks one big question: is light a wave or a stream of particles, and is an electron a particle or a wave? The answer, built from three experiments, is “both”.



HL E.2 questions



Flashcards



Photoelectric effect simulation

What this page covers

- ✓ The photoelectric effect and why it shows that light behaves as particles (photons)
- ✓ The threshold frequency: light below a certain frequency releases no photoelectrons, however intense it is
- ✓ Einstein's explanation using the work function, $E_{\max} = hf - \Phi$, and how $E_{\max}$ is measured with a stopping voltage
- ✓ Which features of the photoelectric effect the classical wave model of light cannot explain
- ✓ The de Broglie wavelength of a particle, $\lambda = \frac{h}{p}$
- ✓ Diffraction of electrons as evidence that matter behaves as waves, including where the first minimum of a scattering pattern lies
- ✓ Wave-particle duality of both light and matter
- ✓ Compton scattering: photons scattered by electrons leave with a longer wavelength, $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$

In one minute

- Light arrives in **photons** of energy $E = hf$. In the **photoelectric effect** one photon gives all its energy to one electron in a metal surface.
- The electron needs at least the **work function** Φ to escape. So nothing happens below the **threshold frequency** $f_0 = \Phi/h$, however bright the light. Above it, $E_{\max} = hf - \Phi$.
- Intensity changes the *number* of photons per second, so it changes the photocurrent — not $E_{\max}$. A graph of $E_{\max}$ against f is a straight line of gradient h .
- Particles have a **de Broglie wavelength** $\lambda = h/p$. Electrons diffract through crystals and around nuclei, so matter behaves as waves. The first minimum of the pattern is at $\theta \approx \lambda/D$.
- In **Compton scattering** an X-ray photon bounces off an electron like a particle in a collision, and leaves with a longer wavelength: $\Delta\lambda = \frac{h}{m_e c} (1 - \cos \theta)$.

Contents

- | | |
|---|------------------------------|
| 1. The photoelectric effect | 7. Diffraction of particles |
| 2. Threshold frequency | 8. Wave-particle duality |
| 3. Einstein's explanation and the work function | 9. Compton scattering |
| 4. Measuring $E_{\max}$: stopping voltage | 10. Exam tips |
| 5. Why the wave model fails | 11. Common misconceptions |
| 6. The de Broglie wavelength | 12. How this topic connects |
| | 13. Check your understanding |

What is the photoelectric effect?

The photoelectric effect is the emission of electrons from the surface of a metal when electromagnetic radiation of high enough frequency falls on it. The electrons released are called photoelectrons.

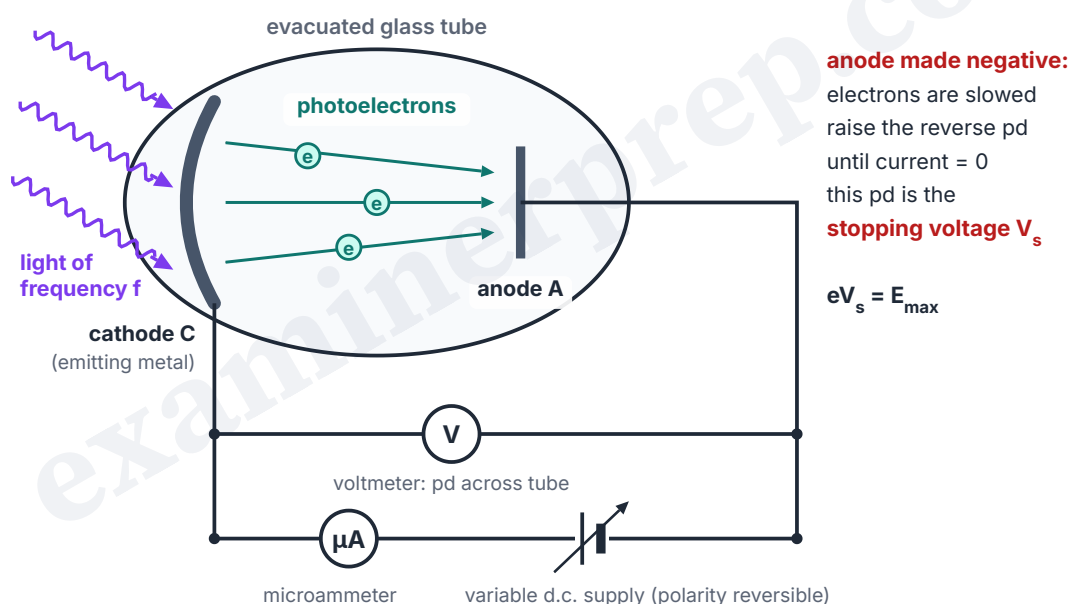
In E.1 you met the **photon**: a quantum (packet) of electromagnetic energy. The energy of one photon depends only on the frequency of the radiation:

$$E = hf$$

- E = energy of one photon (J)
- h = Planck constant, 6.63×10^{-34} J s (data booklet)
- f = frequency of the radiation (Hz)

$E = hf$ is in the data booklet (E.1). Combined with $c = f\lambda$ it gives $E = \frac{hc}{\lambda}$.

Photon energies are tiny, so they are often quoted in electronvolts: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.



A photocell. Light falls on the metal cathode C inside an evacuated tube; photoelectrons cross to the anode A and the microammeter registers a current. Reversing the supply makes the anode negative and slows the electrons down.

In the apparatus above, the tube is evacuated so that photoelectrons are not stopped by collisions with gas molecules. With the anode positive, every photoelectron is collected and the microammeter reads the **photocurrent**. The key observations are:

- For a given metal, there is a **threshold frequency** f_0 . Below it, no electrons are emitted, however intense the light and however long it shines.

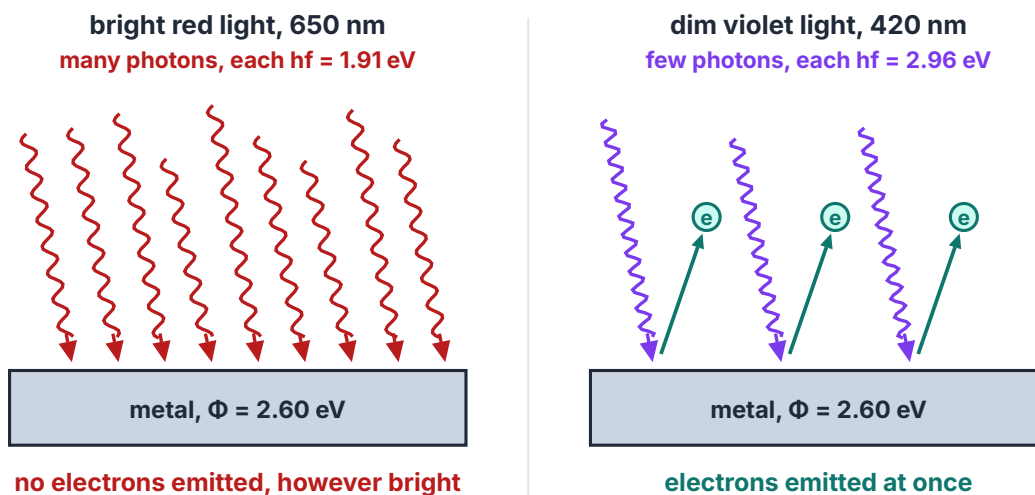
- Above f_0 , emission begins **immediately** (within about 10^{-9} s), even when the light is very dim.
- The **maximum kinetic energy** of the photoelectrons increases with the frequency of the light, but does **not** depend on its intensity.
- The **number** of photoelectrons emitted per second (and so the photocurrent) is proportional to the intensity, for a fixed frequency.

Each of these is explained if light is a stream of particles — photons — and each one is a problem for a pure wave model. That is why the photoelectric effect is **evidence for the particle nature of light**.

What is the threshold frequency?

The threshold frequency f_0 of a metal is the minimum frequency of electromagnetic radiation that can release photoelectrons from its surface.

- An electron absorbs **one photon** at a time and receives **all** of its energy. (The chance that one electron absorbs two photons is negligible at ordinary intensities.)
- If one photon does not carry enough energy for an electron to escape, the electron loses that energy again as it collides with the metal ions — it cannot “save up” energy from several photons.
- So emission depends on the energy of each photon, hf , and therefore on frequency. Making the light brighter only sends more photons per second, each with the same energy.
- Different metals have different threshold frequencies. For many metals f_0 is in the ultraviolet; for the alkali metals (such as caesium) it is in the visible.
- The threshold wavelength is $\lambda_0 = c/f_0$: radiation of *longer* wavelength than λ_0 releases no electrons.



The same metal (work function 2.60 eV) under two lamps. Bright red light delivers many photons per second, but each has only 1.91 eV , so none can release an electron. Dim violet light delivers few photons, but each carries 2.96 eV , so electrons are emitted immediately.

TRY IT YOURSELF

E.2 Photoelectric Effect simulation: change the frequency and the intensity of the light and the metal of the cathode, and watch the threshold frequency, the photocurrent and the stopping voltage respond.

How did Einstein explain the photoelectric effect?

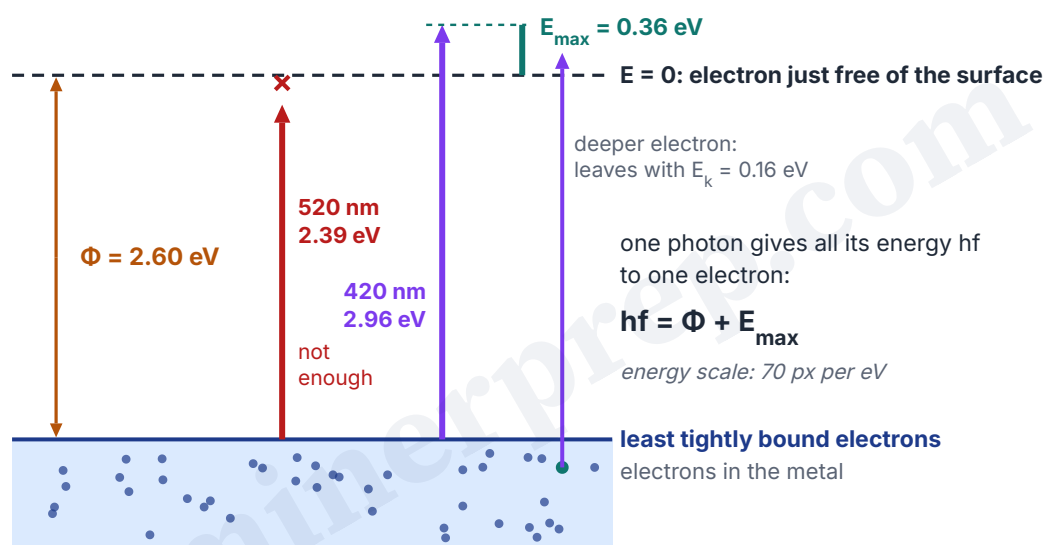
The work function Φ of a metal is the minimum energy needed to remove an electron from the surface of the metal.

Einstein applied conservation of energy to a single photon–electron interaction. The photon's energy hf is used first to release the electron (at least Φ); whatever is left is the electron's kinetic energy. Electrons that need exactly Φ — the least tightly bound, at the surface — leave with the most kinetic energy:

$$E_{\text{max}} = hf - \Phi$$

- E_{max} = maximum kinetic energy of the photoelectrons (J or eV)
- h = Planck constant (J s); f = frequency of the incident radiation (Hz)
- Φ = work function of the metal (J or eV)

- This equation is in the data booklet. Keep every energy in the **same unit**: all in joules, or all in electronvolts.
- Most photoelectrons come from below the surface and lose some energy on the way out, so they leave with *less* than $E_{\max}$. That is why the equation gives the **maximum**.
- At the threshold frequency the photoelectrons have just zero kinetic energy: $0 = hf_0 - \Phi$, so $\Phi = hf_0$ (not in the data booklet; it follows directly).



Energy picture drawn to scale (work function 2.60 eV). A 520 nm photon (2.39 eV) cannot lift even the least tightly bound electron to $E = 0$. A 420 nm photon (2.96 eV) can, leaving it 0.36 eV of kinetic energy; an electron 0.20 eV deeper leaves with only 0.16 eV.

WORKED EXAMPLE 1 Easy · Calculate / State

A metal surface has a work function of 2.28 eV.

(a) Calculate the threshold frequency and the threshold wavelength for this metal. (b) Yellow light of wavelength 570 nm falls on the surface. State and explain whether photoelectrons are emitted. (c) Violet light of wavelength 410 nm is used instead. Calculate the maximum kinetic energy of the photoelectrons, in eV and in J.

$$\text{(a)} \quad \Phi = 2.28 \times 1.60 \times 10^{-19} = 3.65 \times 10^{-19} \text{ J.}$$

$$f_0 = \frac{\Phi}{h} = \frac{3.65 \times 10^{-19}}{6.63 \times 10^{-34}} = 5.50 \times 10^{14} \text{ Hz}$$

$$\lambda_0 = \frac{c}{f_0} = \frac{3.00 \times 10^8}{5.50 \times 10^{14}} = \mathbf{5.45 \times 10^{-7} \text{ m (545 nm)}}$$

$$\text{(b) } E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{570 \times 10^{-9}} = 3.49 \times 10^{-19} \text{ J} = 2.18 \text{ eV.}$$

This is less than the work function (2.28 eV) — equivalently, 570 nm is longer than the threshold wavelength of 545 nm — so **no photoelectrons are emitted**, whatever the intensity.

$$\text{(c) } E = \frac{hc}{\lambda} = \frac{1.989 \times 10^{-25}}{410 \times 10^{-9}} = 4.85 \times 10^{-19} \text{ J} = 3.03 \text{ eV.}$$

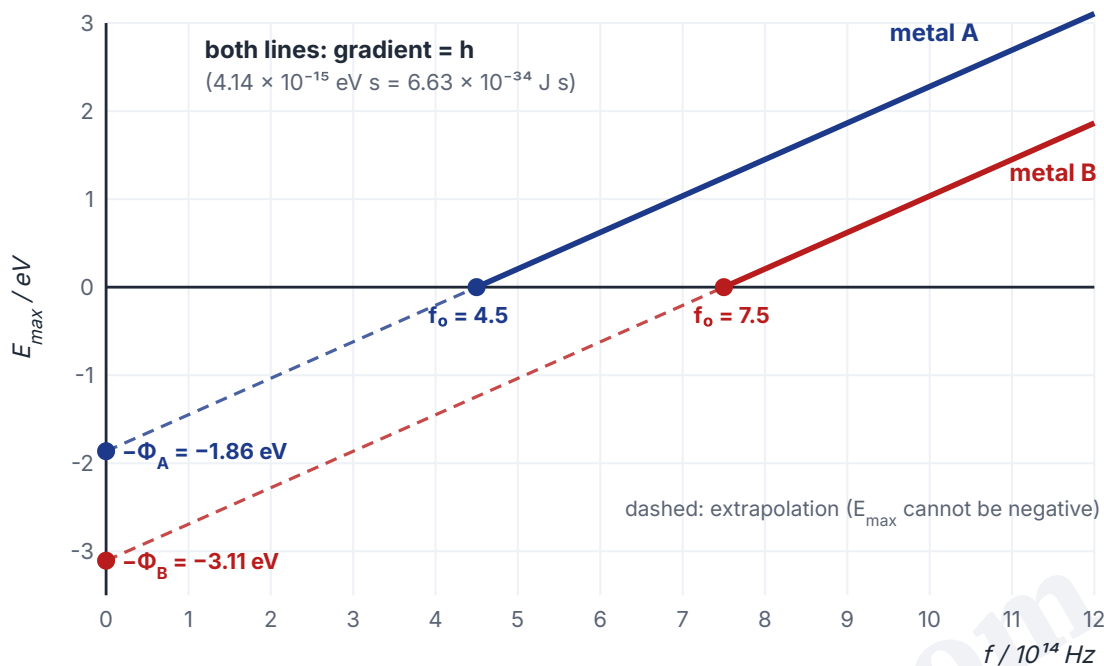
$$E_{\text{max}} = hf - \Phi = 3.03 - 2.28 = \mathbf{0.75 \text{ eV} = 1.2 \times 10^{-19} \text{ J}}$$

Notice that E_{max} is a difference of two larger numbers. Carry three significant figures through the subtraction, or you will lose accuracy in the answer.

The $E_{\text{max}}-f$ graph

Rearranged as $E_{\text{max}} = hf - \Phi$, Einstein's equation has the form $y = mx + c$. A graph of E_{max} against f is a **straight line**:

- **gradient** = h , the Planck constant — the same for every metal, so lines for different metals are **parallel**;
- **intercept on the f axis** = **threshold frequency f_0** ;
- **intercept on the E_{max} axis** = $-\Phi$ (found by extrapolating the line; negative kinetic energies are never measured).



Maximum kinetic energy against frequency for two metals. Both lines have gradient h ; their intercepts give the threshold frequencies (4.5×10^{14} Hz and 7.5×10^{14} Hz) and work functions (1.86 eV and 3.11 eV). The metal with the larger work function has its line further to the right.

How is the maximum kinetic energy measured?

In the photocell, the supply is reversed so that the anode is **negative** relative to the cathode. Photoelectrons now have to move against an electric field to reach the anode, losing kinetic energy as they go. As the reverse potential difference is increased, fewer electrons reach the anode, until even the fastest are turned back and the current falls to zero.

The stopping voltage V_s is the minimum reverse potential difference between anode and cathode that reduces the photocurrent to zero. It is related to the maximum kinetic energy of the photoelectrons by $eV_s = E_{\max}$.

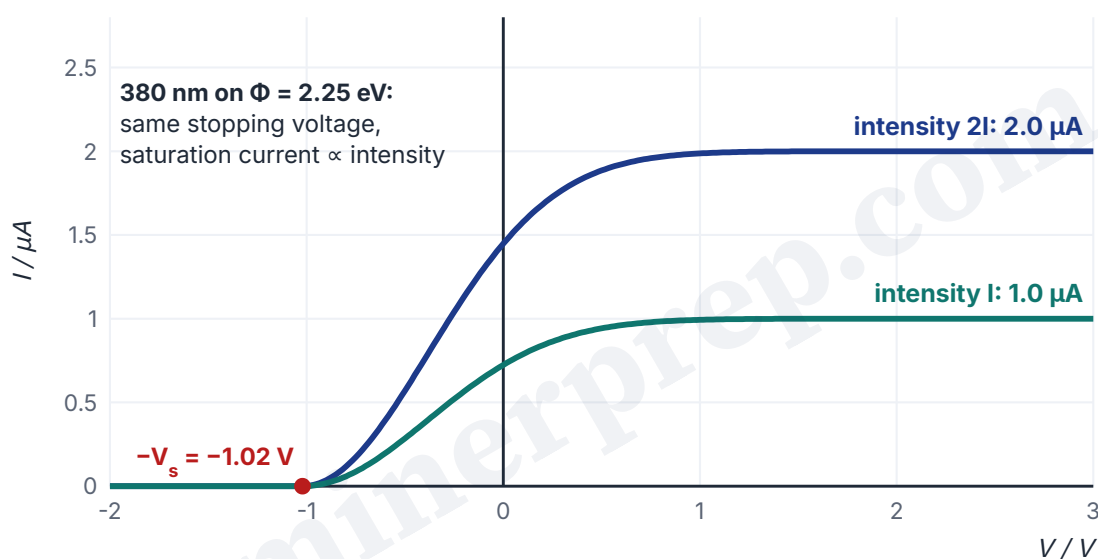
$$eV_s = E_{\max} = hf - \Phi$$

- e = elementary charge, 1.60×10^{-19} C; V_s = stopping voltage (V)
- $eV_s = E_{\max}$ is not in the data booklet: it is the work done on a charge, $W = qV$, set equal to the kinetic energy lost.
- A neat consequence: $E_{\max}$ in eV has the **same number** as V_s in volts. A stopping voltage of 1.5 V means $E_{\max} = 1.5$ eV.

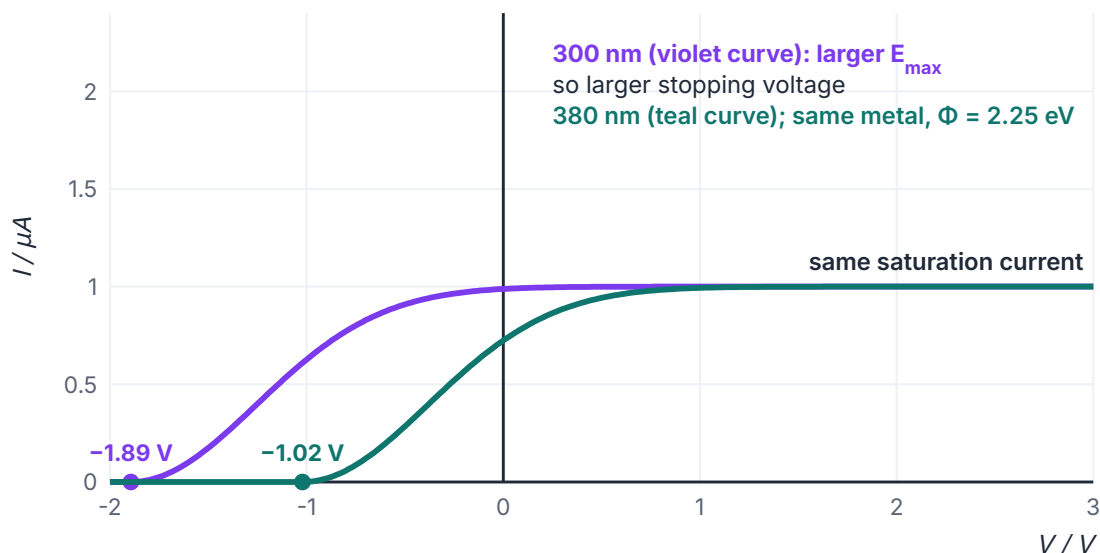
- Plotting V_s against f gives a straight line of gradient $\frac{h}{e}$ and f -intercept f_0 .

Current–potential difference graphs

The two graphs below show the photocurrent I as the anode potential V is varied from negative (retarding) to positive (collecting). At large positive V every emitted electron reaches the anode, so the current levels off at a **saturation current** equal to $e \times$ (number of photoelectrons per second).



Same frequency (380 nm) on the same metal (work function 2.25 eV), two intensities. Doubling the intensity doubles the number of photons per second, so the saturation current doubles from $1.0 \mu\text{A}$ to $2.0 \mu\text{A}$. The stopping voltage, 1.02 V, does not change because each photon still carries the same energy.



Same metal, two frequencies, with the same number of photons arriving per second. The higher frequency (300 nm) gives a larger $E_{\max}$ and so a larger stopping voltage (1.89 V against 1.02 V).

The saturation current is the same.

WORKED EXAMPLE 2 Medium · Calculate / Determine

Ultraviolet radiation of wavelength 254 nm falls on a metal cathode of work function 4.10 eV in a photocell.

- Calculate the energy of one photon, in J and in eV.
- Determine the stopping voltage.
- Calculate the maximum speed of the photoelectrons.
- The radiation delivers 2.0 mW to the cathode and the saturation photocurrent is 1.2 μA . Determine the fraction of incident photons that release a photoelectron.
- The intensity is doubled without changing the wavelength. State the new stopping voltage and the new saturation current.

$$\text{(a)} \quad E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{254 \times 10^{-9}} = 7.83 \times 10^{-19} \text{ J} = 4.89 \text{ eV}$$

$$\text{(b)} \quad E_{\max} = 4.89 - 4.10 = 0.79 \text{ eV, so } eV_s = E_{\max} \text{ gives } V_s = 0.79 \text{ V}$$

$$\text{(c)} \quad E_{\max} = 0.794 \times 1.60 \times 10^{-19} = 1.27 \times 10^{-19} \text{ J.}$$

$$v_{\max} = \sqrt{\frac{2E_{\max}}{m_e}} = \sqrt{\frac{2 \times 1.27 \times 10^{-19}}{9.110 \times 10^{-31}}} = 5.3 \times 10^5 \text{ m s}^{-1}$$

$$\text{(d)} \quad \text{Photons per second} = \frac{P}{E} = \frac{2.0 \times 10^{-3}}{7.83 \times 10^{-19}} = 2.55 \times 10^{15} \text{ s}^{-1}.$$

$$\text{Electrons per second} = \frac{I}{e} = \frac{1.2 \times 10^{-6}}{1.60 \times 10^{-19}} = 7.5 \times 10^{12} \text{ s}^{-1}.$$

$$\text{Fraction} = \frac{7.5 \times 10^{12}}{2.55 \times 10^{15}} = 2.9 \times 10^{-3} \text{ (about 0.3 \%)}$$

(e) Stopping voltage **unchanged at 0.79 V** (photon energy unchanged); saturation current **2.4 μA** (twice as many photons, so twice as many photoelectrons per second) — compare the first current–pd graph above, which shows the same behaviour for a different metal.

Most photons are reflected or absorbed without releasing an electron, so the photocurrent is far smaller than “one electron per photon” would suggest. The speed in (c) is less than 0.2 % of c , so the non-relativistic $E_k = \frac{1}{2}mv^2$ is fine.

WORKED EXAMPLE 3 Medium · Determine (graph)

A student illuminates a metal with monochromatic light of different frequencies f and measures the stopping voltage V_s each time.

$f / 10^{14} \text{ Hz}$	6.0	7.0	8.0	9.0	10.0
V_s / V	0.41	0.83	1.24	1.66	2.07

(a) Use the data to determine a value for the Planck constant. (b) Determine the threshold frequency and the work function of the metal, in eV. (c) The cathode is replaced by one made of a metal of work function 3.00 eV. Determine the stopping voltage when the frequency is $8.0 \times 10^{14} \text{ Hz}$.

Since $eV_s = hf - \Phi$, a graph of V_s against f is a straight line of gradient h/e . Plotting the five points confirms this: equal steps of frequency give equal steps of about 0.41 V in V_s .

$$\text{(a) Gradient} = \frac{2.07 - 0.41}{(10.0 - 6.0) \times 10^{14}} = 4.15 \times 10^{-15} \text{ V s.}$$

$$h = e \times \text{gradient} = 1.60 \times 10^{-19} \times 4.15 \times 10^{-15} = 6.64 \times 10^{-34} \text{ J s}$$

(close to the accepted $6.63 \times 10^{-34} \text{ J s}$)

$$\text{(b) Extend the line to } V_s = 0: f_0 = 10.0 \times 10^{14} - \frac{2.07}{4.15 \times 10^{-15}} = 5.0 \times 10^{14} \text{ Hz}$$

$$\Phi = hf_0 = 6.64 \times 10^{-34} \times 5.01 \times 10^{14} = 3.33 \times 10^{-19} \text{ J} = 2.08 \text{ eV}$$

(The same value is read from the intercept on the V_s axis, -2.08 V, which means $\Phi = 2.08$ eV.)

(c) The new line has the **same gradient** (h/e), so it is parallel to the first, but its V_s -axis intercept is -3.00 V instead of -2.08 V: every stopping voltage is $3.00 - 2.08 = 0.92$ V lower. At 8.0×10^{14} Hz: $V_s = 1.24 - 0.92 = \mathbf{0.32\text{ V}}$
(Check with the accepted h : $\frac{hf}{e} - 3.00 = 3.315 - 3.00 = 0.315$ V; accept 0.31 – 0.32 V. The frequency is above the new threshold, 7.2×10^{14} Hz, so electrons are still emitted.)

Use a best-fit line through all the points rather than two data points if the data are scattered. Here a least-squares fit gives the same gradient, 4.15×10^{-15} V s.

Which features cannot be explained by the wave model of light?

In the classical wave model, light is a continuous wave whose energy is spread evenly over the wavefront, and the energy delivered per second depends on the **intensity** (amplitude squared). The table compares what this model predicts with what is observed.

Observation	Wave-model prediction	Photon explanation
There is a threshold frequency: below f_0 , no emission at any intensity.	Any frequency should work if the light is intense enough, or shines for long enough, because energy keeps arriving.	One photon gives its energy to one electron. If $hf < \Phi$, no single photon can release an electron, and the energy cannot be accumulated.
$E_{\max}$ depends on frequency, not on intensity.	More intense light delivers more energy per second, so the electrons should leave with more kinetic energy.	$E_{\max} = hf - \Phi$ depends only on the energy of one photon, which depends only on f .
Emission is instantaneous, even in very dim light.	In dim light an electron would need a long time to absorb enough energy (see below).	A single photon delivers its whole energy in one event, so emission starts as soon as the first photons arrive.
Photocurrent is proportional to intensity.	Consistent with the wave model — this is the one feature it does not contradict.	Intensity is proportional to the number of photons per second, and so to the number of photoelectrons per second.

How long would the wave model take? Suppose dim light of intensity $1.0 \times 10^{-3} \text{ W m}^{-2}$ falls on the metal in the figure of the red and violet lamps ($\Phi = 2.60 \text{ eV}$), and an electron can collect energy only from an area about the size of one atom, $1.0 \times 10^{-19} \text{ m}^2$. The power reaching it is $1.0 \times 10^{-3} \times 1.0 \times 10^{-19} = 1.0 \times 10^{-22} \text{ W}$, so collecting $4.16 \times 10^{-19} \text{ J}$ would take

$$t = \frac{4.16 \times 10^{-19}}{1.0 \times 10^{-22}} = 4.2 \times 10^3 \text{ s} \approx 70 \text{ minutes}$$

Experimentally (with light above the threshold frequency) the delay is less than about 10^{-9} s . The photon model was accepted because it explained all four observations; the observations **falsified** the idea that light is purely a wave.

What is the de Broglie wavelength?

If waves (light) can behave as particles, can particles behave as waves? In 1924 de Broglie proposed that every moving particle has a wavelength linked to its momentum.

The de Broglie wavelength of a particle is the wavelength associated with it when it behaves as a wave; it equals the Planck constant divided by the particle's momentum: $\lambda = \frac{h}{p}$.

$$\lambda = \frac{h}{p}$$

- λ = de Broglie wavelength (m)
- h = Planck constant (J s)
- p = momentum of the particle (kg m s^{-1}); for a slow particle $p = mv$
- This equation is in the data booklet. It applies to **all** particles (electrons, protons, neutrons, atoms, tennis balls) and also to photons, which carry momentum $p = h/\lambda$.
- If you know the **kinetic energy**, use $E_k = \frac{p^2}{2m}$ (data booklet, A.3): $p = \sqrt{2mE_k}$.
- For a charged particle accelerated from rest through a potential difference V : $E_k = qV$, so $\lambda = \frac{h}{\sqrt{2mqV}}$. This combined form is **not** in the data booklet — build it each time.
- Because h is so small, everyday objects have wavelengths far too small to detect. Wave effects are observable only when λ is comparable to the size of an aperture or obstacle — which, for electrons, means atomic spacings and nuclei.

WORKED EXAMPLE 4 Medium · Show that / Explain

Electrons are accelerated from rest through a potential difference of 150 V.

- (a) Show that the de Broglie wavelength of the electrons is about 1×10^{-10} m. (b) Explain why these electrons are diffracted by a crystal, in which the atoms are about 10^{-10} m apart. (c) Calculate the de Broglie wavelength of a

0.15 kg ball moving at 30 m s^{-1} , and comment on the result. (d) The accelerating potential difference is increased to 600 V. Determine the new de Broglie wavelength of the electrons.

(a) $E_k = eV = 1.60 \times 10^{-19} \times 150 = 2.40 \times 10^{-17} \text{ J}.$

$$p = \sqrt{2m_e E_k} = \sqrt{2 \times 9.110 \times 10^{-31} \times 2.40 \times 10^{-17}} = 6.61 \times 10^{-24} \text{ kg m s}^{-1}.$$

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{6.61 \times 10^{-24}} = 1.00 \times 10^{-10} \text{ m} \approx 1 \times 10^{-10} \text{ m}$$

(b) Significant diffraction happens when the wavelength is **comparable to the spacing** of the diffracting structure. The electron wavelength ($1.0 \times 10^{-10} \text{ m}$) is about the same as the spacing of the atoms, so the regular rows of atoms act like a diffraction grating for the electron waves.

(c) $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{0.15 \times 30} = 1.5 \times 10^{-34} \text{ m}.$ This is about 10^{19} times smaller than a nucleus, so there is no gap or obstacle small enough to diffract the ball: its wave behaviour is undetectable.

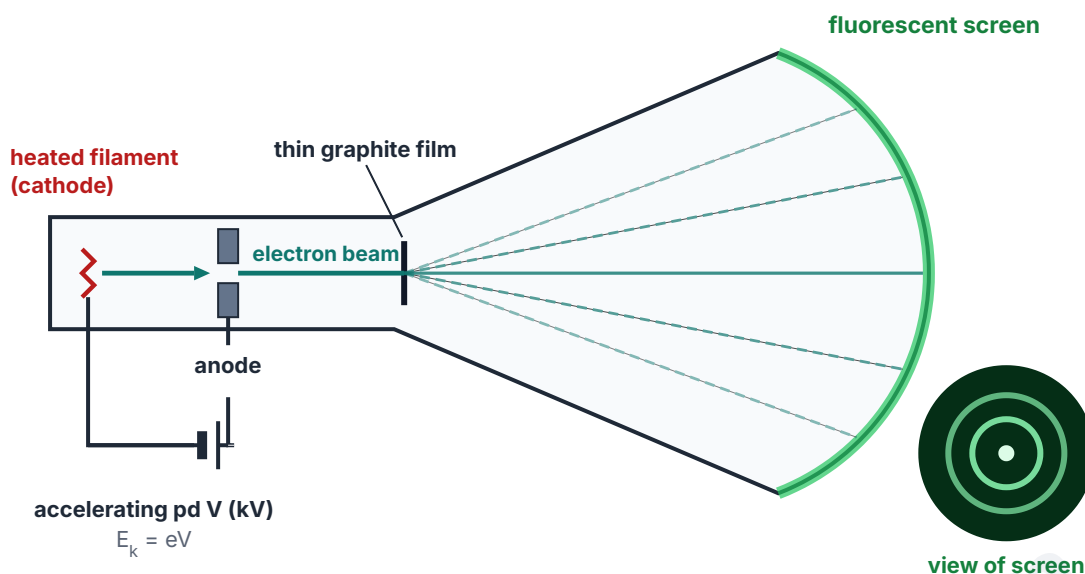
(d) $\lambda \propto \frac{1}{p} \propto \frac{1}{\sqrt{V}}.$ Multiplying V by 4 halves λ : $\lambda = \frac{1.00 \times 10^{-10}}{2} = 5.0 \times 10^{-11} \text{ m}$

Check on (a): the electron speed is $p/m_e = 7.3 \times 10^6 \text{ m s}^{-1}$, about 2 % of c , so the non-relativistic $E_k = p^2/2m$ is valid.

What is the evidence that particles behave as waves?

Diffraction is a wave property. When a beam of electrons (or other particles) is diffracted, producing a pattern of maxima and minima of intensity, the particles are behaving as waves with the de Broglie wavelength.

Electron diffraction by a crystal



An electron diffraction tube. Electrons from a heated filament are accelerated through a few kilovolts, pass through a thin film of graphite and strike a fluorescent screen, where they form concentric rings — a diffraction pattern.

- A beam of electrons passes through a thin layer of graphite (or a metal foil), which contains many tiny crystals in random orientations.
- The screen shows bright **rings** around a central spot. Rings appear only at angles where waves scattered by the regularly spaced layers of atoms interfere constructively — a pattern that particles travelling in straight lines cannot produce.
- Increasing the accelerating voltage increases the momentum, so $\lambda = h/p$ decreases and the rings become **smaller**.
- Measuring the ring angles and the accelerating voltage confirms $\lambda = h/p$ quantitatively. This, not merely "a pattern was seen", is what makes the result convincing evidence.

Diffraction of electrons by a nucleus — a scattering experiment

High-energy electrons scattered by nuclei produce a diffraction pattern just like light passing a small obstacle. The experiment works as follows:

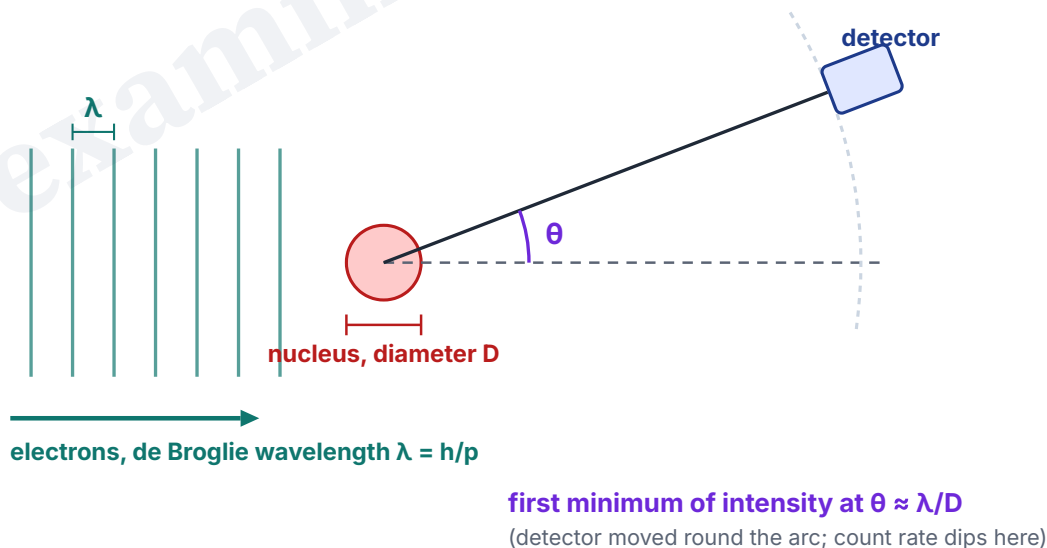
- A beam of electrons, all of the same momentum, is directed at a thin target of one kind of nucleus (for example a thin foil).
- A detector is moved round an arc centred on the target, and the count rate (number of electrons per second) is recorded at each scattering angle θ .

- The intensity falls from the forward direction and passes through a **first minimum**, then rises to a weaker maximum — a diffraction pattern.
- The first minimum is located, as for diffraction at a single slit (C.3), approximately where

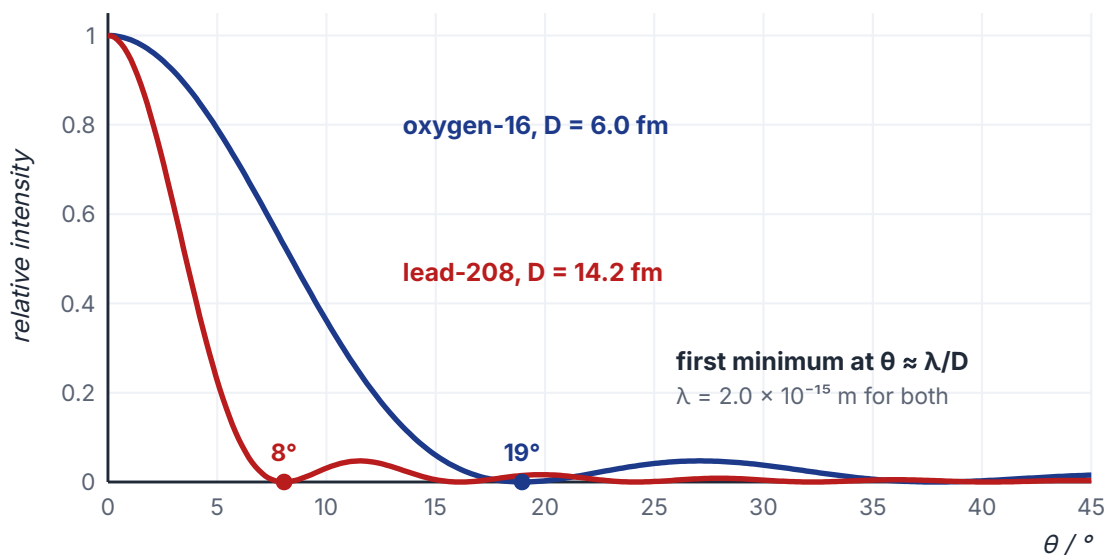
$$\theta \approx \frac{\lambda}{D}$$

- θ = angle of the first minimum from the incident direction (rad)
- λ = de Broglie wavelength of the electrons (m), $\lambda = h/p$
- D = diameter of the nucleus (m)

The data booklet gives $\theta = \frac{\lambda}{b}$ for single-slit diffraction (C.3); here the diameter of the nucleus plays the part of the slit width. To produce a measurable pattern, λ must be comparable to a nuclear diameter (about 10^{-15} – 10^{-14} m), so the electrons need very large momenta. Measuring θ then gives the size of the nucleus — this is how nuclear radii and $R = R_0 A^{1/3}$ (E.1) were established.



Electron waves of wavelength λ diffracted by a nucleus of diameter D . The detector, moved round the arc, records a dip in count rate — the first minimum — at an angle $\theta \approx \lambda/D$.



Relative intensity against scattering angle for the same electron beam ($\lambda = 2.0 \times 10^{-15} \text{ m}$) on oxygen-16 and on lead-208, using the single-slit model. The larger lead nucleus gives its first minimum at a smaller angle (8° against 19°).

WORKED EXAMPLE 5 Hard · Calculate / Show that / Estimate

A beam of electrons, each of momentum $2.2 \times 10^{-19} \text{ kg m s}^{-1}$, is directed at a thin target of calcium-40 nuclei. The Fermi radius is $R_0 = 1.20 \times 10^{-15} \text{ m}$.

(a) Calculate the de Broglie wavelength of the electrons. (b) Show that the diameter of a calcium-40 nucleus is about 8 fm. (c) Estimate the angle at which the first minimum of the scattered intensity is detected. (d) With the same beam, a different target gives a first minimum at 15.0° . Estimate the nucleon number of the nuclei in this target.

$$\text{(a)} \quad \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{2.2 \times 10^{-19}} = 3.0 \times 10^{-15} \text{ m} \quad (3.01 \times 10^{-15} \text{ m})$$

$$\text{(b)} \quad R = R_0 A^{1/3} = 1.20 \times 10^{-15} \times 40^{1/3} = 4.10 \times 10^{-15} \text{ m}.$$

$$D = 2R = 8.21 \times 10^{-15} \text{ m} \approx 8 \text{ fm}$$

$$\text{(c)} \quad \theta \approx \frac{\lambda}{D} = \frac{3.01 \times 10^{-15}}{8.21 \times 10^{-15}} = 0.367 \text{ rad} = 21^\circ$$

$$\text{(d)} \quad 15.0^\circ = 0.262 \text{ rad}. \quad D \approx \frac{\lambda}{\theta} = \frac{3.01 \times 10^{-15}}{0.262} = 1.15 \times 10^{-14} \text{ m, so}$$

$$R = 5.76 \times 10^{-15} \text{ m}.$$

$$A = \left(\frac{R}{R_0} \right)^3 = \left(\frac{5.76 \times 10^{-15}}{1.20 \times 10^{-15}} \right)^3 = \text{about } 110$$

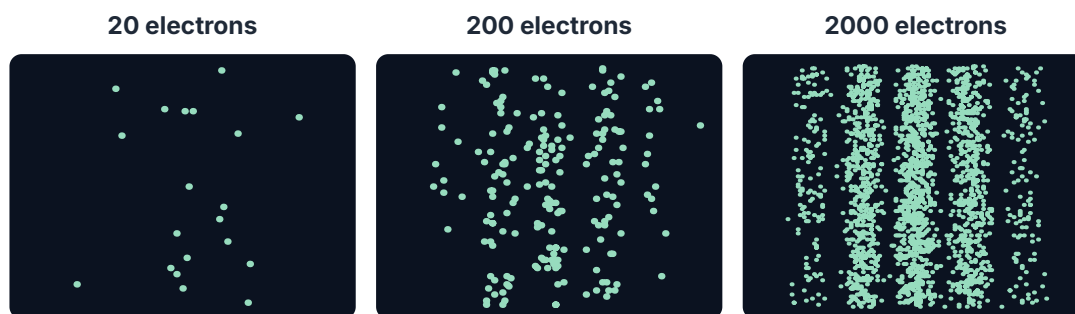
These are estimates: $\theta \approx \lambda/D$ is the small-angle single-slit result, and a nucleus is a sphere with a fuzzy edge, not a slit. Using $\sin \theta = \lambda/D$ in (c) would give 22° — the same to the precision an estimate deserves. The cube in (d) makes A very sensitive to the measured angle.

What is wave–particle duality?

Wave–particle duality is the idea that both electromagnetic radiation and matter show wave properties in some experiments and particle properties in others; neither model alone describes all of their behaviour.

	Evidence of wave behaviour	Evidence of particle behaviour
Light (photons)	diffraction, interference, refraction (C.3)	photoelectric effect; Compton scattering; line spectra from discrete photon energies (E.1)
Matter (e.g. electrons)	electron diffraction by crystals and by nuclei; interference	deflection in electric and magnetic fields as a charged particle with definite mass and charge (D.3); detection as single localised impacts

- The two descriptions are linked by $E = hf$ and $p = h/\lambda$: a particle property (energy, momentum) on one side, a wave property (frequency, wavelength) on the other.
- An entity does not switch between being a wave and a particle; the experiment decides which aspect is revealed. Propagation (travelling, diffracting) shows wave behaviour; interaction (emission, absorption, collision) shows particle behaviour.



each electron arrives at one point (particle) — the fringes only appear as the dots build up (wave)

Electrons sent towards a pair of slits one at a time. Each one arrives as a single dot (particle behaviour), but where the dots land is governed by a wave: after thousands of electrons, interference fringes appear.

What is Compton scattering?

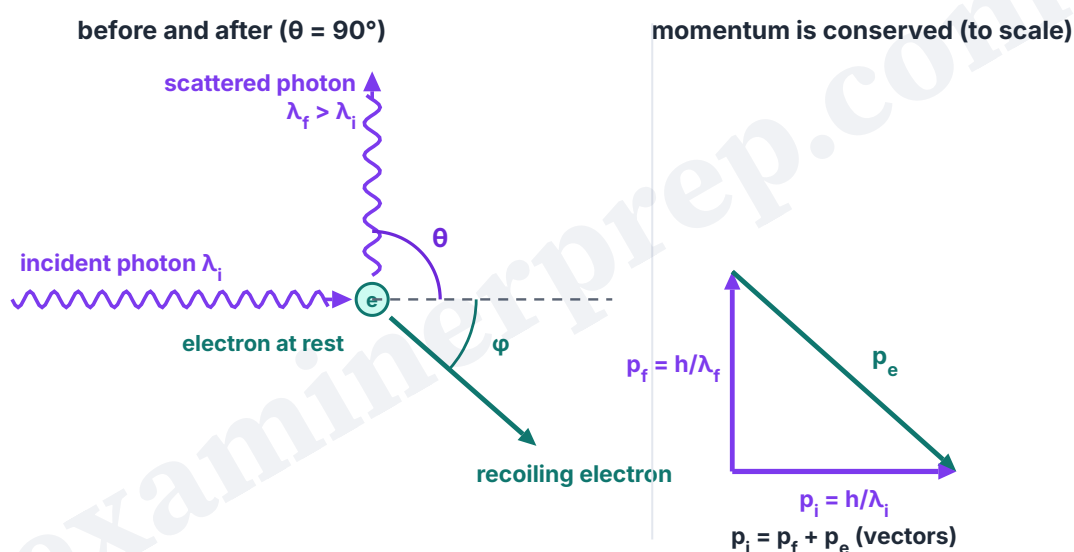
Compton scattering is the scattering of a photon (usually an X-ray or gamma-ray photon) by an electron, in which the photon transfers some of its energy and momentum to the electron and leaves with a longer wavelength.

- When X-rays of a single wavelength λ_i are directed at a target containing loosely bound electrons (such as carbon), the scattered X-rays contain a component of **longer** wavelength λ_f .
- A classical wave would make the electrons oscillate at the frequency of the wave and re-radiate at the **same** frequency, so no change of wavelength is expected. The shift cannot be explained by the wave model.
- Compton treated the event as an **elastic collision between two particles**: a photon of momentum h/λ_i and an electron at rest. Energy and momentum are both conserved. The photon gives some energy to the recoiling electron, so its energy hc/λ falls and its wavelength increases.

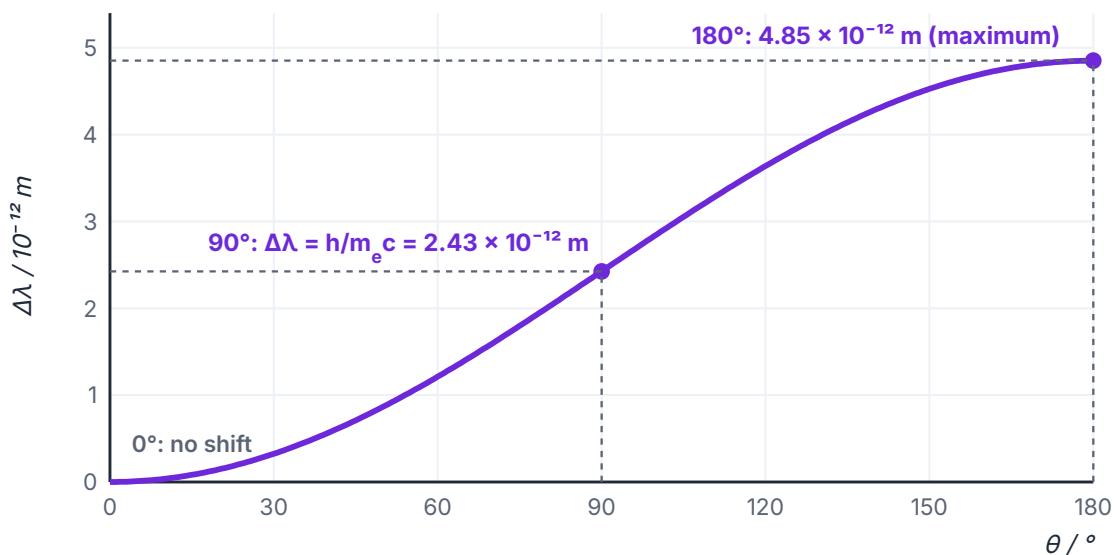
$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_e c} (1 - \cos \theta)$$

- λ_i, λ_f = wavelengths of the incident and scattered photons (m)
- θ = angle through which the photon is scattered
- m_e = rest mass of the electron, 9.110×10^{-31} kg; c = speed of light

- $\frac{h}{m_e c} = 2.43 \times 10^{-12} \text{ m}$ is a constant (the shift at 90°)
- This equation is in the data booklet. Its **derivation is not required**.
- $\Delta\lambda$ does **not** depend on λ_i or on the target material — only on the scattering angle. It is zero at $\theta = 0$ and greatest, $2h/m_e c = 4.85 \times 10^{-12} \text{ m}$, at $\theta = 180^\circ$ (photon bounced straight back).
- Because $\Delta\lambda$ is at most about $5 \times 10^{-12} \text{ m}$, the effect is noticeable only for short wavelengths — X-rays and gamma rays. For visible light ($\sim 5 \times 10^{-7} \text{ m}$), the fractional change is about 10^{-5} and is unobservable in this kind of experiment.



Compton scattering through 90° of X-rays of wavelength $2.00 \times 10^{-11} \text{ m}$. The photon leaves with a longer wavelength and the electron recoils at an angle ϕ below the original direction (42° here). The momentum vectors, drawn to scale, close: the incident momentum equals the vector sum of the final momenta.



The Compton shift against scattering angle, $\Delta\lambda = (h/m_e c)(1 - \cos \theta)$. It is the same for every incident wavelength.

WORKED EXAMPLE 6 Hard · Calculate / Determine / Explain

X-rays of wavelength $7.10 \times 10^{-11} \text{ m}$ are scattered by loosely bound electrons that are initially at rest. Scattered X-rays are detected at 90° to the incident beam.

- (a) Calculate the wavelength of the scattered X-rays. (b) Determine the kinetic energy of a recoiling electron, in eV. (c) Determine the magnitude and direction of the electron's momentum, and confirm that it is consistent with (b). (d) Explain why this experiment is not performed with visible light.

$$\text{(a) } \Delta\lambda = \frac{h}{m_e c} (1 - \cos 90^\circ) = \frac{6.63 \times 10^{-34}}{9.110 \times 10^{-31} \times 3.00 \times 10^8} \times 1 = 2.43 \times 10^{-12} \text{ m.}$$

$$\lambda_f = 7.10 \times 10^{-11} + 0.243 \times 10^{-11} = \mathbf{7.34 \times 10^{-11} \text{ m}}$$

$$\text{(b) } E_i = \frac{hc}{\lambda_i} = \frac{1.989 \times 10^{-25}}{7.10 \times 10^{-11}} = 2.801 \times 10^{-15} \text{ J; } E_f = \frac{1.989 \times 10^{-25}}{7.343 \times 10^{-11}} = 2.709 \times 10^{-15} \text{ J.}$$

$$\text{By conservation of energy, } E_k = E_i - E_f = 9.26 \times 10^{-17} \text{ J} = \mathbf{578 \text{ eV} \approx 580 \text{ eV}}$$

(c) Photon momenta: $p_i = \frac{h}{\lambda_i} = 9.34 \times 10^{-24} \text{ kg m s}^{-1}$ (along the beam);
 $p_f = \frac{h}{\lambda_f} = 9.03 \times 10^{-24} \text{ kg m s}^{-1}$ (at 90°).

Momentum is conserved, so the electron carries p_i along the beam and p_f in the direction opposite to the scattered photon:

$$p_e = \sqrt{p_i^2 + p_f^2} = 1.30 \times 10^{-23} \text{ kg m s}^{-1} \text{ at } \tan^{-1} \frac{9.03}{9.34} = 44^\circ \text{ to the}$$

incident direction, on the opposite side from the scattered photon.

Check: $\frac{p_e^2}{2m_e} = \frac{(1.30 \times 10^{-23})^2}{2 \times 9.110 \times 10^{-31}} = 9.26 \times 10^{-17} \text{ J}$ — the same as (b) ✓

(d) The shift is at most $4.85 \times 10^{-12} \text{ m}$ whatever the wavelength. For these X-rays that is about 7 % of λ_i (at 180°), which is easily measured. For visible light of wavelength 500 nm it would be about 1×10^{-5} of λ_i , too small to detect. Visible photons also have energies (a few eV) comparable to the energy binding electrons in atoms, so the electrons cannot be treated as free.

The electron speed is about $1.4 \times 10^7 \text{ m s}^{-1}$ (under 5 % of c), so $E_k = p^2/2m$ is a good approximation here.

Why is Compton scattering stronger evidence than the photoelectric effect?

The photoelectric effect shows that energy is absorbed in quanta of hf — but a photon is absorbed and disappears, and some features could be argued to depend on how electrons are bound in a metal. In Compton scattering a free photon survives the interaction and behaves exactly like a particle in a collision, carrying **momentum** h/λ as well as energy, with both conserved. The measured shift and its angle dependence match the particle calculation exactly.

Exam tips for E.2

EXAM TIP 1 — ONE PHOTON, ONE ELECTRON

When you explain any feature of the photoelectric effect using photons, say explicitly that **one photon transfers all its energy to one electron**, and that the photon energy depends on **frequency**. Then say what intensity does: it changes the **number of photons per second** and so the number of electrons per second. Answers that only say "light is particles" do not earn the marks.

EXAM TIP 2 — JOULES OR ELECTRONVOLTS, NEVER BOTH

In $E_{\max} = hf - \Phi$, hf comes out in joules while Φ is usually given in eV. Convert one of them before subtracting. Mixing units is the most common way to lose every mark in a photoelectric calculation.

EXAM TIP 3 — READ GRAPHS FOR THE RIGHT QUANTITY

For an $E_{\max}$ - f graph: gradient = h , f -intercept = f_0 , $E_{\max}$ -intercept = $-\Phi$. For a V_s - f graph the gradient is h/e . If you compare two metals, say both the similarity (**same gradient**) and the difference (**different intercepts**). Remember that the metal with the larger Φ has the line further to the right.

EXAM TIP 4 — DE BROGLIE FROM KINETIC ENERGY

Do not write $\lambda = h/mv$ and then use v from an energy you have not converted. Go via momentum: $p = \sqrt{2mE_k}$, with $E_k = qV$ for an accelerated charge. Show the momentum as an intermediate value — it earns a mark even if a later step goes wrong.

EXAM TIP 5 — DESCRIBE THE SCATTERING EXPERIMENT FULLY

When asked to describe a particle-diffraction experiment, include: a beam of particles of **one known momentum**; a **thin target**; a **detector moved through a range of angles**; a plot of intensity against angle showing a **minimum**; the first minimum at $\theta \approx \lambda/D$ with $\lambda = h/p$. For Compton

scattering, the feature the wave model cannot explain is the **change in wavelength** of the scattered radiation.

Common misconceptions

Misconception	Why it is wrong	Correct idea
Brighter light gives faster photoelectrons.	Intensity changes the number of photons per second, not the energy of each.	$E_{\max}$ depends only on f (and Φ); intensity changes the photocurrent.
If light below the threshold frequency shines for long enough, electrons will eventually escape.	An electron cannot store energy from several photons; it loses it in collisions within the metal.	Below f_0 there is no emission at any intensity or duration.
All photoelectrons leave with $hf - \Phi$.	Electrons below the surface lose energy on the way out.	$hf - \Phi$ is the maximum kinetic energy; there is a range from 0 to $E_{\max}$.
The work function is the energy of the photon needed to release an electron at any frequency.	It is a property of the metal surface, not of the light.	Φ is the minimum energy to remove an electron from the surface; $\Phi = hf_0$.
Only light has a wavelength; electrons are just particles.	Electrons diffract with a measured wavelength equal to h/p .	All matter has a de Broglie wavelength; it is detectable only when λ is comparable to the obstacle or gap.
In Compton scattering, the shift depends on the incident wavelength.	$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$ contains no λ_i .	The shift depends only on θ ; the <i>fractional</i> shift is large only for short wavelengths.

How this topic connects

Particles that diffract: de Broglie wavelength and gap size

Every particle has a de Broglie wavelength $\lambda = h/p$. Diffraction — the spreading of a wave through a gap or around an obstacle — is significant when the wavelength is comparable to the size of that gap or obstacle (C.3). Electrons accelerated through around 100 V have $\lambda \sim 10^{-10}$ m, matching the atomic spacing in a crystal, so a crystal diffracts them like a grating. Electrons of very high momentum have $\lambda \sim 10^{-15}$ m and are diffracted by nuclei. The pattern builds up particle by particle: each one arrives at a single point, and the wave determines where the arrivals collect.

Compton scattering versus a snooker-ball collision: what carries over and what does not

Similar: both are elastic collisions in which total momentum and total energy are conserved; the incident body transfers some energy and momentum to the target, which recoils, and the vector momenta must close in a triangle (A.2).

Different: the photon has no rest mass and always travels at c , so it cannot slow down — it loses energy by **increasing its wavelength** (lower frequency), not by losing speed. Its momentum is h/λ , not mv . A ball that hits a stationary ball of the same mass head-on stops; a photon never stops — even at 180° it bounces back with the longest possible wavelength.

Why Compton scattering makes a stronger case for photons than the photoelectric effect

The photoelectric effect demonstrates energy quantisation, but the photon is absorbed and the result involves the binding of electrons in a solid. Compton scattering shows a photon behaving as a particle *before and after* an interaction with a single free electron, with both its energy (hf) and its **momentum** (h/λ) conserved exactly as in a mechanical collision. Its quantitative prediction — a shift depending only on angle, with the constant $h/m_e c$ — was confirmed precisely, a result no wave model can reproduce.

Tools & inquiry — determining the Planck constant

- Use a photocell with filters (or LEDs) to select several frequencies. For each, increase the reverse pd until the photocurrent just reaches zero and record V_s . Plot V_s against f : gradient = h/e , f -intercept = f_0 (as in worked example 3).
- Finding the exact pd at which the current reaches zero is difficult, because the current approaches zero gradually. This gives a **random** uncertainty in each V_s ; repeat readings and use error bars.
- A filter that lets through a band of frequencies (not a single one) makes V_s correspond to the *highest* frequency passed; contact potentials between the electrodes shift every V_s by the same amount — a **systematic** error that changes the intercept but not the gradient, so h is still reliable.
- Real photoelectric apparatus is expensive, so simulations are widely used to collect data — useful practice in processing, but remember that a simulation only returns the model it was built on.
- **NOS — falsification and paradigm shift:** a few clear observations (threshold frequency, instantaneous emission) were enough to falsify the idea that light is purely a wave. The replacement, wave–particle duality, did not throw away the wave model; it limited where each model applies.

Key equations

Equation	Use	In data booklet?
$E = hf$	energy of one photon	yes (E.1)
$c = f\lambda$	convert between wavelength and frequency	yes (C.2, as $v = f\lambda$)
$E_{\max} = hf - \Phi$	Einstein's photoelectric equation	yes
$\Phi = hf_0$	work function from threshold frequency	no — set $E_{\max} = 0$
$eV_s = E_{\max}$	stopping voltage	no — from $W = qV$
$\lambda = \frac{h}{p}$	de Broglie wavelength; photon momentum	yes
$E_k = \frac{p^2}{2m}$	momentum from kinetic energy	yes (A.3)
$\theta \approx \frac{\lambda}{D}$	first diffraction minimum (single-slit form $\theta = \lambda/b$)	yes (C.3), as $\theta = \lambda/b$
$R = R_0 A^{1/3}$	nuclear radius, used with diffraction data	yes (E.1)
$\Delta\lambda = \frac{h}{m_e c} (1 - \cos \theta)$	Compton shift	yes

Check your understanding

1. Light of wavelength 620 nm falls on a metal of work function 2.3 eV. A student says that making the light ten times brighter will release photoelectrons. Evaluate this claim.

$$\text{Photon energy} = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{620 \times 10^{-9}} = 3.21 \times 10^{-19} \text{ J} =$$

2.0 eV, less than 2.3 eV. Each electron absorbs one photon, so no electron can escape. Brighter light only delivers more photons per second, each still

with 2.0 eV, so the claim is wrong: no photoelectrons are released at any intensity.

2. The stopping voltage for photoelectrons from a surface is 1.35 V. Calculate the maximum speed of the photoelectrons.

$$E_{\max} = eV_s = 1.60 \times 10^{-19} \times 1.35 = 2.16 \times 10^{-19} \text{ J. } v = \sqrt{\frac{2E_{\max}}{m_e}} = \sqrt{\frac{2 \times 2.16 \times 10^{-19}}{9.110 \times 10^{-31}}} = 6.9 \times 10^5 \text{ m s}^{-1}.$$

3. Calculate the de Broglie wavelength of a proton moving at $2.0 \times 10^5 \text{ m s}^{-1}$.

$$\lambda = \frac{h}{m_p v} = \frac{6.63 \times 10^{-34}}{1.673 \times 10^{-27} \times 2.0 \times 10^5} = 2.0 \times 10^{-12} \text{ m } (1.98 \times 10^{-12} \text{ m}).$$

4. X-ray photons of wavelength $2.50 \times 10^{-11} \text{ m}$ are Compton-scattered through 60° . Calculate the scattered wavelength and the energy given to the electron, in keV.

$$\Delta\lambda = 2.43 \times 10^{-12} \times (1 - \cos 60^\circ) = 1.21 \times 10^{-12} \text{ m, so } \lambda_f = 2.62 \times 10^{-11} \text{ m.}$$

$$\text{Energy to the electron} = hc \left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f} \right) = 3.68 \times 10^{-16} \text{ J} = 2.30 \text{ keV (out of an incident 49.7 keV).}$$

5. In an electron diffraction tube, the accelerating potential difference is increased from 2.5 kV to 5.0 kV. State and explain what happens to the rings on the screen.

The electrons gain more kinetic energy and momentum. Since $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e eV}}$, the wavelength falls by a factor $\sqrt{2.5/5.0} = 0.71$ (from $2.46 \times 10^{-11} \text{ m}$ to $1.74 \times 10^{-11} \text{ m}$). A shorter wavelength is diffracted through

smaller angles, so the rings become **smaller** (their radii shrink by about the same factor).

Practise E.2 with exam-style questions

Original Paper 1A, 1B and 2 questions on quantum physics, with full mark schemes. E.2 is HL only, so there is no SL bank for this sub-topic. Tick the questions you want and print them as your own test.

[HL E.2 questions](#)

[Full question bank](#)

Revise and explore

- [Flashcards](#) — choose IB Physics and filter to E.2 Quantum physics. Free, no sign-up.
- [E.2 Photoelectric Effect simulation](#)
- [Full IB Physics question bank \(all sub-topics, SL & HL\)](#)

[← Previous: E.1 Structure of the atom](#) · [Next: E.3 Radioactive decay →](#)