

IB PHYSICS SL & HL · IGCSE · PHYSICS TOOLKIT · REVISION NOTES

Scalars and Vectors

Which quantities need a direction, which do not, and how to add, subtract and resolve the ones that do.

Evan Kotronis

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Underpins every topic in Theme A (Space, time and motion) and the field topics in Theme D · SL and HL · also covers IGCSE Physics

What this page gives you: a complete list of the scalar and vector quantities you will meet, a reliable test for deciding which is which, and the three vector skills that turn up in almost every mechanics question: adding, subtracting and resolving.

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1. Scalars and vectors: the definitions

SCALAR

Magnitude only

A size and a unit are the whole answer.

"The mass of the trolley is 2.4 kg."

Nothing is missing.

VECTOR

Magnitude and direction

A size, a unit **and** a direction are needed.

"A force of 12 N." Incomplete. *"12 N vertically upwards"* is complete.

A scalar quantity is defined as:

a quantity that has magnitude only

A vector quantity is defined as:

a quantity that has both magnitude and direction

How vectors are written and drawn

- **On a diagram:** an arrow. Its **length** is proportional to the magnitude and it **points** in the direction of the vector
- **In print:** a bold symbol, $\mathbf{F}$; **by hand:** an arrow or a line above or below the symbol, $\vec{F}$
- The plain italic symbol F means the **magnitude** only, which is itself a scalar
- **In one dimension** the direction is carried by the **sign**: choose a positive direction, and a negative value then means "the other way"
- **In two dimensions** give an angle from a named reference: "30° above the horizontal", "40° east of north", or a bearing

EXAM TIP

The quickest test for any quantity: **does turning it round change the physics?** Push a box with 20 N to the left instead of to the right and it moves the other way, so force is a vector. A 2 kg mass is the same 2 kg whichever way you turn it, so mass is a scalar.

2. Scalar quantities: the full list

Every quantity below is completely described by a number and a unit. The examples are typical values you could meet in a question.

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SCALAR	SYMBOL	SI UNIT	EXAMPLE
Distance	d	m	One lap of a running track: 400 m
Speed	v	m s^{-1}	A cyclist's speedometer reads 8.5 m s^{-1}
Time	t	s	A 100 m sprint lasts 10.8 s
Mass	m	kg	A bag of flour: 1.0 kg, on the Earth or on the Moon
Volume	V	m^3	A 500 ml bottle holds $5.0 \times 10^{-4} \text{ m}^3$
Density	ρ	kg m^{-3}	Water: 1000 kg m^{-3}
Temperature	T	K	A cold morning: 271 K (-2°C)
Energy (all forms)	E	J	Kinetic energy of a thrown ball: 45 J
Work done	W	J	Lifting a suitcase onto a shelf: 180 J
Power	P	W	A kettle element: 2.2 kW
Pressure	p	Pa	Atmospheric pressure: $1.0 \times 10^5 \text{ Pa}$
Electric charge	q	C	Charge on an electron: $-1.60 \times 10^{-19} \text{ C}$
Electric current	I	A	A phone charger delivers 2.0 A (see section 5)
Potential difference	V	V	Across a torch bulb: 3.0 V
Resistance	R	Ω	A fixed resistor: 470Ω
Frequency	f	Hz	Mains supply in Europe: 50 Hz
Period	T	s	A pendulum swings with period 1.6 s
Wavelength	λ	m	Green light: $5.3 \times 10^{-7} \text{ m}$

Gravitational / electric potential	V_g, V_e	$\text{J kg}^{-1}, \text{V}$	At the Earth's surface $V_g \approx -6.3 \times 10^7 \text{ J kg}^{-1}$
Activity · half-life	$A, T_{1/2}$	Bq, s	A school source: 37 kBq; half-life of a sample: 8.0 days

3. Vector quantities: the full list

Every quantity below needs a direction. Notice how many of them are **forces** or are built from forces.

VECTOR	SYMBOL	SI UNIT	EXAMPLE (WITH ITS DIRECTION)
Displacement	s	m	A hiker ends 3.2 km north-east of the car park
Velocity	v	m s^{-1}	A train moving at 25 m s^{-1} due west
Acceleration	a	m s^{-2}	Free fall: 9.8 m s^{-2} vertically downwards
Force	F	N	A 15 N push along a table, to the right
Weight	W	N	A 60 kg student: 590 N towards the centre of the Earth
Normal force · tension · friction · drag	N, T, F_f, F_d	N	Friction of 4.0 N opposing a sliding book
Momentum	p	kg m s^{-1}	A 0.16 kg ball at 30 m s^{-1} towards the batter: 4.8 kg m s^{-1}
Impulse	J	N s	A kick gives 6.0 N s along the direction of the kick
Gravitational field strength	g	N kg^{-1}	9.8 N kg^{-1} downwards near the Earth's surface
Electric field strength	E	N C^{-1}	Between charged plates: $2.0 \times 10^4 \text{ N C}^{-1}$ from + plate to – plate
Magnetic flux density	B	T	Near a bar magnet: 0.05 T from N pole towards S pole
Centripetal acceleration	a	m s^{-2}	A car on a bend: 3.0 m s^{-2} towards the centre of the curve
Torque HL	τ	N m	12 N m turning a wheel clockwise
Angular momentum HL	L	$\text{kg m}^2 \text{ s}^{-1}$	A spinning skater; direction along the axis of rotation

COMMON ERROR

Writing $g = -9.8 \text{ m s}^{-2}$ and calling the minus sign "because gravity is negative". The sign only appears because you chose **upwards as positive**. With downwards positive, $g = +9.8 \text{ m s}^{-2}$. State your positive direction first, every time.

4. Scalar–vector pairs

Several quantities come in pairs that sound alike. In each pair the vector carries a direction and the scalar does not, and the two can have very different values.

SCALAR	VECTOR PARTNER	HOW THEY DIFFER	EXAMPLE
Distance	Displacement	Distance is the whole path; displacement is the straight line from start to finish	Out and back on a 2.5 km jog: distance 5.0 km, displacement 0
Speed	Velocity	Speed is the rate of change of distance; velocity is the rate of change of displacement	A satellite in a circular orbit: speed constant, velocity always changing
Mass	Weight	Mass is the amount of matter; weight is the gravitational force on it, $W = mg$	A 5.0 kg rock: weight 49 N on the Earth, 8.1 N on the Moon, mass 5.0 kg on both
Gravitational potential	Gravitational field strength	Potential is energy per unit mass; field strength is force per unit mass	Both describe a field: one as an energy map, one as a force map

WORKED EXAMPLE 1 — CHANGE IN SPEED VS CHANGE IN VELOCITY

[3]

A tennis ball hits a wall at 8.0 m s^{-1} and rebounds along the same line at 6.0 m s^{-1} .

Determine (a) the change in speed and (b) the change in velocity of the ball.

Step 1: Speed is a scalar, so simply subtract the magnitudes

$$\text{change in speed} = 6.0 - 8.0 = -2.0 \text{ m s}^{-1} \text{ (the ball is } 2.0 \text{ m s}^{-1} \text{ slower)}$$

Step 2: Velocity is a vector, so choose a positive direction

- Take **towards the wall as positive**: $u = +8.0 \text{ m s}^{-1}$, $v = -6.0 \text{ m s}^{-1}$

Step 3: Change in velocity = final – initial

$$\Delta v = (-6.0) - (+8.0) = -14.0 \text{ m s}^{-1}, \text{ i.e. } \mathbf{14.0 \text{ m s}^{-1} \text{ away from the wall}}$$

EXAM TIP

Rebound questions reward the vector answer. The change in velocity (14.0 m s^{-1}) is much larger than the change in speed (2.0 m s^{-1}), and it is the change in **velocity** that you need for momentum, impulse and force.

5. Tricky cases: a sign is not a direction

- Many scalars can be **negative**. The minus sign tells you something about the value, not a direction in space:
 - **Temperature**: $-15 \text{ }^{\circ}\text{C}$ is below the zero of the Celsius scale
 - **Charge**: $-1.6 \times 10^{-19} \text{ C}$ is a negative charge, not a charge “pointing backwards”
 - **Work done**: negative work means energy is transferred *out of* the body, e.g. by friction
 - **Potential**: gravitational potential is negative everywhere because its zero is defined at infinity
- **Electric current is a scalar**, even though we draw it with arrows and talk about its “direction” round a circuit

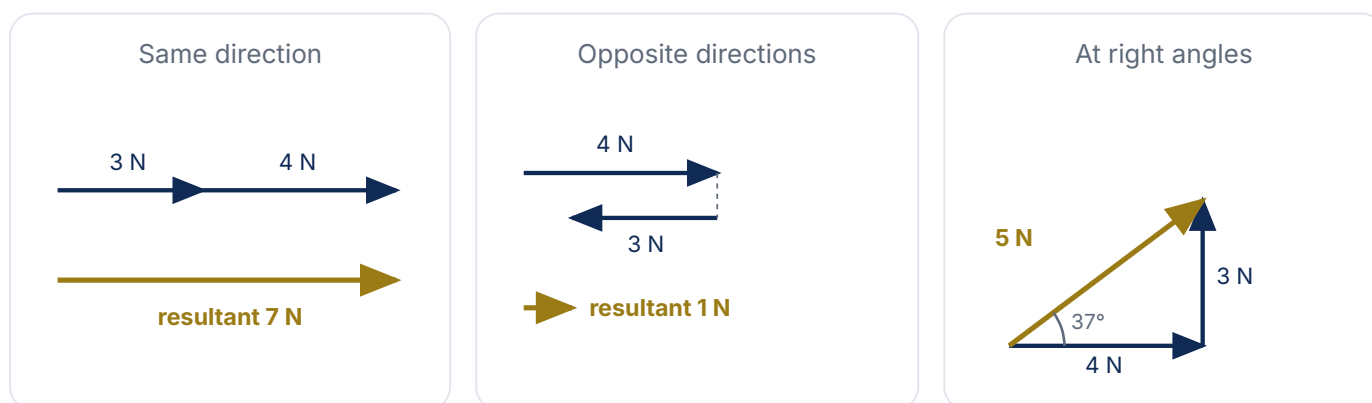
- Currents meeting at a junction add as ordinary numbers: $2.0\text{ A} + 3.0\text{ A} = 5.0\text{ A}$, whatever the angle between the wires
- A true vector would need the angle, so current does not follow vector addition
- **Pressure is a scalar.** At a point in a fluid it acts equally in all directions. The *force* it produces on a surface ($F = pA$) is a vector, acting at right angles to that surface
- **Energy is always a scalar**, even kinetic energy of a moving body. Two balls with 10 J each have 20 J in total, whichever way they are moving

COMMON ERROR

"Current is a vector because it has a direction." Having a sense round a circuit is not enough; a vector must also **add by vector rules**. Current does not, so it is classed as a scalar.

6. Adding and subtracting vectors

- Scalars add like ordinary numbers. Vectors add **tip-to-tail**: draw the second arrow starting where the first one ends
- The **resultant** is the single arrow from the tail of the first to the tip of the last
- The same two vectors give different resultants depending on the angle between them



A 3 N force and a 4 N force can give any resultant from 1 N to 7 N. Only at right angles is it 5 N.

The two cases you calculate

- **Along one line:** choose a positive direction, give each vector a sign, and add the signed values
- **At right angles:** use Pythagoras for the magnitude and trigonometry for the direction
 - $\text{magnitude} = \sqrt{A^2 + B^2}$ · $\text{angle} = \tan^{-1}(\text{opposite} \div \text{adjacent})$, measured from a named direction
- **At any other angle:** resolve each vector into perpendicular components (section 7), add the components, then recombine
- **Subtracting** a vector is adding its reverse: $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$, where $-\mathbf{B}$ has the same length as $\mathbf{B}$ but points the opposite way

WORKED EXAMPLE 2 — A CROSSWIND

[4]

A light aircraft flies with a velocity of 240 km h^{-1} due north relative to the air. A wind blows at 70 km h^{-1} due east.

Determine the resultant velocity of the aircraft relative to the ground.

Step 1: The two velocities are perpendicular, so use Pythagoras for the magnitude

$$v = \sqrt{(240^2 + 70^2)} = \sqrt{62\,500} = 250 \text{ km h}^{-1}$$

Step 2: Find the direction from north

$$\theta = \tan^{-1}(70 \div 240) = 16.3^\circ$$

Step 3: State magnitude and direction together

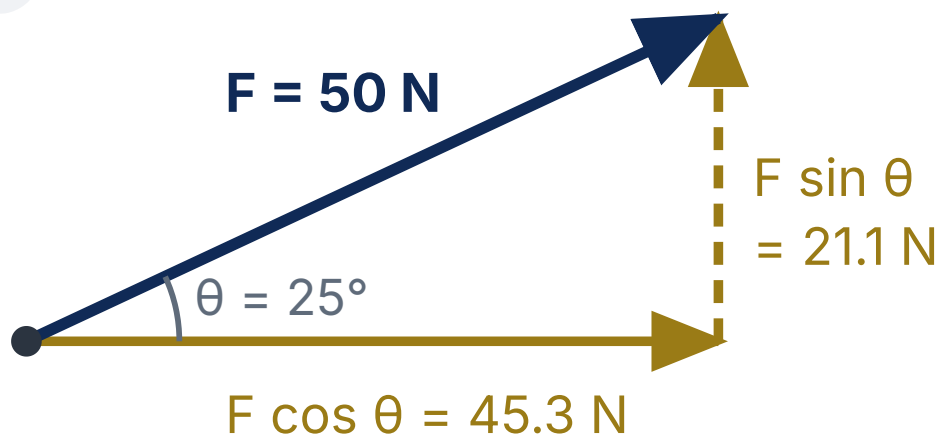
250 km h^{-1} at 16° east of north

EXAM TIP

With the command term *determine* for a vector, the direction carries a mark. A bare " 250 km h^{-1} " is a speed, and it is incomplete as a velocity.

7. Resolving a vector into components

- **Resolving** replaces one vector by two perpendicular vectors that have the same combined effect
- For a vector of magnitude F at angle θ to the horizontal:
 - horizontal component $F_x = F \cos \theta$
 - vertical component $F_y = F \sin \theta$
- The component **next to** the angle uses cos; the component **opposite** the angle uses sin. If the angle is given from the vertical, the two swap
- The components are independent: a horizontal force cannot change vertical motion. This is what makes projectile and inclined-plane problems solvable



The two gold components together have exactly the same effect as the single 50 N force.

WORKED EXAMPLE 3 — PULLING A SLEDGE**[4]**

A child pulls a sledge with a rope. The tension in the rope is 50 N at 25° above the horizontal.

Calculate (a) the horizontal and (b) the vertical component of the tension.

Step 1: The horizontal side is next to the 25° angle, so use cos

$$F_x = 50 \cos 25^\circ = \mathbf{45.3 \text{ N}} \text{ (forwards)}$$

Step 2: The vertical side is opposite the angle, so use sin

$$F_y = 50 \sin 25^\circ = \mathbf{21.1 \text{ N}} \text{ (upwards)}$$

Step 3: Check

$$\sqrt{(45.3^2 + 21.1^2)} = 50.0 \text{ N } \checkmark$$

- Only the 45.3 N component pulls the sledge forwards. The 21.1 N component partly lifts it, which reduces the normal force and so reduces friction

COMMON ERROR

Using sin and cos the wrong way round when the angle is measured from the vertical. Do not memorise "horizontal = cos". Look at where the angle is: the component **next to** the angle always takes cos.

8. Multiplying: when a vector becomes a scalar

COMBINATION	RESULT	EXAMPLES
scalar $\times$ scalar	Scalar	Power $\times$ time = energy $\cdot E = Pt$
scalar $\times$ vector	Vector in the same direction (reversed if the scalar is negative)	$\mathbf{p} = m\mathbf{v} \cdot \mathbf{F} = m\mathbf{a} \cdot \mathbf{W} = mg \cdot \mathbf{F} = q\mathbf{E}$
vector $\div$ scalar	Vector	$\mathbf{v} = \Delta\mathbf{s} \div \Delta t \cdot \mathbf{a} = \Delta\mathbf{v} \div \Delta t$
vector $\cdot$ vector (component along the other)	Scalar	Work done $W = Fs \cos \theta$: force and displacement are both vectors, work is a scalar

- The last row explains why work and energy are scalars: $W = Fs \cos \theta$ keeps only the part of the force that lies **along** the displacement, and the result has no direction
- In worked example 3, if the sledge moves 12 m horizontally, the work done by the rope is $50 \times 12 \times \cos 25^\circ = \mathbf{544 \text{ J}}$. The vertical component does no work because the sledge does not move vertically

EXAM TIP

Use units as a check. Momentum (kg m s^{-1}) is mass $\times$ velocity, so it must be a vector. Kinetic energy ($\text{kg m}^2 \text{s}^{-2} = \text{J}$) contains v^2 , and squaring removes the direction, so it is a scalar.

9. Quick-check questions

1. Sort these into scalars and vectors: impulse, density, weight, power, electric field strength, half-life, drag, wavelength.

Vectors: impulse, weight, electric field strength, drag. **Scalars:** density, power, half-life, wavelength.

2. Two forces of 6.0 N and 8.0 N act on the same object. State the largest and smallest possible resultant.

Largest 14.0 N (same direction); smallest 2.0 N (opposite directions). At right angles the resultant is 10.0 N.

3. A car travels round a roundabout at a steady 9.0 m s^{-1} . Explain why it is accelerating.

Velocity is a vector. The direction of motion is changing, so the velocity is changing, and a changing velocity is an acceleration (towards the centre), even though the speed is constant.

4. A student says temperature must be a vector because it can be negative. Explain the mistake.

A negative temperature is a value below the zero of the scale, not a direction in space. Temperature has magnitude only, so it is a scalar.

5. A ball of mass 0.20 kg moving at 5.0 m s^{-1} to the right bounces back at 5.0 m s^{-1} . Determine the change in momentum.

Taking right as positive: $\Delta p = 0.20 \times (-5.0) - 0.20 \times (+5.0) = -2.0 \text{ kg m s}^{-1}$, i.e. 2.0 kg m s^{-1} to the left. The change in speed is zero, but the change in momentum is not.

6. A force of 30 N acts at 60° to the vertical. Calculate its horizontal and vertical components.

The angle is next to the vertical, so: vertical = $30 \cos 60^\circ = 15 \text{ N}$; horizontal = $30 \sin 60^\circ = 26 \text{ N}$.

7. Two wires carry 1.5 A and 2.5 A into a junction at an angle of 90° . State the current in the third wire leaving the junction, with a reason.

4.0 A. Current is a scalar, so currents at a junction add arithmetically; the angle between the wires is irrelevant.

8. Explain why a 1.0 kg mass has the same mass on the Moon but a different weight.

Mass is a scalar property of the object itself. Weight is the gravitational force $W = mg$, a vector that depends on the local field strength, which is smaller on the Moon (about 1.6 N kg^{-1}), so the weight drops to about 1.6 N.

10. Checklist

Before the exam, make sure you can tick every box.

- ☐ I can define a **scalar** and a **vector** and give at least five examples of each.
- ☐ I can name the vector partner of distance, speed and mass, and explain the difference in each pair.
- ☐ I can explain why **current, pressure, energy and temperature** are scalars.
- ☐ I can choose a **positive direction** and use signs for vectors along one line.
- ☐ I can add two perpendicular vectors, giving **magnitude and direction**.
- ☐ I can find the range of possible resultants of two vectors.
- ☐ I can **resolve** a vector into perpendicular components, whichever way the angle is given.
- ☐ I can say whether the product of two quantities is a scalar or a vector.

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