

THEME E · NUCLEAR AND QUANTUM PHYSICS

SL &amp; HL

≈ 35 min read

# IB Physics E.3 Radioactive Decay — Notes (SL & HL)

Written by Evan Kotronis for the IB Diploma Physics course (first assessment 2025). Everything outside the purple boxes is for SL and HL. E.3 has a lot of additional HL content — evidence for the strong force, the neutron–proton ratio, nuclear energy levels, the neutrino and the exponential decay law — and each part sits in a purple **HL** box next to the SL idea it extends.

 SL E.3 questions HL E.3 questions Flashcards Radioactive decay simulation

## What this page covers

- ✓ Isotopes and nuclide notation
- ✓ Mass defect and nuclear binding energy, using  $E = mc^2$  with masses in kg, u and MeV  $c^{-2}$
- ✓ How the binding energy per nucleon changes with nucleon number, and how to read the curve
- ✓ The strong nuclear force: short-range and attractive between nucleons
- ✓ Why radioactive decay is random and spontaneous
- ✓ What happens to the nucleus in  $\alpha$ ,  $\beta^-$ ,  $\beta^+$  and  $\gamma$  decay, and how to write the decay equations
- ✓ Neutrinos and antineutrinos
- ✓ How penetrating and how ionising  $\alpha$ ,  $\beta$  and  $\gamma$  radiation are, and how this decides real-life uses
- ✓ Activity, count rate and half-life; changes over whole numbers of half-lives; correcting for background radiation
- ✓ The evidence for the strong nuclear force (HL)

- ✓ How the neutron-to-proton ratio affects the stability of a nuclide (HL)
- ✓ Why the binding energy per nucleon is roughly constant above  $A \approx 60$  (HL)
- ✓  $\alpha$  and  $\gamma$  spectra as evidence for discrete nuclear energy levels (HL)
- ✓ The continuous  $\beta$  spectrum as evidence for the neutrino (HL)
- ✓ The decay constant,  $N = N_0 e^{-\lambda t}$ ,  $A = \lambda N$  and  $T_{1/2} = \frac{\ln 2}{\lambda}$ , for any time interval (HL)
- ✓ When  $\lambda$  can be treated as the probability that a nucleus decays per unit time (HL)

### In one minute

- A nuclide is written  ${}_Z^A\text{X}$ . Isotopes have the same proton number  $Z$  but different neutron numbers  $N$ .
- A nucleus has less mass than its separated nucleons. This **mass defect**  $\Delta m$  corresponds to the **binding energy**  $\Delta m c^2$ ;  $1 \text{ u} = 931.5 \text{ MeV } c^{-2}$ .
- Binding energy per nucleon peaks at about 8.8 MeV near  $A \approx 60$  (iron and nickel): fusion of light nuclei and fission of heavy nuclei both release energy.
- Decay is **random** (which nucleus decays, and when, cannot be predicted) and **spontaneous** (unaffected by temperature, pressure or chemistry).
- $\alpha$ :  $A - 4$ ,  $Z - 2$ .  $\beta^-$ :  $Z + 1$ , plus electron and antineutrino.  $\beta^+$ :  $Z - 1$ , plus positron and neutrino.  $\gamma$ : no change in  $A$  or  $Z$ .
- Penetration  $\alpha < \beta < \gamma$ ; ionising ability  $\alpha > \beta > \gamma$ . Half-life is the time for the activity (or number of undecayed nuclei) to halve. Subtract background *before* halving.
- HL:  $N = N_0 e^{-\lambda t}$ ,  $A = \lambda N$ ,  $T_{1/2} = \ln 2 / \lambda$ . A graph of  $\ln N$  (or  $\ln A$ ) against  $t$  is a straight line of gradient  $-\lambda$ .

### Contents

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| 6. Activity, count rate and half-life              | 13. Key equations                |
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## What is a nuclide, and what are isotopes?

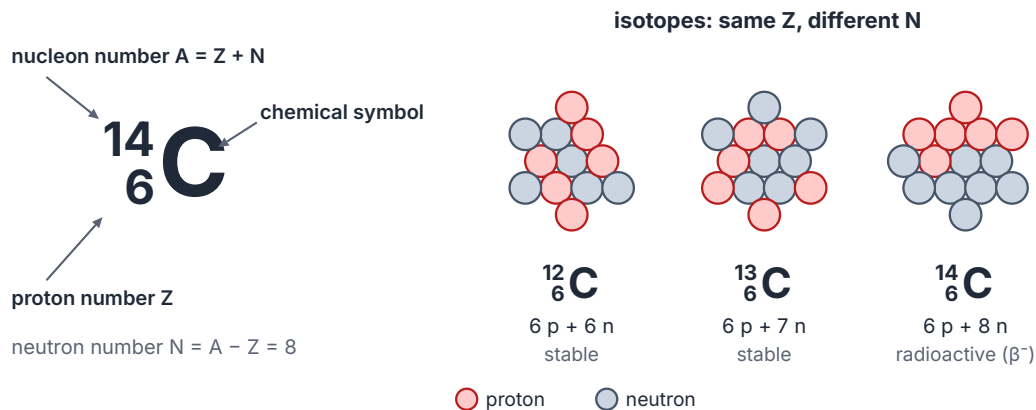
The nucleus contains protons and neutrons, together called **nucleons**. A particular nucleus — a **nuclide** — is described by two numbers:

- **proton number**  $Z$ : the number of protons (this fixes the element and its chemistry);
- **nucleon number**  $A$ : the total number of protons and neutrons, so the **neutron number** is  $N = A - Z$ .

The nuclide is written  ${}_Z^AX$ , for example  ${}_6^{14}\text{C}$  (6 protons, 8 neutrons). It is also written carbon-14 or C-14.

Isotopes are nuclides of the same element: they have the same proton number but different numbers of neutrons, and so different nucleon numbers.

- Isotopes have the same chemical properties (same number of electrons in the neutral atom) but different masses and often very different nuclear stability: C-12 and C-13 are stable, while C-14 is radioactive.
- The same notation is used for particles in nuclear equations:  ${}_1^1\text{p}$ ,  ${}_0^1\text{n}$ ,  ${}_{-1}^0\text{e}$  (electron),  ${}_{+1}^0\text{e}$  (positron),  ${}_2^4\alpha$  or  ${}_2^4\text{He}$ .



*Nuclide notation for carbon-14, and the three naturally occurring isotopes of carbon. All have  $Z = 6$ ; only the number of neutrons differs.*

## What holds the nucleus together?

Protons repel each other electrically, and the nucleus is tiny (radius of order  $10^{-15}$  m), so this repulsion is enormous. Gravity between nucleons is far too weak to oppose it. Something else must hold the nucleons together.

The strong nuclear force is an attractive force that acts between nucleons (proton–proton, neutron–neutron and proton–neutron). It is very strong but has a very short range, of the order of the size of a nucleus (a few  $\times 10^{-15}$  m).

- Within its range it is much stronger than the electric repulsion between protons, which is why stable nuclei exist.
- Beyond a few femtometres it is negligible, so a nucleon is attracted only by its **nearest neighbours**, whereas the electric repulsion acts between **every** pair of protons in the nucleus.
- It acts on neutrons as well as protons. Neutrons therefore add attraction without adding repulsion — the key to nuclear stability (see the HL box in section 3).

### HL What is the evidence for the strong nuclear force?

- **Nuclei exist at all.** Two protons  $1.0 \times 10^{-15}$  m apart repel with an electric force of about 230 N, while their gravitational attraction is only about  $1.9 \times$

$10^{-34}$  N — some  $10^{36}$  times smaller. A stable nucleus therefore needs a much stronger attractive force acting at this range.

- **It is short-range.** In high-energy  $\alpha$ -scattering experiments (E.1), the results agree with Rutherford's purely electric prediction until the  $\alpha$  particle gets within a few femtometres of the nucleus; then the scattering deviates. A second force "switches on" only at nuclear distances.
- **It saturates.** Nuclear radii follow  $R = R_0 A^{1/3}$  (E.1), so every nucleus has about the same density: adding nucleons makes the nucleus bigger, not more tightly packed. Together with the near-constant binding energy per nucleon for  $A > 60$  (section 8), this shows each nucleon is bound only to its near neighbours — the signature of a short-range force.

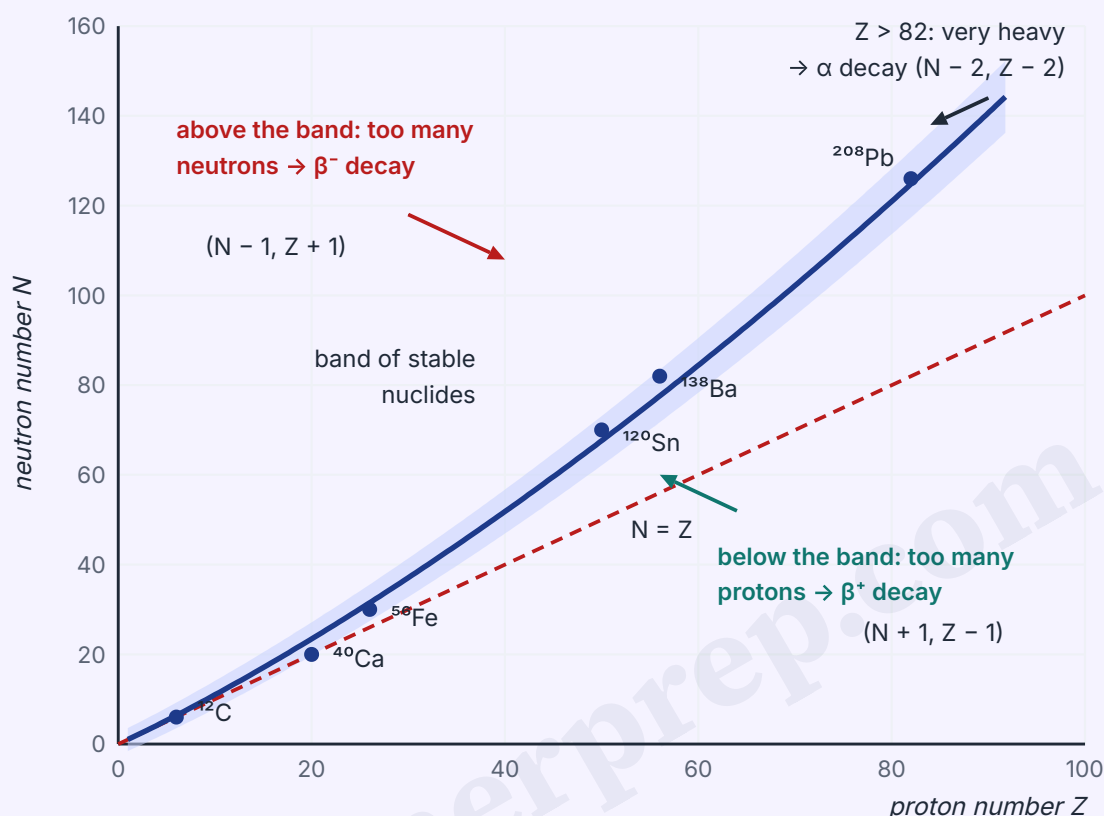
## Why do nuclei decay, and can we predict when?

An unstable nucleus becomes more stable by emitting radiation: an  $\alpha$  particle, a  $\beta$  particle, or  $\gamma$  radiation. This is **radioactive decay**.

Radioactive decay is random: it is impossible to predict which nucleus in a sample will decay next, or when a given nucleus will decay; each nucleus has a constant probability of decaying in a given time. It is spontaneous: the rate of decay cannot be changed by external conditions such as temperature, pressure or chemical combination.

- Evidence for randomness: the count from a source measured over equal short intervals **fluctuates** about a mean value.
- Because a sample contains an enormous number of nuclei (often more than  $10^{15}$ ), the **fraction** that decays in a given time is very predictable, even though individual decays are not. This is why half-life is a well-defined constant for each nuclide.
- A useful model: throw a large number of dice and remove every six each round. You cannot predict which dice go, but about one-sixth go each round, and the number left falls exponentially.

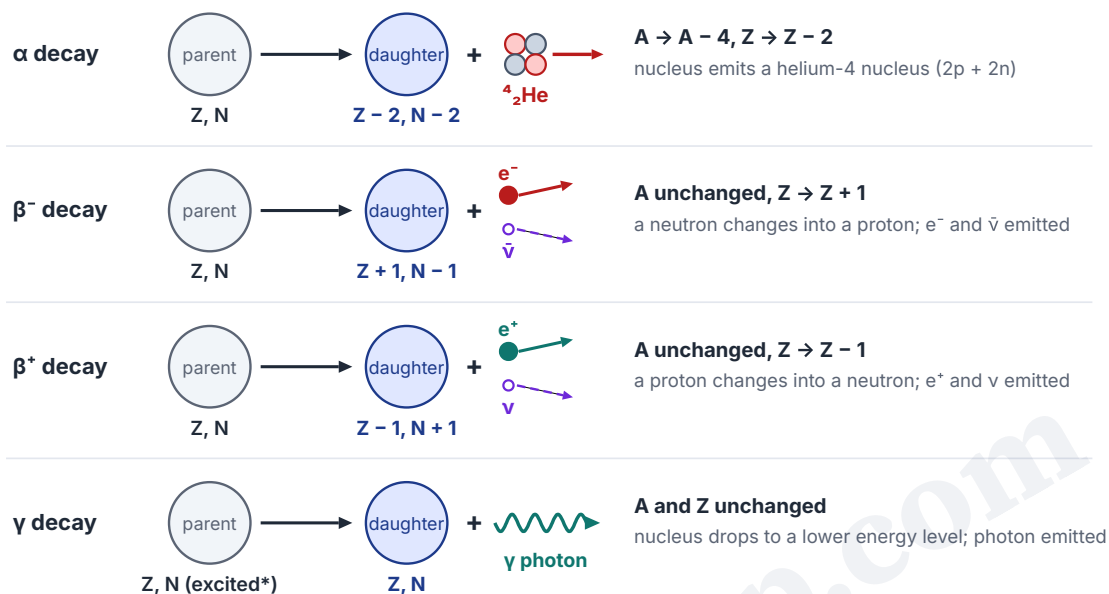
## HL How does the neutron-to-proton ratio affect stability?



Neutron number against proton number. Stable nuclides lie in a narrow band (shaded) that follows  $N = Z$  for light nuclei but curves above it for heavy nuclei: lead-208 has  $N/Z = 126/82 \approx 1.5$ . The arrows show the direction in which each decay moves an unstable nuclide.

- For light nuclides ( $Z \lesssim 20$ ) stability needs  $N \approx Z$ , i.e.  $N/Z \approx 1$ .
- As  $Z$  increases, the electric repulsion grows (every proton repels every other proton), but the strong attraction only acts between neighbours. Extra neutrons add strong attraction without adding repulsion, so heavy stable nuclei need  $N/Z$  up to about 1.5.
- **Neutron-rich** nuclides (above the band) decay by  $\beta^-$ :  $N$  falls by 1 and  $Z$  rises by 1, moving towards the band.
- **Proton-rich** nuclides (below the band) decay by  $\beta^+$ :  $Z$  falls by 1 and  $N$  rises by 1.
- **Very heavy** nuclides ( $Z > 82$ ) have so many protons that the long-range electric repulsion is too large for the short-range strong force to balance; they usually decay by  $\alpha$  emission, which reduces both  $Z$  and  $N$  by 2.

# What happens to the nucleus in $\alpha$ , $\beta$ and $\gamma$ decay?



Changes in the nucleus in the four types of decay. Nucleon number and charge are conserved in every case.

| Decay     | General equation                                                         | Change in $A$ | Change in $Z$ | Change in $N$ |
|-----------|--------------------------------------------------------------------------|---------------|---------------|---------------|
| $\alpha$  | ${}^A_Z\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\alpha$       | -4            | -2            | -2            |
| $\beta^-$ | ${}^A_Z\text{X} \rightarrow {}^A_{Z+1}\text{Y} + {}^0_{-1}e + \bar{\nu}$ | 0             | +1            | -1            |
| $\beta^+$ | ${}^A_Z\text{X} \rightarrow {}^A_{Z-1}\text{Y} + {}^0_{+1}e + \nu$       | 0             | -1            | +1            |
| $\gamma$  | ${}^A_Z\text{X}^* \rightarrow {}^A_Z\text{X} + \gamma$                   | 0             | 0             | 0             |

- **Balancing rule:** the top numbers (nucleon number) and the bottom numbers (charge) must each add up to the same total on both sides.
- **$\beta^-$  decay:** inside the nucleus a neutron changes into a proton,  ${}^1_0\text{n} \rightarrow {}^1_1\text{p} + {}^0_{-1}e + \bar{\nu}$ . The electron is created in the decay — it was not in the nucleus beforehand.
- **$\beta^+$  decay:** a proton changes into a neutron,  ${}^1_1\text{p} \rightarrow {}^1_0\text{n} + {}^0_{+1}e + \nu$ . The positron is the antiparticle of the electron: same mass, opposite charge.
- **$\gamma$  decay** usually follows  $\alpha$  or  $\beta$  decay, which often leave the daughter nucleus in an excited state (marked \*). The nucleus loses energy as a high-energy

photon; its composition does not change.

## Neutrinos and antineutrinos

- Every  $\beta^-$  decay emits an **antineutrino**  $\bar{\nu}$ , and every  $\beta^+$  decay emits a **neutrino**  $\nu$ .
- Both are uncharged, have an extremely small mass and interact so weakly with matter that almost all of them pass straight through the Earth. They carry away energy and momentum.
- They have zero charge and zero nucleon number, so they do not change the balancing — but leaving them out of an equation is an error.

### WORKED EXAMPLE 1 Easy · State / Deduce

Write down the decay equations for: (a) radium-226 ( $Z = 88$ ) emitting an  $\alpha$  particle to form radon (Rn); (b) phosphorus-32 ( $Z = 15$ ), a  $\beta^-$  emitter used in medicine, decaying to sulfur (S); (c) fluorine-18 ( $Z = 9$ ), a  $\beta^+$  emitter used in PET scans, decaying to oxygen (O). (d) State the number of neutrons in the parent and the daughter in (b).

(a)  ${}_{88}^{226}\text{Ra} \rightarrow {}_{86}^{222}\text{Rn} + {}_2^4\alpha$  check:  $226 = 222 + 4$ ;  $88 = 86 + 2$

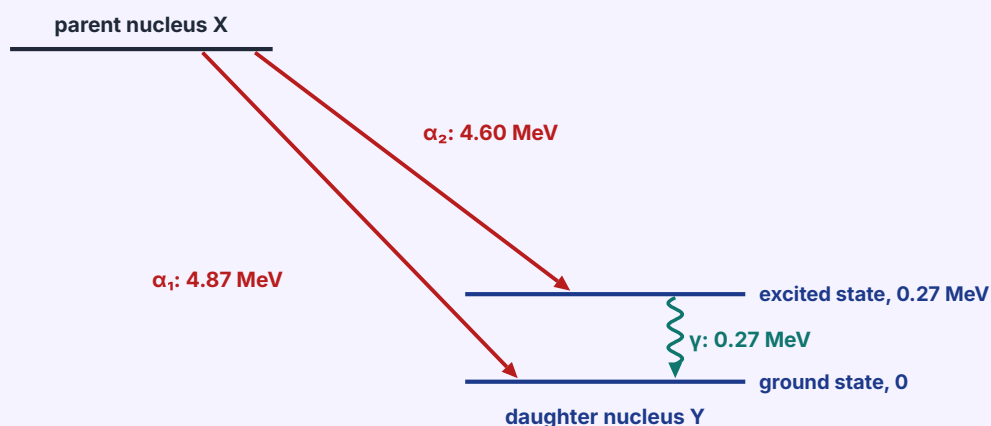
(b)  ${}_{15}^{32}\text{P} \rightarrow {}_{16}^{32}\text{S} + {}_{-1}^0\text{e} + \bar{\nu}$  check:  $15 = 16 + (-1)$

(c)  ${}_{9}^{18}\text{F} \rightarrow {}_{8}^{18}\text{O} + {}_{+1}^0\text{e} + \nu$  check:  $9 = 8 + 1$

(d) P-32:  $N = 32 - 15 = 17$ ; S-32:  $N = 32 - 16 = 16$ . One neutron has become a proton.

### HL What do $\alpha$ and $\gamma$ spectra tell us about the nucleus?

When the  $\alpha$  particles from one nuclide are measured, they do not have a spread of energies: they have one or a few **sharply defined** energies (a line spectrum). The  $\gamma$  photons that follow also have sharply defined energies. This is the nuclear equivalent of the atomic line spectra of E.1.

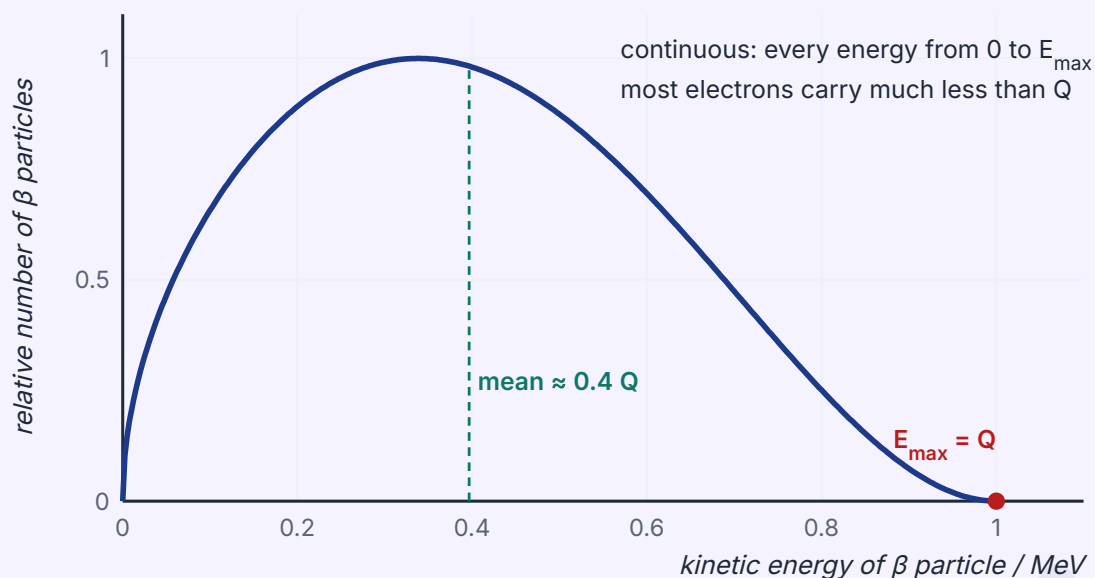


$$\alpha_1 - \alpha_2 = 4.87 - 4.60 = 0.27 \text{ MeV} = \gamma \text{ energy} \rightarrow \text{the nucleus has discrete energy levels}$$

*A nuclide X can decay by  $\alpha$  emission either directly to the ground state of Y or to an excited state of Y, which then emits a  $\gamma$  photon. The difference between the energies released in the two  $\alpha$  routes equals the  $\gamma$ -photon energy. (Illustrative values.)*

- $\alpha$  decay is a two-body decay: momentum conservation fixes how the energy released is shared between the  $\alpha$  particle and the daughter (worked example 4), so each  $\alpha$  line corresponds to one particular final state of the daughter nucleus.
- The energy differences between  $\alpha$  lines match the energies of the  $\gamma$  photons emitted afterwards (after allowing for the small recoil energy of the daughter nucleus). Both can only be explained if the nucleus has **discrete energy levels**.
- Nuclear level spacings are typically  $10^4$ – $10^6$  eV, compared with a few eV for atomic levels, so the photons are  $\gamma$  rays rather than visible light (a 0.27 MeV photon has wavelength  $4.6 \times 10^{-12} \text{ m}$ ).

### HL Why does the $\beta$ spectrum point to the neutrino?



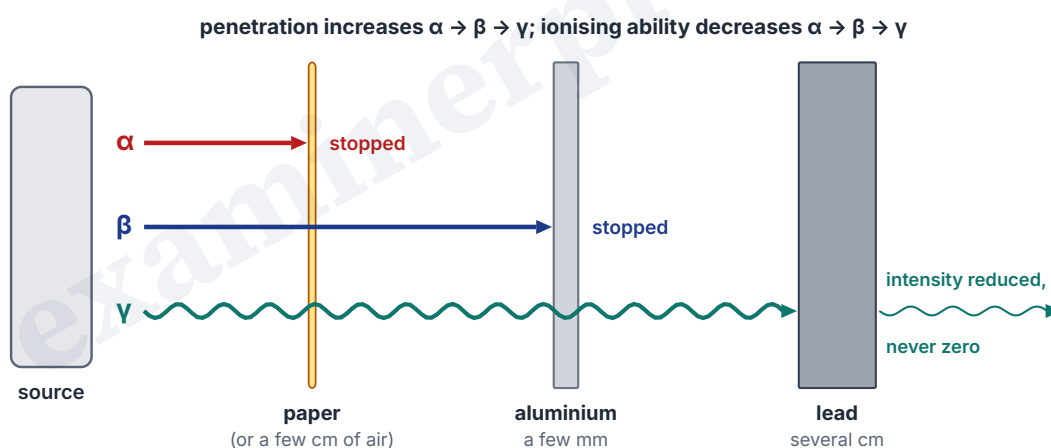
Energy spectrum of  $\beta$  particles from a nuclide whose decay releases  $Q = 1.0$  MeV (shape only). The  $\beta$  particles have a continuous range of energies from zero up to a maximum equal to  $Q$ .

- If  $\beta$  decay produced only the daughter nucleus and an electron, it would be a two-body decay like  $\alpha$  decay, and every electron would have the same energy.
- Instead the spectrum is **continuous**. Since the energy released by each decay is fixed (the nuclear levels are discrete), the "missing" energy must be carried by a third particle.
- This particle must be neutral and almost massless, because it was not detected and the maximum  $\beta$  energy equals the whole energy released. It is the (anti)neutrino. Its proposal (Pauli, 1930) saved the conservation of energy and momentum; it was detected directly only in 1956.

## How penetrating and how ionising are $\alpha$ , $\beta$ and $\gamma$ radiation?

All three radiations are **ionising**: as they pass through matter they knock electrons out of atoms. Each ionisation takes energy, so the more strongly a radiation ionises, the faster it loses energy and the shorter its range.

| Property         | $\alpha$                                  | $\beta$ ( $\beta^-$ / $\beta^+$ )                        | $\gamma$                                                                                 |
|------------------|-------------------------------------------|----------------------------------------------------------|------------------------------------------------------------------------------------------|
| Nature           | helium-4 nucleus ( $2p + 2n$ )            | electron / positron                                      | high-energy photon                                                                       |
| Charge           | $+2e$                                     | $-e$ / $+e$                                              | 0                                                                                        |
| Mass             | $\approx 4 \text{ u}$                     | $\approx 0.000549 \text{ u}$                             | 0                                                                                        |
| Ionising ability | strong                                    | moderate                                                 | weak                                                                                     |
| Penetration      | a few cm of air; stopped by paper or skin | up to about 1 m of air; stopped by a few mm of aluminium | intensity reduced by several cm of lead or metres of concrete; never completely absorbed |



*Penetrating power:  $\alpha$  is stopped by paper,  $\beta$  by a few millimetres of aluminium, while  $\gamma$  is only partly absorbed even by several centimetres of lead.*

Because  $\alpha$  and  $\beta$  particles are charged, they are deflected by electric and magnetic fields (D.3):  $\alpha$  and  $\beta^-$  in opposite directions ( $\beta^+$  the same way as  $\alpha$ ), while  $\gamma$  rays are not deflected — one way to identify the radiation.

## Choosing an isotope for a job

Real applications depend on two things: **which radiation** (penetration) and **how long the source lasts** (half-life).

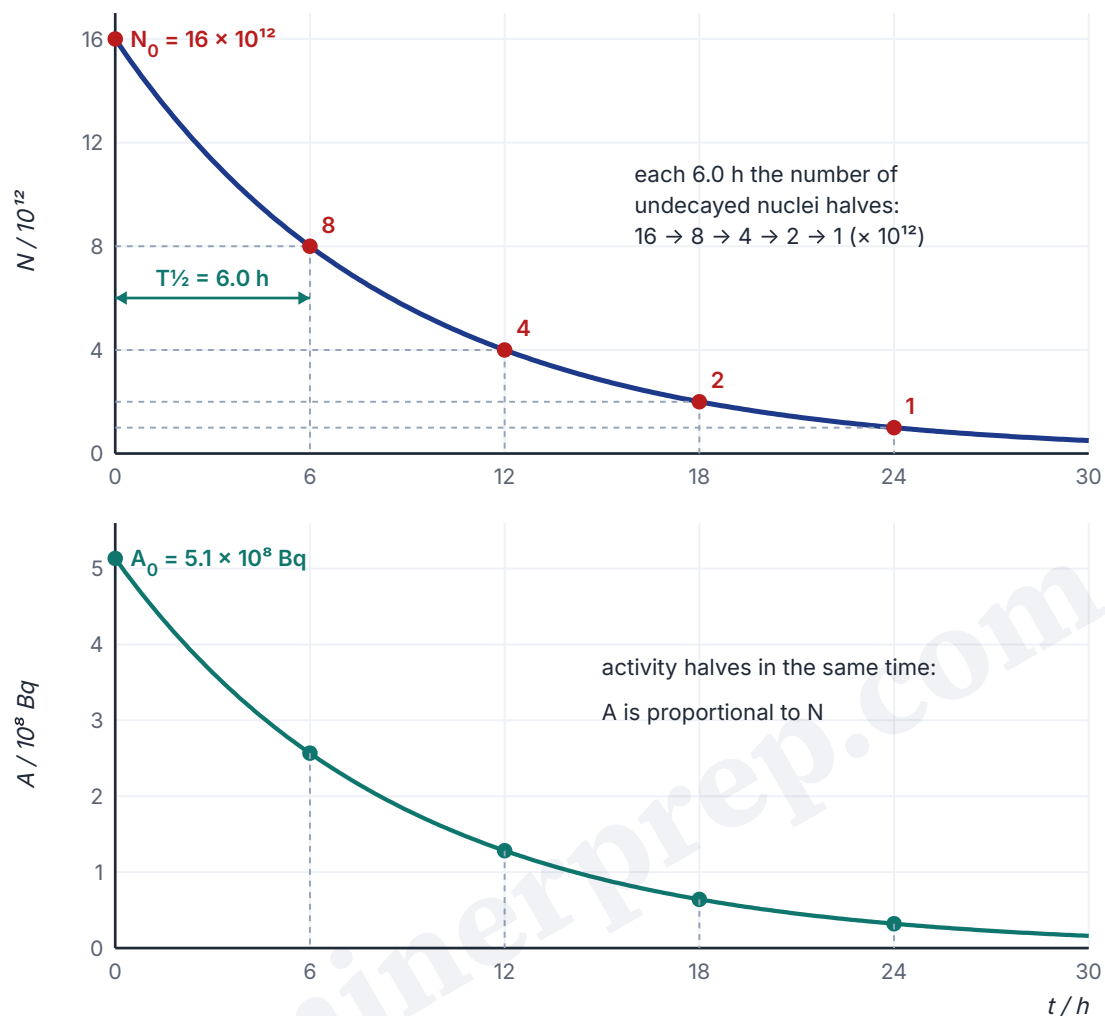
| Application                                         | Radiation needed and why                                                                                                                                      | Half-life needed and why                                                                          |
|-----------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| Medical imaging tracer (e.g. Tc-99m)                | $\gamma$ : passes out of the body to an external camera; weakly ionising, so a low dose to tissue                                                             | a few hours: long enough to take the images, short enough that activity in the patient soon falls |
| Detecting leaks in underground pipes                | $\gamma$ : must pass through soil to a detector at the surface                                                                                                | hours to days: lasts for the test, then decays so the water supply is not contaminated            |
| Monitoring thickness of paper or thin plastic sheet | $\beta$ : partly absorbed, so the count rate is sensitive to small changes in thickness ( $\alpha$ would be stopped completely; $\gamma$ would hardly change) | years: count rate should not fall noticeably during production                                    |
| Monitoring thickness of steel sheet                 | $\gamma$ : only $\gamma$ penetrates centimetres of metal                                                                                                      | years                                                                                             |
| Radiocarbon dating of once-living material          | $\beta^-$ from C-14, which all living things take in                                                                                                          | 5730 years: comparable to the ages being measured (hundreds to tens of thousands of years)        |

## What are activity, count rate and half-life?

| Quantity   | Symbol and unit                      | Meaning                                                                                                                                                          |
|------------|--------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Activity   | $A$ (Bq; 1 Bq = 1 decay per second)  | the number of nuclei in a sample that decay per unit time                                                                                                        |
| Count rate | $C$ ( $s^{-1}$ or counts per minute) | the number of decays recorded per unit time by a detector; smaller than the activity because radiation goes in all directions and not every particle is detected |
| Half-life  | $T_{1/2}$ (s, h, y...)               | the time taken for the number of undecayed nuclei (and so the activity) of a sample to fall to half its initial value                                            |

The half-life of a radioactive nuclide is the time taken for half of the undecayed nuclei in a sample to decay — equivalently, the time for the activity to halve.

- Half-life is the same whatever the starting amount: it takes 6.0 h to go from 16 to 8 units, and another 6.0 h to go from 8 to 4.
- After  $n$  half-lives the fraction of nuclei left undecayed is  $\left(\frac{1}{2}\right)^n$ :  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ ,  $\frac{1}{16}$  ... The activity and the (corrected) count rate fall by the same factor.
- The fraction that has *decayed* after  $n$  half-lives is  $1 - \left(\frac{1}{2}\right)^n$ : after 3 half-lives,  $\frac{7}{8}$  have decayed.
- The decay products (daughter nuclei) remain in the sample; the *mass* of the sample barely changes.



Number of undecayed nuclei (top) and activity (bottom) against time for a sample of  $1.6 \times 10^{13}$  nuclei of a nuclide with half-life 6.0 h. Both curves halve every half-life; neither ever reaches zero.

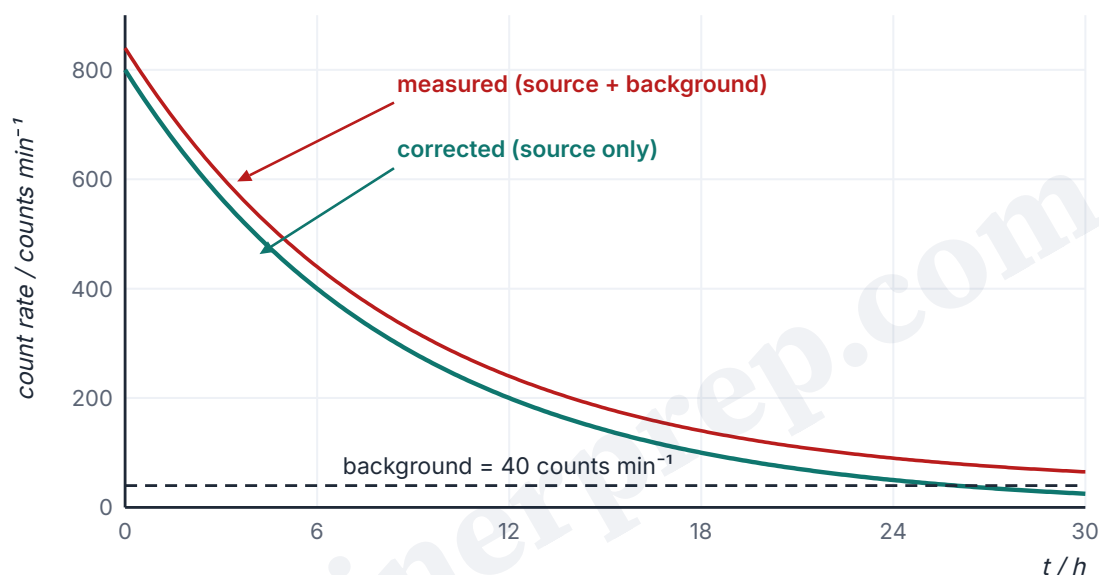
### TRY IT YOURSELF

**E.3 Radioactive decay simulation:** watch individual nuclei decay at random while the total number left follows a smooth half-life curve. Change the starting number and see how the randomness becomes less obvious for large samples.

## Background radiation

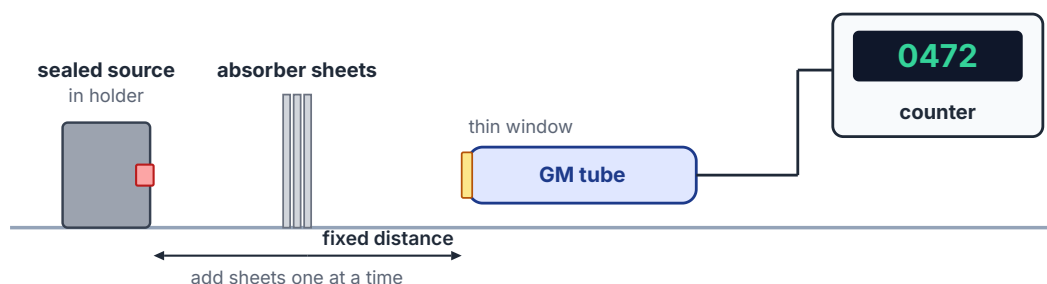
A detector always records some counts even with no source present. This **background radiation** comes from natural sources (radon gas in the air, rocks and building materials, cosmic rays, food) and artificial sources (medical uses, fallout).

- Measure the background count rate first, with the source removed, over a long time (to reduce the random uncertainty).
- **Corrected count rate = measured count rate – background count rate.** Only the corrected count rate halves every half-life.
- The measured count rate never falls below the background: as the source decays, the measured curve levels off at the background value.



Measured and corrected count rate from a technetium-99m source, with background 40 counts min<sup>-1</sup> (worked example 2). Only the corrected count rate falls by the same factor in equal times; the measured count rate levels off at the background.

1. remove source: measure background count rate
2. count for a fixed time for each absorber thickness
3. subtract background from every reading



Using a Geiger-Müller tube and counter to measure count rate. Keeping the source-tube distance fixed and changing only the absorber shows which radiation a source emits; removing the absorber and recording counts at regular intervals gives the half-life.

### WORKED EXAMPLE 2 Medium · Determine (graph) / Predict / Suggest

A technetium-99m source is placed near a detector. The measured count rate at  $t = 0$  is  $840 \text{ counts min}^{-1}$ . The background count rate is  $40 \text{ counts min}^{-1}$ . The graph above shows the count rate over the next 30 hours.

(a) Use the graph to determine the half-life of Tc-99m. (b) Predict the measured count rate after 24 h. (c) Suggest two reasons why Tc-99m is a good choice as a medical tracer.

**(a)** Corrected count rate at  $t = 0$ :  $840 - 40 = 800 \text{ counts min}^{-1}$ . It falls to  $400 \text{ counts min}^{-1}$  at  $t = 6.0 \text{ h}$ , so  **$T_{1/2} = 6.0 \text{ h}$** . (Using the measured curve,  $840 \rightarrow 420$ , gives the wrong answer.)

**(b)**  $24 \text{ h} = 4$  half-lives. Corrected:  $800 \times \left(\frac{1}{2}\right)^4 = 50 \text{ counts min}^{-1}$ .

Measured =  $50 + 40 = \mathbf{90 \text{ counts min}^{-1}}$ .

Halving the measured value four times ( $840/16 = 52.5$ ) is a common error — the background does not decay.

**(c)** It emits  $\gamma$  rays, which pass out of the body to the camera and are only weakly ionising (low dose). Its  $6.0 \text{ h}$  half-life is long enough to prepare the patient and take the images but short enough that the activity soon becomes very small.

## What are mass defect and binding energy?

Mass and energy are equivalent. Whenever the energy of a system changes, its mass changes by

$$E = mc^2$$

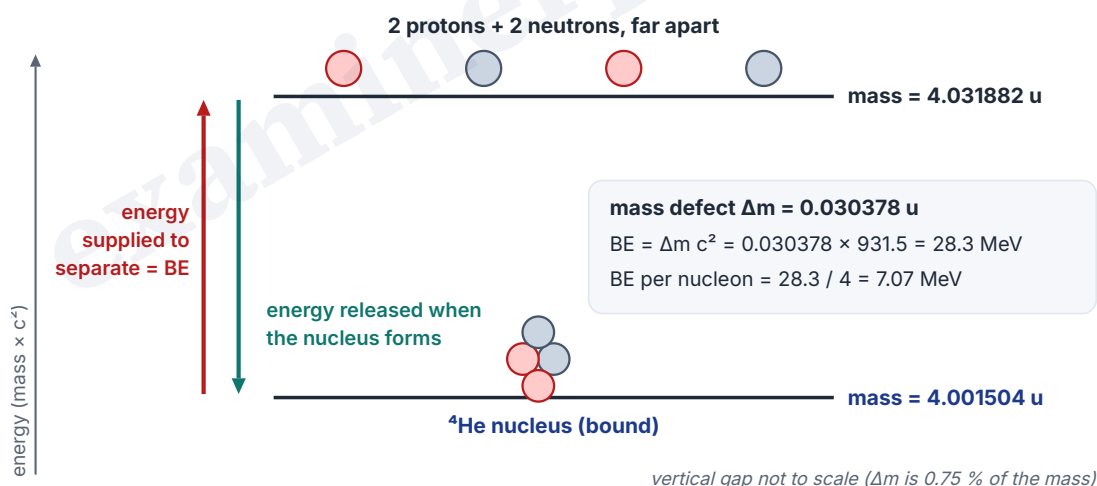
- $E$  = energy (J),  $m$  = mass (kg),  $c$  = speed of light in a vacuum,  $3.00 \times 10^8 \text{ m s}^{-1}$ . In the data booklet.
- In nuclear physics energies are in MeV ( $1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$ ) and masses in unified atomic mass units:  **$1 \text{ u} = 1.661 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV c}^{-2}$**  (data booklet). A mass in u multiplied by 931.5 gives the equivalent energy in MeV directly.

The mass defect of a nucleus is the difference between the total mass of its separated nucleons and the mass of the nucleus. The binding energy is the energy needed to separate a nucleus completely into its individual nucleons; it equals the mass defect  $\times c^2$ .

$$\Delta m = Zm_p + (A - Z)m_n - m_{\text{nucleus}}$$

$$\text{binding energy} = \Delta m c^2$$

- $m_p = 1.007276 \text{ u}$ ,  $m_n = 1.008665 \text{ u}$  (data booklet). These forms are **not** in the data booklet as written — they follow from the definitions and  $E = mc^2$ .
- The same amount of energy is **released** when the nucleus forms from separate nucleons; that is where the “missing” mass went.
- A larger binding energy *per nucleon* means the nucleons are more tightly bound — a more stable nucleus.
- Binding energy is not energy “stored in” the nucleus. A bound nucleus has *less* energy than its separate nucleons.



Mass defect of helium-4. The bound nucleus has less mass (and energy) than its four separated nucleons; the difference, 0.030378 u, corresponds to a binding energy of 28.3 MeV, or 7.07 MeV per nucleon.

### WORKED EXAMPLE 3 Medium · Show that / Calculate

The mass of a carbon-12 nucleus is 11.996706 u.

- Show that the binding energy per nucleon of carbon-12 is about 7.7 MeV.
- Calculate the mass defect in kg and the binding energy in J.

**(a)** Mass of separated nucleons =  $6 \times 1.007276 + 6 \times 1.008665 = 12.095646 \text{ u}$ .

$$\Delta m = 12.095646 - 11.996706 = 0.098940 \text{ u}$$

$$\text{Binding energy} = 0.098940 \times 931.5 = 92.16 \text{ MeV}$$

$$\text{Per nucleon: } \frac{92.16}{12} = \mathbf{7.68 \text{ MeV}} \approx 7.7 \text{ MeV}$$

$$\mathbf{(b)} \Delta m = 0.098940 \times 1.661 \times 10^{-27} = \mathbf{1.64 \times 10^{-28} \text{ kg}}$$

$$E = \Delta m c^2 = 1.643 \times 10^{-28} \times (3.00 \times 10^8)^2 = \mathbf{1.48 \times 10^{-11} \text{ J}}$$

Converting  $1.48 \times 10^{-11} \text{ J}$  back gives 92.4 MeV rather than 92.2 MeV: the data-booklet constants are rounded to 3–4 s.f. Either route is accepted; keep to one route within a calculation.

#### WORKED EXAMPLE 4 Hard · Determine / Explain

Polonium-210 decays by  $\alpha$  emission to lead-206. The nuclear masses are: Po-210, 209.936758 u; Pb-206, 205.929447 u;  $\alpha$  particle, 4.001504 u. The polonium nucleus is initially at rest.

(a) Determine the energy released in the decay, in MeV. (b) Explain why almost all this energy is carried by the  $\alpha$  particle, and estimate its kinetic energy.

$$\mathbf{(a)} \text{ Mass of products} = 205.929447 + 4.001504 = 209.930951 \text{ u.}$$

$$\Delta m = 209.936758 - 209.930951 = 0.005807 \text{ u}$$

$$E = 0.005807 \times 931.5 = \mathbf{5.41 \text{ MeV}} \text{ (5.409 MeV)}$$

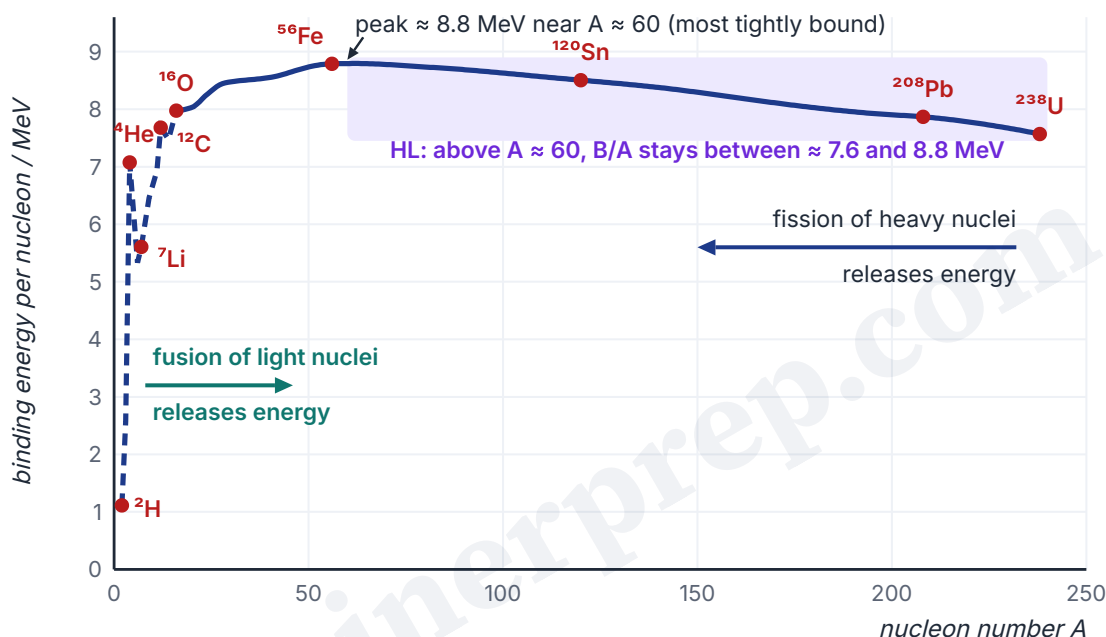
**(b)** The total momentum is zero before the decay, so the  $\alpha$  particle and the lead nucleus move off with equal and opposite momenta  $p$  (A.2). Since

$E_k = \frac{p^2}{2m}$ , the kinetic energy is inversely proportional to the mass: the light  $\alpha$  particle takes the larger share.

$$E_{k,\alpha} = 5.409 \times \frac{206}{206 + 4} = \mathbf{5.31 \text{ MeV}}; \text{ the recoiling lead nucleus has only } 0.10 \text{ MeV.}$$

Because the energy shares are fixed by momentum conservation, every  $\alpha$  particle from this decay has the same energy — which is why  $\alpha$  spectra consist of sharp lines (HL box in

## How does binding energy per nucleon vary with nucleon number?



Binding energy per nucleon against nucleon number for selected nuclides. The curve rises steeply for light nuclei, peaks at about 8.8 MeV near  $A \approx 60$ , then falls slowly. Helium-4 lies well above its neighbours.

- **Light nuclei** (small  $A$ ): few neighbours per nucleon, so the binding energy per nucleon is small and rises steeply with  $A$ . He-4 is unusually tightly bound for its size, which is why  $\alpha$  particles are emitted as a unit.
- **Peak** near  $A \approx 60$  (Fe-56, Ni-62, about 8.8 MeV): these are the most stable nuclides.
- **Heavy nuclei**: the curve falls slowly, because the long-range electric repulsion between the many protons grows faster than the short-range attraction.

### Interpreting the curve

A nuclear reaction releases energy if the products have a **higher total binding energy** than the reactants — i.e. if the products sit **higher on the curve**. The released energy equals the increase in total binding energy (binding energy per

nucleon  $\times$  number of nucleons, summed over the products, minus the same for the reactants).

- **Fusion** of light nuclei moves up the left-hand side of the curve (E.5).
- **Fission** of a heavy nucleus into two medium nuclei moves up the right-hand side (E.4).
- Neither process releases energy for nuclides at the peak, which is why iron accumulates in the cores of massive stars.

#### HL Why is the curve almost flat above $A \approx 60$ ?

From  $A \approx 60$  to  $A \approx 240$  the nucleon number quadruples, yet the binding energy per nucleon only falls from about 8.8 MeV to about 7.6 MeV. The total binding energy is therefore roughly proportional to  $A$ .

- This is what a **short-range** force predicts: each nucleon is attracted only by the few nucleons touching it, so its binding energy does not grow as more nucleons are added further away.
- If every nucleon attracted every other one (as with a long-range force), the binding energy per nucleon would keep rising roughly in proportion to  $A$  — it does not.
- The slow fall at large  $A$  is due to the electric repulsion, which *is* long-range and acts between all pairs of protons.

## HL: What is the radioactive decay law?

#### HL The decay constant and exponential decay

Because each nucleus has the same constant probability of decaying, the rate of decay is proportional to the number of undecayed nuclei present. The constant of proportionality is the **decay constant**  $\lambda$ .

The decay constant  $\lambda$  is the constant in  $A = \lambda N$ : the activity of a sample divided by the number of undecayed nuclei. Only when  $\lambda t$  is small is it

approximately equal to the probability that a given nucleus decays in unit time.

$$N = N_0 e^{-\lambda t} \qquad A = \lambda N = \lambda N_0 e^{-\lambda t} \qquad T_{1/2} = \frac{\ln 2}{\lambda}$$

- $N$  = number of undecayed nuclei at time  $t$ ,  $N_0$  = number at  $t = 0$ ,  $A$  = activity (Bq),  $\lambda$  = decay constant ( $\text{s}^{-1}$ , or  $\text{h}^{-1}$ ,  $\text{y}^{-1}$ ),  $T_{1/2}$  = half-life. All three are in the data booklet.
- $A_0 = \lambda N_0$ , so  $A = A_0 e^{-\lambda t}$ ; a corrected count rate, being proportional to the activity, also follows  $C = C_0 e^{-\lambda t}$ .
- The half-life equation comes from setting  $N = \frac{1}{2}N_0$ :  $\frac{1}{2} = e^{-\lambda T_{1/2}} \Rightarrow \lambda T_{1/2} = \ln 2$ .
- These equations work for **any** time, not just whole numbers of half-lives. Keep the units consistent: if  $\lambda$  is in  $\text{y}^{-1}$ ,  $t$  must be in years; for an activity in Bq,  $\lambda$  must be in  $\text{s}^{-1}$ .

### Is $\lambda$ the probability of decay per second?

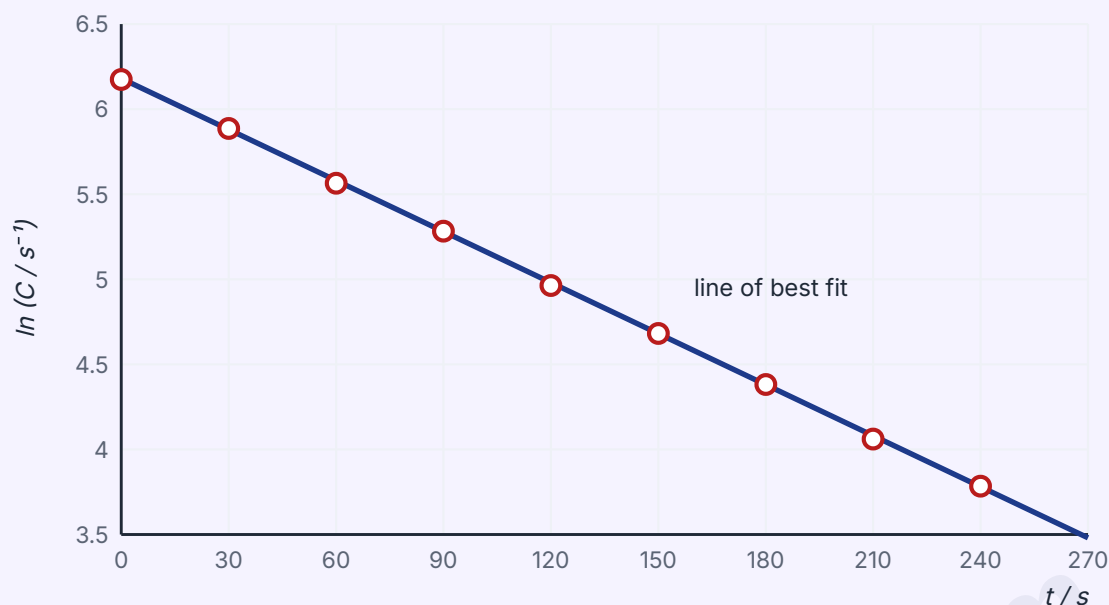
The probability that a given nucleus decays within a time  $t$  is  $1 - e^{-\lambda t}$ . For small  $\lambda t$ ,  $e^{-\lambda t} \approx 1 - \lambda t$ , so the probability is  $\approx \lambda t$ . The decay constant is therefore the probability of decay per unit time **only when  $\lambda t$  is small** (worked example 6(c)). Over one half-life,  $\lambda t = \ln 2 = 0.693$ , but the true probability of decay is 0.50.

### Finding $\lambda$ from a graph

Taking natural logarithms of  $N = N_0 e^{-\lambda t}$ :

$$\ln N = \ln N_0 - \lambda t$$

A graph of  $\ln N$  — or  $\ln A$ , or  $\ln$  (corrected count rate) — against  $t$  is a straight line with gradient  $-\lambda$  and intercept  $\ln N_0$ . This uses every data point, which is far more reliable than reading a single half-life off a curve.



*ln C against t for the data in worked example 6, with the line of best fit drawn through the points.*

**WORKED EXAMPLE 5** HL · Medium · Determine / Calculate

The C-14 activity of carbon from a living tree is 0.25 Bq per gram of carbon. A wooden tool from an archaeological site has a C-14 activity of 0.080 Bq per gram of carbon. The half-life of C-14 is 5730 years.

(a) Calculate the decay constant of C-14 in  $\text{y}^{-1}$ . (b) Determine the age of the tool. (c) Calculate the number of C-14 nuclei in each gram of carbon in the tool today. ( $1 \text{ y} = 3.16 \times 10^7 \text{ s}$ )

$$\text{(a)} \quad \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{5730} = 1.21 \times 10^{-4} \text{ y}^{-1}$$

$$\text{(b)} \quad A = A_0 e^{-\lambda t} \Rightarrow t = \frac{1}{\lambda} \ln \frac{A_0}{A} = \frac{1}{1.21 \times 10^{-4}} \ln \frac{0.25}{0.080} = 9.4 \times 10^3 \text{ years (9420 y)}$$

Sense check: 9420 y is 1.64 half-lives — between 1 (0.125 Bq  $\text{g}^{-1}$ ) and 2 (0.0625 Bq  $\text{g}^{-1}$ ), and 0.080 lies between those values.

$$\text{(c)} \quad \lambda \text{ in } \text{s}^{-1}: \frac{0.693}{5730 \times 3.16 \times 10^7} = 3.83 \times 10^{-12} \text{ s}^{-1}.$$

$$N = \frac{A}{\lambda} = \frac{0.080}{3.83 \times 10^{-12}} = 2.1 \times 10^{10} \text{ nuclei}$$

**WORKED EXAMPLE 6** HL · Hard · Determine (graph) / Calculate / Comment

A student measures the corrected count rate  $C$  from a short-lived nuclide every 30 s.

|                          |      |      |      |      |      |      |      |      |      |
|--------------------------|------|------|------|------|------|------|------|------|------|
| $t / \text{s}$           | 0    | 30   | 60   | 90   | 120  | 150  | 180  | 210  | 240  |
| $C / \text{s}^{-1}$      | 480  | 360  | 261  | 197  | 143  | 108  | 80   | 58   | 44   |
| $\ln(C / \text{s}^{-1})$ | 6.17 | 5.89 | 5.56 | 5.28 | 4.96 | 4.68 | 4.38 | 4.06 | 3.78 |

(a) Use the graph of  $\ln C$  against  $t$  above to determine the decay constant and the half-life. (b) Determine the corrected count rate at  $t = 0$  given by the best-fit line. (c) Calculate the probability that a particular nucleus decays (i) in the next 1.0 s, (ii) in the next half-life, and comment on your answers.

**(a)** Read two widely separated points on the line (not data points):

(0, 6.18) and (240 s, 3.78). Gradient of the best-fit line =

$$\frac{3.78 - 6.18}{240 - 0} = -0.0100 \text{ s}^{-1}, \text{ so } \lambda = \mathbf{0.0100 \text{ s}^{-1}}.$$

$$T_{1/2} = \frac{\ln 2}{0.0100} = \mathbf{69 \text{ s}} \text{ (69.3 s)}$$

**(b)** Intercept =  $\ln C_0 = 6.18 \Rightarrow C_0 = e^{6.18} = \mathbf{483 \text{ s}^{-1}}$  ( $\approx 4.8 \times 10^2 \text{ s}^{-1}$ )

**(c)(i)**  $P = 1 - e^{-0.0100 \times 1.0} = \mathbf{0.00995}$  ( $\approx 0.010$ ) — almost exactly  $\lambda \times 1.0 \text{ s} = 0.0100$ .

**(ii)**  $P = 1 - e^{-\ln 2} = \mathbf{0.50}$ , but  $\lambda T_{1/2} = 0.693$ .

Comment:  $\lambda$  gives the probability of decay per unit time only when  $\lambda t \ll 1$  (here  $\lambda t = 0.01$ ). For longer times the simple product  $\lambda t$  overestimates the probability, because nuclei that have already decayed cannot decay again.

## Exam tips for E.3

### EXAM TIP 1 — BACKGROUND FIRST, THEN HALVE

In any count-rate question, subtract the background **before** applying half-lives, then add it back if the question asks for the *measured* count rate.

Questions with background are often set so that the "halve the measured value" answer is one of the multiple-choice distractors.

### EXAM TIP 2 — COMPLETE DECAY EQUATIONS

A  $\beta$  decay equation needs the correct symbol with both numbers ( ${}^0_{-1}\text{e}$  or  ${}^0_{+1}\text{e}$ ) **and** the correct neutrino: antineutrino with  $\beta^-$ , neutrino with  $\beta^+$ . Check both totals — nucleon number and charge — before moving on. A missing or wrong (anti)neutrino is the most frequently lost mark.

### EXAM TIP 3 — DEFINE PRECISELY

Half-life is the time for the **number of undecayed nuclei** (or the activity) to halve — not "the time for half the mass to go" or "half the time for the sample to decay". Binding energy is the energy needed to **separate** a nucleus into its nucleons, not the energy stored in it. Randomness and spontaneity are different ideas; "random" on its own does not explain why temperature has no effect.

### EXAM TIP 4 — MASS-ENERGY ARITHMETIC

Keep masses in u to six decimal places until the end — the mass defect is a small difference between large numbers, and early rounding destroys it. Then multiply by 931.5 for MeV. Energy released is always quoted as a positive value; check that the products are the lighter side.

### EXAM TIP 5 — HL: UNITS OF $\lambda$ AND LOG GRAPHS

Use  $\lambda$  in  $\text{s}^{-1}$  whenever an activity in Bq appears ( $A = \lambda N$ ); a value in  $\text{y}^{-1}$  must be converted first. On a  $\ln A$ - $t$  graph, quote the gradient with its unit

and a negative sign, then state  $\lambda = -\text{gradient}$ . Remember that  $\ln$  of a quantity has no unit.

## Common misconceptions

| Misconception                                                                | Why it is wrong                                                         | Correct idea                                                                                                                                          |
|------------------------------------------------------------------------------|-------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| After two half-lives the sample has all decayed.                             | Each half-life halves what is <i>left</i> , not the original amount.    | After two half-lives $\frac{1}{4}$ remains; the number never reaches exactly zero.                                                                    |
| A nucleus with a large binding energy has a lot of energy stored in it.      | Binding energy is energy that must be <i>supplied</i> to pull it apart. | Larger binding energy per nucleon means a lower-energy, more stable nucleus.                                                                          |
| $\beta^-$ particles are electrons that were orbiting the atom.               | The electron comes from the nucleus.                                    | It is created when a neutron in the nucleus changes into a proton.                                                                                    |
| Heating a radioactive sample makes it decay faster.                          | Nuclear decay depends only on the nucleus, not on its surroundings.     | Decay is spontaneous; its rate is unaffected by temperature, pressure or chemical state.                                                              |
| $\gamma$ radiation is the most dangerous because it is the most penetrating. | Damage depends on ionisation where the radiation is absorbed.           | Outside the body $\gamma$ and $\beta$ are the bigger hazards; inside the body $\alpha$ is the most damaging because it is the most strongly ionising. |
| A sample's mass halves every half-life.                                      | Decayed nuclei become daughter nuclei, which stay in the sample.        | It is the number of <i>undecayed parent</i> nuclei that halves.                                                                                       |

## How this topic connects

### Gravity and electric forces alone cannot hold a nucleus together

No. For two protons  $1.0 \times 10^{-15}$  m apart, Coulomb's law (D.2) gives a repulsion of about 230 N, while Newton's law of gravitation (D.1) gives an attraction of only about  $1.9 \times 10^{-34}$  N — about  $10^{36}$  times smaller. Neutrons are uncharged, so they feel no electric force, but also add almost no gravitational attraction. With only these two forces every nucleus with more than one proton would fly apart. The existence of stable nuclei (and of a binding energy of several MeV per nucleon) requires the strong nuclear force.

### **Missing energy in $\beta$ decay: how conservation laws predicted the neutrino**

Physicists measuring  $\beta$  decay in the 1920s found that the electrons had a continuous range of energies, even though each decay released the same total energy. Rather than give up conservation of energy and momentum, Pauli proposed that an unseen, neutral, very light particle carried away the missing energy. This prediction told experimenters what to look for: a particle that interacts very rarely, so huge numbers would be needed. In 1956 Cowan and Reines detected antineutrinos from a nuclear reactor, confirming the prediction — a classic example of a conservation law guiding discovery.

### **Atomic and nuclear photons: same physics, very different energies**

Both are photons emitted when a system drops between discrete energy levels, with  $E = hf$  (E.1). The difference is the energy scale: atomic electron transitions involve a few eV (visible and ultraviolet light), while nuclear transitions involve keV to MeV ( $\gamma$  rays), because the strong force binds nucleons far more tightly than the electric force binds electrons. A 0.27 MeV  $\gamma$  photon has a wavelength of  $4.6 \times 10^{-12}$  m, about  $10^5$  times shorter than visible light. Each nuclide has its own characteristic  $\gamma$  energies, just as each element has its own line spectrum.

#### **Tools & inquiry — determining a half-life (NOS: randomness and prediction)**

- **Background first:** record the background count with the source well away for several minutes and calculate the background count rate. Subtract it from every reading.
- **Short half-life** (seconds to hours, e.g. a protactinium generator or a short-lived source): record counts in equal time intervals, find the

corrected count rate, and either read successive halving times from a curve (SL) or plot  $\ln C$  against  $t$  and use  $\lambda = -\text{gradient}$  (HL).

- **Long half-life** (years or more): the count rate does not visibly change. Instead measure the activity  $A$  and find the number of nuclei  $N$  from the mass of the sample, then  $\lambda = A/N$  and  $T_{1/2} = \ln 2/\lambda$  (HL).
- **Random uncertainty:** because decay is random, a count of  $n$  has a statistical spread of about  $\sqrt{n}$ . Counting for longer (larger  $n$ ) reduces the percentage uncertainty.
- **Systematic errors:** keep the source–detector distance fixed; a GM tube's dead time causes very high count rates to be underestimated.
- **Safety:** handle sources with tongs, keep them at a distance, minimise exposure time and return them to their lead-lined box.
- **NOS — models with dice:** rolling many dice and removing the sixes each round reproduces exponential decay with a predictable “half-life” even though each die is unpredictable. It shows how statistical laws arise from random individual events.

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## Key equations

| Equation                                                                         | Use                                                            | Level      | In data booklet?                 |
|----------------------------------------------------------------------------------|----------------------------------------------------------------|------------|----------------------------------|
| $E = mc^2$                                                                       | mass–energy equivalence;<br>binding energy and energy released | SL &<br>HL | yes                              |
| $1 \text{ u} = 1.661 \times 10^{-27} \text{ kg}$<br>$= 931.5 \text{ MeV c}^{-2}$ | converting nuclear masses                                      | SL &<br>HL | yes (constants)                  |
| $\Delta m = Zm_p + (A - Z)m_n - m_{\text{nucleus}}$                              | mass defect                                                    | SL &<br>HL | no — from the definition         |
| $N = A - Z$                                                                      | neutron number                                                 | SL &<br>HL | no                               |
| fraction left $= \left(\frac{1}{2}\right)^n$                                     | after $n$ whole half-lives                                     | SL &<br>HL | no                               |
| $N = N_0 e^{-\lambda t}$                                                         | undecayed nuclei at any time                                   | HL         | yes                              |
| $A = \lambda N = \lambda N_0 e^{-\lambda t}$                                     | activity                                                       | HL         | yes                              |
| $T_{1/2} = \frac{\ln 2}{\lambda}$                                                | half-life and decay constant                                   | HL         | yes                              |
| $\ln N = \ln N_0 - \lambda t$                                                    | straight-line graph, gradient $-\lambda$                       | HL         | no — take $\ln$ of the decay law |

## Check your understanding

**1. Iodine-131 has a half-life of 8.0 days. What fraction of a sample remains undecayed after 24 days, and what fraction has decayed?**

24 days = 3 half-lives, so  $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$  remains undecayed and  $\frac{7}{8}$  has decayed.

**2. Sodium-22 ( $Z = 11$ ) decays by  $\beta^+$  emission to neon (Ne). Write the decay equation.**

${}_{11}^{22}\text{Na} \rightarrow {}_{10}^{22}\text{Ne} + {}_{+1}^0\text{e} + \nu$ . Nucleon number:  $22 = 22 + 0$ ; charge:  $11 = 10 + 1$ .

**3. A source gives a measured count rate of  $520 \text{ counts min}^{-1}$ ; the background is  $20 \text{ counts min}^{-1}$ . Predict the measured count rate after two half-lives.**

Corrected:  $520 - 20 = 500$ ; after two half-lives  $500/4 = 125$ . Measured =  $125 + 20 = 145 \text{ counts min}^{-1}$ .

**4. A factory monitors the thickness of paper with a radioactive source on one side and a detector on the other. State and explain which type of radiation should be used.**

$\beta$ .  $\alpha$  would be completely stopped by the paper, so the count would not change with thickness;  $\gamma$  would pass through almost unaffected.  $\beta$  is partly absorbed, so small changes in thickness give measurable changes in count rate. A long half-life keeps the count rate steady over the production run.

**5. (HL) A sample contains  $1.0 \mu\text{g}$  of cobalt-60 (half-life 5.27 years).**

**Determine its activity. ( $1 \text{ y} = 3.16 \times 10^7 \text{ s}$ )**

$$N = \frac{1.0 \times 10^{-6} \text{ g}}{\frac{60 \text{ g mol}^{-1}}{0.693}} \times 6.02 \times 10^{23} = 1.0 \times 10^{16}. \lambda = \frac{0.693}{5.27 \times 3.16 \times 10^7} = 4.2 \times 10^{-9} \text{ s}^{-1}. A = \lambda N = 4.2 \times 10^7 \text{ Bq}.$$

### Practise E.3 with exam-style questions

Original Paper 1A, 1B and 2 questions on radioactive decay, with full mark schemes. Tick the questions you want and print them as your own test.

**SL E.3 questions**

**HL E.3 questions**

**Revise and explore**

- [Flashcards](#) — choose IB Physics and filter to E.3 Radioactive decay. Free, no sign-up.
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SL students: E.2 is HL only — the previous SL page is [E.1 Structure of the atom](#).

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